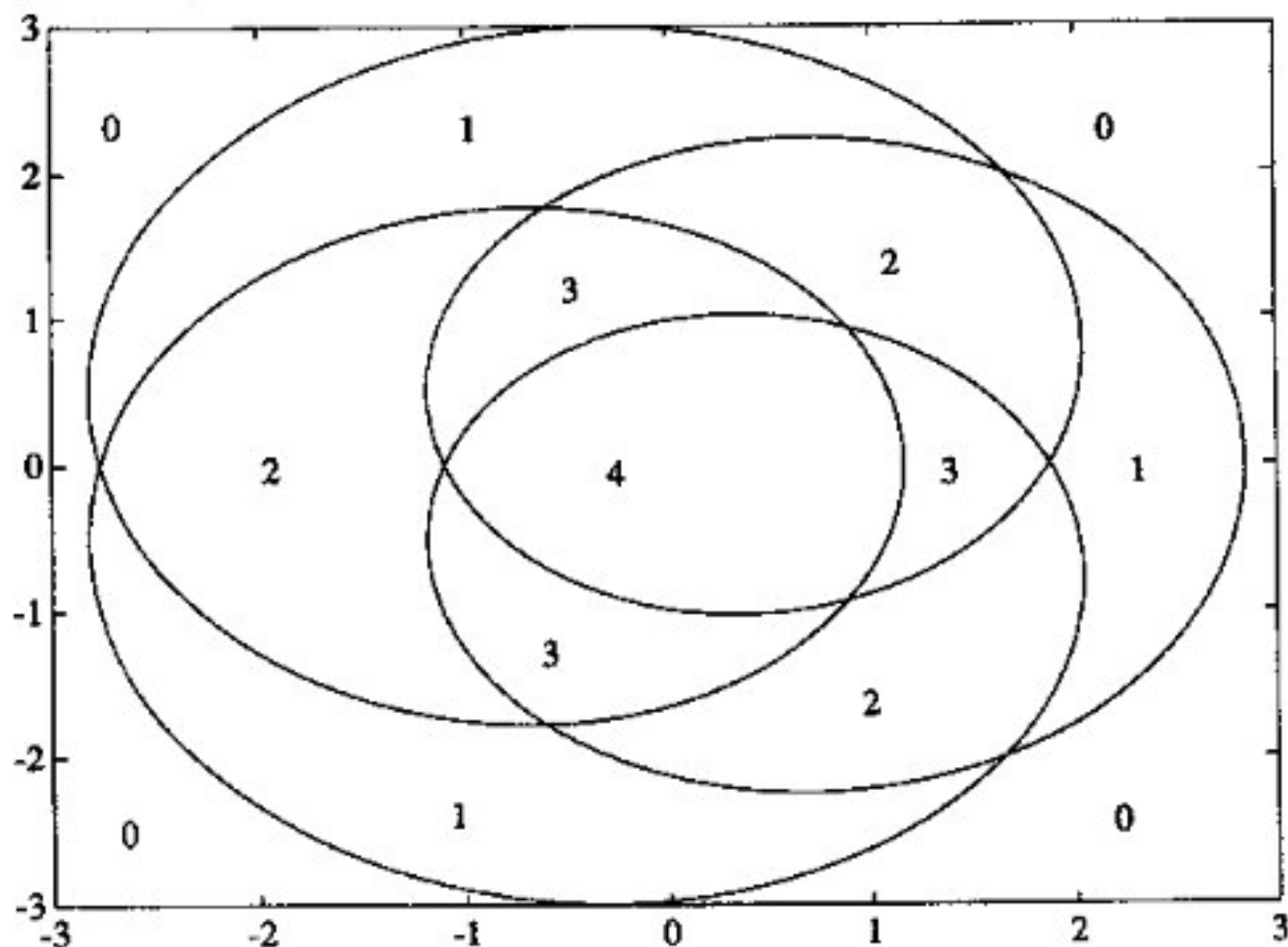


Complex analysis: winding numbers



The curve above (taken anticlockwise) winds about points according to the numbers shown in the diagram.

It is in fact the image of the unit circle  $\gamma$  under the function  $f(z) = 2z^4 - \sin z$ , and hence, by the argument principle  $2z^4 - \sin z = w$  has either 0, 1, 2, 3 or 4 solutions inside the unit circle (counting multiplicity), according to where  $w$  is.

This is because

$$n(f \circ \gamma, w) = \frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z) - w} dz$$

and  $f'(z)/(f(z) - w)$  has a residue of  $M$  at an  $M$ -fold zero of  $f(z) - w$ .