We are looking at differential algebraic subgroups of $\left( U, + \right)$ where $U$ is a $\Delta$-closed field for $\Delta = \{ \partial_1, \ldots, \partial_m \}$.

**Motivation:** “Zilber conjecture” for regular types in $DCF_{0,m}$. That is, a regular type in $DCF_{0,m}$ is either $\not\perp$ to generic of a field of constants or is locally modular.

**True** in finite dimension.

In [Moosa, Pillay, Scanlon], this problem is reduced to the case where the regular type is the generic type of a $\Delta$-subgroup of the additive group. (Proved using differential arc spaces.)

$Y$ is an irreducible affine $\Delta$-variety over $K \subseteq U$. Let $a \in Y(U)$ be a generic of $Y$.

- The type associated to $Y$ is $p_Y := tp(a/K)$.
- The $\Delta$-type of $Y$ is denoted $\tau_Y$ and we define the $\Delta$-type of $p_Y$ to be $\tau_Y$.
- The Kolchin polynomial of $Y$ is $\omega_Y(x) = \sum_{i=0}^{\tau_a} C(X+i) \, \text{choose} \, i$ Then $a_\tau \neq 0$ and is called the **typical $\Delta$-dimension**.

**Fact** (McGrail, Pong): $U(p_Y) \leq \text{RM}(p_Y) \leq \Delta - \text{dim}(p_Y) \leq \text{RD}(p_Y) < \omega^\tau(a_\tau + 1)$.

Note that there are no nontrivial lower bounds. It need not be true that $\omega^\tau a_\tau \leq U(p_Y)$. Suer gives counter-examples with $\tau = 1$ coming from subgroups of the additive group.

**Proposition** ($m = 2$) Let $G \leq (U, +)$ be a subgroup defined by $\delta_1(y) = f(y)$ with $f \in U[\delta_2]$ of order $k$. Then $p_G$ has $\Delta$-type 1 and typical dimension $k$ but $U$-rank $\omega$.

**N.B.:** Call $Y$ **$\Delta$-type minimal** if for every proper $\Delta$-subvariety have a smaller $\Delta$-type.

**Theorem:** Let $G \leq (U, +)$ have $\Delta$-type one and be $\Delta$-type minimal. ($\implies U(p_G) = \omega$ and $p_G$ is regular) Then $p_G$ is nonorthogonal to the generic of a definable field iff the typical $\Delta$-dimension of $G$ is one.
Fact: Every definable subfield of $U$ is the field of constants of finitely many definable derivations of the form $\sum a_i \delta_i$. Moreover, nonorthogonality of $G$ to the field $F$ is witnessed by a definable, surjective homomorphism $G \rightarrow F$.

Theorem: ($m=2$) The group $\{ x \in U : \delta_1(x) = \delta_2^2(x) \}$ is modular.

Modularity of the group $G$ means (assuming that $G$ is finite dimensional) that every definable subset of $G \times \cdots \times G$ is a finite Boolean combination of a definable subgroup. In the infinite dimensional case, this definition works “modulo small.” Technically, one needs to discuss local $p$-weight, semiregular types, et cetera.

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