Vortices
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Vortices are found in many natural phenomena. Our weather is determined by the cyclonic and anticyclonic vortices in the atmosphere, and tornadoes are examples of extreme vortices. Vortex eddies and whirlpools occur in the ocean on a range of different sizes. Vortices are commonly seen in rivers and streams, where it is clear that they can interact with each other to produce complex patterns. On very small scales, quantum vortices can occur in liquid helium. Despite their very different sizes, the same fundamental mathematics governs all vortex motion, and it is these mathematical ideas that form the basis of the project.

Vortices have been studied by mathematicians for over a hundred years, and it is still a very active area of modern research. Studying vortices turns out to be a good way of understanding fluid flow. The project can either be done as a level 3 project or as an MMath project.

The project will start by considering models of vortices. The simplest is the line vortex, but ring vortices (smoke rings) are also of interest. More realistic models in which the vortex core has a specific form, e.g. the Rankine vortex and the Hill spherical vortex, will be studied. All vortices obey some key mathematical theorems, such as Kelvin’s circulation theorem. The next topic will be vortex interactions. Elegant mathematical solutions are available for arrays of vortices, for example vortices located at the vertices of a regular polygon. The stability of these arrays, i.e. whether the configuration breaks up or not, can also be treated mathematically. For those interested in computer simulations, this could be developed further to study other arrays of vortex interactions; for those more interested in analytical methods, the stability of infinite arrays of line vortices is a classic problem in applied mathematics.

For the MMath project, the study of vortices could proceed in a number of ways, depending on the interest of the student. For those interested in fluid dynamics and particularly hydrodynamic stability, the stability of vortices with finite cores, Crow instability and Widnall instability, are topics with both important applications (the Crow instability can occur in the trailing vortices left by aircraft on take-off) and very beautiful experiments (particularly the break-up of ring vortices by the Widnall instability). Another possible area would be to study vortex formation and development, with applications in geophysical fluid dynamics, such as tornadoes. For those interested in quantum mechanics, quantum vortices (the circulation round the vortex has to be a multiple of Planck’s constant) have interesting properties, and an introduction to the simplest models used to study of superfluid mechanics (viscosity vanishes completely in liquid helium cooled below $4.2^\circ$K) could be undertaken.