On Resurging a Bore-Soliton-Splash Rogue Wave

Onno Bokhove

School of Mathematics, University of Leeds
with M. Robinson (Oxford), E. Gagarina & W. Zweers (Twente)
1. Introduction

Rogue waves are anomalously high waves defined relative to a significant wave height $H_s$.

- Index (Khariff et al. ’09, Dysthe et al. ’08):
  \[ AI = \frac{H_{rw}}{H_s} > 2 \quad \text{or} \quad AI = \frac{\eta_{rw}}{H_s} > 1.25 \quad (1) \]
- Relevance in maritime & coastal engineering — ship design & safety offshore structures
- Pyramidal rogue wave (Faulkner 2001):

Fig.1. Pyramidal wave off south Japan
Introduction


- interference of waves with various frequencies and phases: nonlinear Schrödinger equation, solitons
- currents opposing waves
- spatial wave focussing due to refraction or wave caustics, or wave focussing in coastal convergences
- episodic waves generated elsewhere
- spilling breakers on the deep ocean
- crossing seas, nearly standing waves with pyramidal waves.
Here, emphasis will lie on rogue waves & wave focussing:

- in crossing seas
- due to coastal or submarine convergences.

Moreover, (rogue) wave energy devices based on wave focussing will be introduced or discussed:

- TapChan or tapered channel device
- Oscillating Water Column (OWC) device with wind turbine
- New device with a more direct energy conversion?
Introduction: idea

- Turn the investigation of pyramidal rogue waves around.
- How can we create the highest man-made rogue wave in an idealized geometry, given certain restrictions.
  - stowage in \textit{contractions, hydraulics}
  - wave impact against sea walls (Peregrine ARFD 2003)
- wave impact in closing contractions, pilot in \textit{Roombeek}
2. Bore-Soliton-Splash

- **Water channel**: $L = 43.5 \pm 0.25\text{m}$, $D = 1.20\text{m}$, $w = 2 \pm 0.05\text{m}$

- **Sluice** compartment & gate, lifted with $2.5\text{m/s}$
- **V-shaped or linear convergence** at other end with $c = 2.7\text{m}$
- Start at rest with water levels $h_0 \in [0.32, 0.47]\text{m}$ and $h_1 \in [0.67, 1.02]\text{m}$.
### 7+2 Cases of which 3 repeats (reproducibility):

<table>
<thead>
<tr>
<th>#</th>
<th>( h_0 ) (m) ±0.01m</th>
<th>( h_1 ) (m) ±0.01m</th>
<th>( H_s ) ±0.05m</th>
<th>( H_{rw} ) ±0.5m</th>
<th>Peak</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.32</td>
<td>0.67</td>
<td>-</td>
<td>0.6</td>
<td>-</td>
<td>bore</td>
</tr>
<tr>
<td>2</td>
<td>0.38</td>
<td>0.74</td>
<td>-</td>
<td>2.5</td>
<td>-</td>
<td>good splash</td>
</tr>
<tr>
<td>3</td>
<td>0.41</td>
<td>0.9</td>
<td>0.35</td>
<td>3.25</td>
<td>2\textsuperscript{nd}</td>
<td>thin jet cf. 6 &amp; 8</td>
</tr>
<tr>
<td>4</td>
<td>0.47</td>
<td>1.0</td>
<td>0.35</td>
<td>1</td>
<td>2\textsuperscript{nd}</td>
<td>bore &amp; low</td>
</tr>
<tr>
<td>5</td>
<td>0.41</td>
<td>1.02</td>
<td>0.40</td>
<td>1.5</td>
<td>1\textsuperscript{st}</td>
<td>bore &amp; low</td>
</tr>
<tr>
<td>6</td>
<td>0.41</td>
<td>0.9</td>
<td>0.35</td>
<td>3.5</td>
<td>2\textsuperscript{nd}</td>
<td>BSS cf. 3 &amp; 8</td>
</tr>
<tr>
<td>7</td>
<td>0.45</td>
<td>0.8</td>
<td>0.35</td>
<td>2.5</td>
<td>2\textsuperscript{nd}</td>
<td>good splash</td>
</tr>
<tr>
<td>8</td>
<td>0.41</td>
<td>0.9</td>
<td>0.35</td>
<td>3.5</td>
<td>2\textsuperscript{nd}</td>
<td>BSS &amp; ++</td>
</tr>
<tr>
<td>9</td>
<td>0.43</td>
<td>0.9</td>
<td>0.45</td>
<td>1.8</td>
<td>1\textsuperscript{st}</td>
<td>bubble</td>
</tr>
</tbody>
</table>
Phenomenology

- Estimated wave amplitudes $H_s \in [0.35, 0.45] m$
- After opening sluice gates three solitary waves emerge: large, medium and small
- First highest soliton often breaks into spilling breaker
- **Variety of outcomes**: minor/major reflections, resonances between waves, smooth waves, sheets, pyramidal waves
- $H_{rw} \in [0.6, 3.5] m$. 

Estimated wave amplitudes $H_s \in [0.35, 0.45] m$
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Bore-Soliton-Splash

Highest Cases 3, 6 & 8:

- $H_s = 0.35\text{m} & H_{rw} = [3.25, 3.5, 3.5]\text{m}$
- *bore soliton splash*
- Truly rogish:

$$AI = \frac{H_{rw}}{H_s} = \frac{3.5}{0.35} \approx 10.$$
Smooth Case 9: No Rogue Wave

Take $h_0 = 0.43\text{m}$ instead of $h_0 = 0.41\text{m}$, same $h_1 = 0.9\text{m}$:
Smooth Case 9: No Rogue Wave
Smooth Case 9: No Rogue Wave
Smooth Case 9: No Rogue Wave
Smooth Case 9: No Rogue Wave
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Smooth Case 9: No Rogue Wave
Smooth Case 9: No Rogue Wave
Bore-Soliton-Splash: Ingredients

- Nonlinearity.
- Dispersion.
- Solitons.
- Spilling breaker or bore.
- Pyramidal wave or vertical jet.
- Collapse of wave into droplets/bubbles: two phases.
Roadmap

- Modeling BSS with **KP-equation** (Kadomtsev-Petviashvili):
  - dispersion & nonlinearity.
- Modeling BSS with **SPH** (Smoothed Particle Hydrodyn.):
  - sluice gate motion, bore & splash
- Bores in **Boussinesq models**:
  - dispersion & nonlinearity
- On air-water **single-phase mixture theory**:
  - smooth & broken waves, splash
- **Geometric wave modeling**:
  - accurate balance dispersion & nonlinearity; limit case
- **Rogue Wave Energy** device.
2. Bore-Soliton-Splash with KP?

- **Kadomtsev-Petviashvili equation** is unidirectional dispersive wave equation with weak lateral or \( y\)-dependence (Kodama 2010):

\[
\partial_x \left( \partial_t \eta + \frac{3}{2} \eta \partial_x \eta + \frac{1}{6} \partial_{xxx} \eta \right) + \frac{1}{2} \partial_{yy} \eta = 0 \tag{2}
\]

- Free surface deviation \( \eta(x, t) \) (scaled).
When 2 KdV sech²-solitons with $A = 2c$ & phase speed $c$:

$$\eta(x, t) = 2c \text{sech}^2\left(\frac{\sqrt{3c}}{2}(x - x_0 - ct)\right)$$  \hspace{1cm} (3)

approach each other under an angle $2\theta$ (or one soliton grazes a wall with slant $\theta$): then there is an angle $\theta_{max}$ for which $A_{max} = 4$ (Yeh & Kodama JFM 2011):
2. Bore-Soliton-Splash with KP?

- For 3 KdV solitons with amplitude $A$, there is “V-shaped channel geometry” with angles $\theta_{max}$ for which $A_{max} = 9$
- Exact KP solution by Kodama (idea, Dresden '11):

- Fine in open sea, but inconsistent in water wave channel!
- Solution: numerics/asymptotics bidirectional Boussinesq model.
3. Bore-Soliton-Splash with SPH?

Brute-force simulation with Smoothed Particle Hydrodynamics:

- Non-leaky sluice gate made of stiff particles.
- Laterally periodic narrow slice before convergence.
- Copies of these slices are fed into the channel just before the convergence.
Bore-Soliton-Splash with SPH?

SPH bore soliton splash:
splash low, 2.88m, no solitons, too much dissipation (Yim et al. JWPCOE 2008, Gomez et al. JHR 2013).
4. Bores in Boussinesq Models

Derived a new water wave model (Cotter & B. 2010, Gagarina et al. 2013c):

- full water wave dispersion & potential flow as limit
- depth-averaged shallow water equations as other limit
- bore relations (if any) derived variationally (Clebsch):
New Water Wave Model

- Dynamics of surface variables $u^*(x, y, t), h(x, y, t)$:

  \[ \partial_t h + \nabla \cdot (h \vec{u}) = 0 \]  
  \[ \partial_t u^* + \nabla B + qh\vec{u}^\perp = 0, \]  

  where $h \vec{u} = hu^* + \int_b^{b+h} \nabla_H \varphi dz$, $PV \ q = (\partial_x v^* - \partial_y u^*)/h$  
  & $B = \frac{1}{2}|u^*|^2 + g (h + b) - \frac{1}{2}((\partial_z \varphi)^2_s(1 + |\nabla_H (h + b)|^2)$

- Interior dynamics velocity potential difference $\varphi$:

  \[ \nabla^2 \varphi + \nabla \cdot u^* = 0 \]  

- At the free surface $z = h(x, y, t) + b(x, y)$, the potential is $\varphi = 0$. 

where $h \vec{u} = hu^* + \int_b^{b+h} \nabla_H \varphi dz$, $PV \ q = (\partial_x v^* - \partial_y u^*)/h$
5. Single-Phase Mixture Theory

- Dispersive bores difficult to model (numerically)?
- Hence, consider instead **incompressible two-phase** (here air & water) system.
- Bulk density $\rho = \rho_l + \rho_g \equiv \rho_0 l \varphi + \rho_0 g (1 - \varphi)$.
- Bulk velocity $\rho u = \rho_l u_l + \rho_g u_g$.
- Equations of motion with a very **simple closure** are:

\[
\begin{align*}
\partial_t \rho_l + \nabla \cdot (\rho_l u_l) &= 0 \\
\rho_l (\partial_t u_l + u_l \cdot \nabla u_l) &= -\varphi \nabla p + \rho_l g - \rho_l c (u_l - u) \\
\partial_t \rho_g + \nabla \cdot (\rho_g u_g) &= 0 \\
\rho_g (\partial_t u_g + u_g \cdot \nabla u_g) &= -(1 - \varphi) \nabla p + \rho_g g - \rho_g c (u_g - u).
\end{align*}
\]
Single-Phase Mixture Equations

- Scale species hydrostatically in local pancakes of wave breaking.
- Horizontal velocity equals bulk velocity.
- Vertical velocity has segregation terms:

\[
\begin{align*}
    w_l - w &= -K(1 - \varphi) \quad \text{and} \\
    w_g - w &= K \varphi / \tilde{\rho} \quad \text{with} \quad K = g(1 - \tilde{\rho})/c. 
\end{align*}
\]

- Density ratio \(\tilde{\rho} = \rho_{0g}/\rho_{0l}\).
Now add **non-hydrostatic terms** to bulk flow only in vertical momentum equation.

Mixed phases unmix **hydrostatically**:

\[
\begin{align*}
\partial_t (\rho u_H) + \nabla \cdot (\rho uu_H + p) &= 0 \\
\partial_t (\rho w) + \nabla \cdot (\rho uw) &= -\partial_z p - \rho g \\
\partial_t \varphi + \nabla \cdot (u \varphi) - \partial_z (K \varphi (1 - \varphi)) &= 0 \\
\nabla_H \cdot u_H + \partial_z (w + K \varphi (1 - \varphi) (1 - \tilde{\rho}) / \tilde{\rho}) &= 0 \\
\rho &= \rho_0 g (1 - \varphi) + \rho_0 l \varphi \\
K &= g (1 - \tilde{\rho}) / c \quad \text{with} \quad \tilde{\rho} = \rho_0 g / \rho_0 l.
\end{align*}
\]

When phases separated then **potential-flow limit** is enclosed.
6. Geometric Wave Modeling

- When phases separated then potential-flow limit is enclosed.
- To model solitons accurately, a numerical method with good dispersion and no wave damping is required.
- Reason to study & use geometrical wave modeling, e.g., variational principles for water waves.

\[ \nabla^2 \phi = 0 \]

Figure: Sketch & definitions variables in wave tank.
Geometric Wave Modeling: Variational Principle


\[ 0 = \delta \int_0^T \mathcal{L}[\phi, \phi_s, h, t] dt \]

\[ = \delta \int_0^T \int_{x_w(t)}^L \int \phi_s \partial_t h - \frac{1}{2}g((h + b)^2 - b^2) + gH_0 h \]

\[ - \int_b^{b+h} \frac{1}{2} |\nabla \phi|^2 dz dx dy - \int_b^{b+h} \int \frac{dx_w}{dt} \phi_w dz dy dt \]

- with potential \( \phi = \phi(x, y, z, t) \) such that \( u = \nabla \phi \)
- free-surface \( \phi_s(x, y, t) \equiv \phi(x, y, z = h + b, t) \) at \( \partial D_s \)
- piston wave-maker \( x_w(t) \) with \( \phi_w \equiv \phi(x_w, y, z, t) \)
- non-autonomous due to piston wave-maker
- Canonical Hamiltonian system with explicit time dependence.
Simulation (2D) of waves in MARIN’s

- wave tank:

- Include workings of wave-maker.
FEM formulation of Miles’ variational principle.

- FEM test/basis functions \( \tilde{\phi}_j(x, z, t), \hat{\phi}_k(x, t) \) with \( i, j \) in \( D \); \( k, l \) at free surface \( \partial D_s \) & \( m \) at wave maker.

- Substitute \( \phi_h(x, z, t) = \phi_j(t) \tilde{\phi}_j(x, z, t), \)
  \( h_h(x, t) = h_k(t) \hat{\phi}_k(x, t) \) in VP

\[
0 = \delta \int_0^T \mathcal{L} [\phi_j, h_k, t] dt \tag{8}
\]

\[
= \delta \int_0^T M_{kl} \phi_k \frac{dh_l}{dt} - \phi_k D_{kl} \frac{dh_l}{dt} - \ldots
\]

\[
- \frac{1}{2} g (h_k + b_k - H) M_{kl} (h_l + b_l - H)
\]

\[
- \frac{1}{2} \phi_i A_{ij} \phi_j - w_m(t) \phi_m dt. \tag{9}
\]

- \( M_{kl}, D_{kl}, A_{ij} \) \& \( w_m \) depend on \( \{ h_k(t), t \} \): mesh movement.
Geometric Wave Modeling: Stable Time Integrator

Goal: to derive stable variational time integrators with Discontinuous Galerkin FEM

- Compare water wave FEM with toy damped oscillator (Olver)

\[ 0 = \delta \int_0^T \left( p \frac{dq}{dt} - H(p, q) \right) e^{\gamma t} dt \]

\[ \delta(pe^{\gamma t}) : \quad \frac{dq}{dt} = p = \frac{\partial H}{\partial p} \]

\[ \delta q : \quad \frac{dp}{dt} + \gamma p = -(q + q^3) = -\frac{\partial H}{\partial q} \quad (10) \]

with energy/Hamiltonian \( H = H(p, q) = \frac{1}{2} p^2 + \frac{1}{2} q^2 + \frac{1}{4} q^4 \)

- Note the integrating factor s.t.:

\[ \frac{d(pe^{\gamma t})}{dt} = -(q + q^3)e^{\gamma t}. \quad (11) \]
Dynamics becomes linear in long-time limit. The transformation

\[ q = Q e^{-\gamma t/2} \quad p = P e^{-\gamma t/2} \]  \hspace{1cm} (12)

shows from

\[ 0 = \delta \int_0^T P \frac{dQ}{dt} - \tilde{H} dt \]

that for \( t \to \infty \)

\[ \frac{d\tilde{H}}{dt} = 0 \quad \text{with} \]

\[ \tilde{H}(P, Q, t) = \frac{1}{2} P^2 + \frac{1}{2} \gamma PQ + \frac{1}{2} Q^2 + \frac{1}{4} Q^4 e^{-\gamma t} \]

\[ = (\frac{1}{2} p^2 + \frac{1}{2} \gamma pq + \frac{1}{2} q^2 + \frac{1}{4} q^4) e^{\gamma t}. \]
**Goal:** to derive stable variational time integrators with Discontinuous Galerkin FEM

- Finite elements in time.
- $p_h = p_j \phi_j(t)$ and $q_h = q_j \phi_j(t)$ expanded in piecewise continuous fashion, e.g.:

  ![Diagram](image)

  - What to do with derivative $pdq/dt$ at the jumps? since $p \frac{dq}{dt} \neq \frac{dQ}{dt}$ with $Q = Q(p, q)$.
  - “No” staggered C-grid in $t$: crux lies in choice numerical flux!
Continuous to Discontinuous Galerkin FEM

- Introduce even & odd numbered elements
- Introduce continuous FEM: $p_h^*, q_h^*$
- Even basis functions: quadratic in $p_h^*$ & linear in $q_h^*$

\[
q_h^* = q_L^* + \tau(q_R^* - q_L^*), \\
p_h^* = p_L^* + (p_R^* - p_L^*) \times ((4 - 6\alpha)\tau + (6\alpha - 3)\tau^2), (13)
\]

with reference element $\tau \in [0, 1]$ & $t = t^n + \tau(t^{n+1} - t^n)$

- Let even elements shrink to 0 in size
- Idea is to define the $\int p_h dq_h$ at the discontinuities.
The integral over $p_h dq_h/dt$ over an even element becomes

$$
\int_{0}^{1} p_h \frac{dq_h}{d\tau} d\tau = (\alpha p_L^* + \beta p_R^*)(q_R^* - q_L^*). \quad (14)
$$
Limit from Continuous to Discontinuous Galerkin FEM

- Using limit CGFEM to DGFEM, discrete variational principle is

\[
0 = \delta \sum_{n=0}^{N-1} \int_{t_n}^{t_{n+1}} \left( \frac{dq_h}{dt} - H(p_h, q_h, t) \right) e^{\gamma t} dt + \sum_{n=-1}^{N} \int_{0}^{1} \phi_p(\tau; p_L, p_R) \frac{d\phi_q}{d\tau}(\tau; q_L, q_R)
\]

- For quadratic & linear paths (\(\gamma = 0\))

\[
\phi_p = p_L + 2a_1\tau + 3a_2\tau^2 \quad \text{&} \quad \phi_q = q_L + \tau(q_R - q_L) \quad \text{s.t.}:
\]

\[
\int_{0}^{1} \phi_p \frac{d\phi_q}{d\tau} d\tau = (\alpha p_L e^{\gamma t_L} + \beta p_R e^{\gamma t_R})(q_R - q_L)
\]

with \(0 \leq \alpha, \beta \leq 1\), i.e., jump in \(q\) \times weighted mean in \(p\).
Non-Conservative Products: Symplectic Euler

- **Recap:**

\[
0 = \delta \sum_{n=0}^{N-1} \int_{t_n}^{t_{n+1}} \left( p_h \frac{dq_h}{dt} - H(p_h, q_h, t) \right) e^{\gamma t} dt
\]

\[
+ \sum_{n=0}^{N} \left( \alpha p_L e^{\gamma t_L} + \beta p_R e^{\gamma t_R} \right) (q_R - q_L)
\]

- **Symplectic Euler (SE) for piecewise constant basis functions:**

\[
p_h e^{\gamma t} = p_{n+1} e^{\gamma t_{n+1}}, \quad q_h = q^n \quad \text{&} \quad \alpha = 1, \beta = 0:
\]

\[
L_h(p_h, q_h) = \sum_{n=0}^{N-1} -\Delta t_n H(p_{n+1}, q^n, t_{n+1}) e^{\gamma t_{n+1}}
\]

\[
+ \sum_{n=0}^{N} (q_{n+1} - q^n) p_{n+1} e^{\gamma t_{n+1}}
\]
Non-Conservative Products: Symplectic Euler

- First order in time symplectic Euler (SE) for toy model:

\[
\delta q^n: \quad p^{n+1} = e^{-\gamma \Delta t_n} p^n - \Delta t_n (q^n + (q^n)^3)
\]

\[
\delta(p^{n+1} e^{\gamma t^{n+1}}): \quad q^{n+1} = q^n + \Delta t_n p^{n+1}.
\]

- Compare SE w. forward/backward Euler & midpoint:

\[
p^{n+1}_{FE} = p^n - \Delta t_n (q^n + (q^n)^3) - \Delta t_n \gamma p^n
\]

\[
p^{n+1}_{BE} = p^n - \Delta t_n (q^n + (q^n)^3) - \Delta t_n \gamma p^{n+1}
\]

\[
p^{n+1}_{MP} = p^n - \Delta t_n (q^n + (q^n)^3) - \frac{\Delta t_n}{2} \gamma (p^n + p^{n+1})
\]
Non-Conservative Products: Symplectic Euler

- Comparison (blue: SE, red: SE-FE, black: SE-BE):
Time DGFEM: Remarks

- Rederived second-order schemes: Stormer-Verlet & modified midpoint, extended to non-autonomous case
- Explore other $\alpha$’s & quadratures
- Explore other basis functions
- New schemes?
- **Optimization criteria:**
  - linear stability analysis
  - verification of (pseudo-)symplecticity
  - time reversibility
  - dispersion analysis
  - numerical tests.
Applied 2nd-order Stormer-Verlet approach to variational FEM formulation (Gagarina et al. 2013b):

\[ 0 = \delta \int_{0}^{T} \mathcal{L}[\phi_j, h_k, t] dt \]

\[ = \delta \int_{0}^{T} \phi_k M_{kl} \frac{dh_l}{dt} dt - \phi_k D_{kl} \frac{dh_l}{dt} dt - \ldots \]

\[ - \frac{1}{2} g(h_k + b_k - H) M_{kl} (h_l + b_l - H) \]

\[ - \frac{1}{2} \phi_i A_{ij} \phi_j - w_{m}(t) \phi_m dt. \]  

New stable time scheme for forced waves.
NCP: Stormer-Verlet Simulations

- *Driven wave focusing* in MARIN’s wave tank.
- Entire *wave tank* MARIN with false vertical wall instead of *beach*.
- Comparison with measured data MARIN:

![Graphs showing FFT results for the wave tank simulations.](image-url)
7. Wave Energy Device

Combine geometric wave focussing with linear dynamo:

- Dynamo above water on a mast attached to floater.
- More direct energy conversion!
- **Mechanically robust?**
- Working *prototype* (using flash lights charged by shaking).
8. Conclusions

- We explored *bore soliton splash*.
- Portable *toy* Bore-Soliton-Splash.
- Wave, floater & dynamo modeling for new device, using KP-equations, etc.
- Mixture model and numerical integrators under investigation for wave & ship dynamics:
Conclusions: Tohoku Tsunami

- Vertical run-up Tohoku tsunami highest in Onagawa Bay: 42m for a rough estimate of 7.5m high incoming waves.
- Ratio run-up/wave height: $AI \approx 5.5$ (shallow water rogue waves Nikolkina & Didenkulova 2011).
- Recall BSS/incoming wave height is $AI \approx 10$. 
References

- **www**: presentation, movies & eprints?
- Gagarina, Ambati, Nurijanyan & Bokhove 2013a: time (dis)continuous Galerkin FEM. In preparation.
- Gagarina, Ambati, Van der Vegt & Bokhove 2013b: Submitted JCP.
Time DGFEM: Stormer-Verlet

- **Stormer-Verlet (SV)** for piecewise linear basis functions with $\alpha = 1$, $\beta = 0$ and $\zeta \in [-1, 1]$:

$$
\begin{align*}
p_h &= p_l^{n+1} \frac{1 - \zeta}{2} + p_r^{n+1} \frac{1 + \zeta}{2} \\
q_h &= q^{n+1/2} (1 + \zeta) - q^n \zeta.
\end{align*}
$$

(19)
Time DGFEM: Stormer-Verlet

- **Discrete Lagrangian** $\gamma = 0$:

$$L_h(p_h, q_h) = \sum_{n=-1}^{N} (q^{n+1} + q^n - 2q^{n+1/2})p_r^{n+1}$$

$$+ \sum_{n=0}^{N-1} (p_i^{n+1} + p_r^{n+1})(q^{n+1/2} - q^n)$$

$$- \frac{\Delta t_n}{2} \left( H(p_i^{n+1}, q^{n+1/2}) + H(p_r^{n+1}, q^{n+1/2}) \right)$$

- **Stormer-Verlet** with $p_i^{n+1} = p_r^n$ continuous & $q$ remains DG:

$$\delta p_i^{n+1} : q^{n+1/2} = q^n + \Delta t p_i^{n+1}/2$$

$$\delta q^{n+1/2} : p_r^{n+1} = p_i^{n+1} - \frac{\Delta t}{2} \left( (q^{n+1/2})^3 + (q^{n+1/2})^3 \right)$$

$$\delta p_r^{n+1} : q^{n+1} = q^{n+1/2} + \Delta t p_r^{n+1}/2$$
New Rogue Wave Energy Device

**TapChan** device, tapered channel (Falcao 2010):
- At coasts, small tides, large fairly unidirectional waves
- Natural converging inlets & pools: hydropower.
- Operational in Norway till storm wrecked it ...
Wave Energy Device

**OWC**, Oscillating Water Column (Falcao 2010):
- Convergence with trapped air: compressing or decompressing
- Place Wells or wind turbine in “blow hole”.
- Less efficient but very robust.
IPS wave buoy (Falcao 2010):

- Multidirectional seas, open sea.
- Dynamo under water.