Wetropolis flood demonstrator: on extreme rainfall & river flooding

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1. Introduction

In Hebden Bridge ("Science of floods"), a town in Yorkshire that has seen a lot of sustained and flash floods over the last decade, I addressed the following questions:

- Is it going to rain more in the future?
- Can we define extreme precipitation & flooding events?
- How (well) can we predict heavy precipitation & floods?
- How (well) can we mitigate and control flooding?
- How can we elucidate the above in an interactive, conceptual table-top demonstration “Wetropolis”?
I will elucidate the answers to the above and use these in the design and modelling of an interactive, conceptual table-top demonstration and experiment on rainfall, flooding as well as flood mitigation and control. The design will serve 2 purposes:

- as **public demonstration** of the concepts of flooding and
- **simplified test environment** on the science of flood modeling:
  - reliable estimation of extreme flooding events
  - improve short-term forecasts using data assimilation
  - multi-scale modelling of street-sewer networks
  - rational mathematical strategies for flood control.
1. Introduction

I will identify mathematical elements involved to model:

- “random” rain supply, channel/river flow,
- the hydrology of “Hele-Shaw” valleys (illustrating effects porous moors, fast run-off from tarmac, and the function of reservoirs),
- flow in bypass canals, and
- control features such as weirs and sluice gates as well as
- data assimilation using a few measurement points.

Work in progress . . .
2. Is it going to rain more in the future?

Definitions:

- **Return period**: if 100 years of *daily rainfall* data were available, an event with a 1 in 20 year return period would be expected to occur 5 times in that data set.

- In any year such an event has on average a 5% chance to happen on one day per year.

- **Extreme events**: events with a (longer) return period of 1 in 5, 1 in 10, 1 in 20, 1 in 30, 1 in 50 or 1 in 100 years.
Is it going to rain more in the future?

Similarly, Michael Sanderson’s UKCP09 report Met Office 2010 (data 1960–2006), for UK:

- No, annual mean daily rainfall has not increased significantly since 1766.
- But proportion of winter rainfall in heavy rainfall events has increased across UK in last 45 years.

- Summer rainfall has decreased except in NE England and Northern Scotland.
- “Although summer rainfall may decrease, ... concentrated into a number of intense downpours from storms”.
3. Define extreme precipitation & flooding events?

**Caution:**
- The **uncertainty** in the return period for a 1 in 100 year event will be **larger** than in a 1 in 20 year event.
- **Uncertainty** will be **smaller** for larger data sets.
Challenges and basic problem (Coles, 2001):

- There are very few observations “in the tail of the distribution”, i.e. for extremes with long return periods.
- Estimates required beyond largest observed data values.
- E.g., Aire River gauge at Armley/Leeds had highest record of 4.03m prior to 26-12-2015; how to estimate return period for the Boxing Day flood with 5.21m at this gauge?

- Standard statistical techniques work well when there are a lot of data, but don’t work well for extremes.
Wong, Maraun, . . . Kent 2014, J. Climate; MSc Kent:
Modelling Extreme Rainfall

- Fit of Γ– & generalised Pareto distributions (Ziggurat):

\[ h(r; \beta) = c_\beta ((1 - w(r; m, \tau))f(r; \gamma, \lambda) + w(r; m, \tau)g(r; \xi, \sigma_\mu = 0)) \]  (1)

(a) The bulk of the distribution
(b) The extreme upper tail of the distribution
4. How (well) can we predict heavy precipitation and (overland) floods?

Distinguish between downpours & sustained rainfall:

- **Downpours** (short ca. 1 hour) and **flash floods**, e.g., *Hebden Bridge July 2012*
  
  http://www.bbc.co.uk/news/uk-18778840
How (well) can we predict heavy precipitation and (overland) floods?

- Sustained rainfall and flooding, e.g.
  Boxing Day rainfall and floods 26-12-2015
  *Kildwick; Bingley; Armley*

- *Aire River Flood & Aire River Kirkstall The Forge*
How (well) can we predict heavy precipitation and (overland) floods?

<table>
<thead>
<tr>
<th>SITE</th>
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<tr>
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<td>NORTH YORKSHIRE</td>
<td>97</td>
</tr>
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<td>MYERSCOUGH</td>
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</tr>
<tr>
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<tr>
<td>DISFORTH AIRFIELD</td>
<td>NORTH YORKSHIRE</td>
<td>60.8</td>
</tr>
</tbody>
</table>

This wet spell has added to the heavy rainfall through the rest of the month to make December 2015 already the wettest on record in parts of the UK.

Figure 2: [http://blog.metoffice.gov.uk](http://blog.metoffice.gov.uk) Rainfall summary 25-27 December 2015.
How (well) can we predict heavy precipitation and (overland) floods?

- **Downpours**, their location & amount are very difficult to predict.

- **Numerical Weather Prediction** (NWP is computer modelling from e.g. the Met Office) cannot handle those cases (well), due to lack of computer power and lack of insights in the physics of precipitation.

- Also, **NWP uses/assimilates lots of data** to bring the computer model back to reality.

- **Sustained rainfall and river flooding** is by and large well-predicted: the Met Office and Environment Agency do a reasonably good job e.g. Aire River floods.

- Prediction of localised surface water/brook flooding is more uncertain/less good, due to a lack of data.
5. How (well) can we mitigate flooding?

- Create multiple **storage buffers** upstream.
  - Lowers peak values but broadens the flood peaks.
  - May work less well for large water volumes, including consecutive heavy rainfall events.
- Create **more permeable surfaces**, green soft-surface gardens and absorbing roofs in urban areas, wadis.
- Use **bypass canals as overflow channels** to lessen peaks. Does require maintenance and alteration to canal system.
How (well) can we mitigate flooding?

- **Sustainable flood plain management**: designated flood plains (pay owners), houses on stilts, houses on mounts with surrounding canals/wadis.
- Legislation & “Waterschappen” (**Water Governance**).
6. Wetropolis: conceptual table-top demo?

A demo to highlight the concepts of flooding:

- Use *vertical cells* as conceptual valleys:
  - spongey material = moor
  - hard surface = urban asphalt,
  - adjustable weirs for storage in reservoirs, etc.
Wetropolis: conceptual table-top experiment?

- Plan-view, *mathematical design* before construction:

![Wetropolis control map](https://www.facebook.com/resurging.flows)

- [https://www.facebook.com/resurging.flows](https://www.facebook.com/resurging.flows)
Wetropolis: conceptual table-top experiment?
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Introduction
Rain in future?
Extreme precipitation & flooding?
Predict heavy precipitation & floods?
Mitigate flooding?

Wetropolis
Simulator
Discussion
Wetropolis: conceptual table-top experiment?
7. Design Hydro- & Meteorological Simulator

Prototype 0.0 with one river, one canal, one “Hele-Shaw” moor as valley, one reservoir as valley:

- **River**: modelled 1D, straight with depth $h_r(x, t)$ with $x \in [0, L_x]$, e.g., $L_x = 4.211$ m
- **River**: simple kinematic model, bottom slope balances Manning friction
- **Canal**: modelled 1D, straight as compartment depths $h_{3c}(t)$ for $x \in [0, L_{3c}]$, $h_{2c}(t)$ for $x \in [L_{3c}, L_{2c}]$ and $h_{1c}(t)$ for $x \in [L_{2c}, L_{1c}]$
- **Canal**: closed at $x = 0$, weir between canals 3&2 at $x = L_{3c}$, weir between canals 2 & 1 at $x = L_{2c}$, weir between canal 1 and river at $x = L_{1c}$. 
Prototype zero with one river, one canal, one porous moor as valley, one reservoir as valley:

- **“Hele-Shaw” moor:** porous cell with Darcy flow and ground water level \( h_m(y, t) \) orthogonal to canal; \( \gamma_m \in [0, 1] \) flows via weir into canal and \( 1 - \gamma_m \) into the river at \( x = L_w < L_{1c} \)

- **Reservoir:** “Hele-Shaw” cell with depth \( h_{res}(t) \) & weir height \( P_{wr}(t) \).

- **Controls:** splitter and weir heights \( \gamma_m(t), P_{1w}(t), P_{2w}(t) \).
Design Hydro- & Meteorological Simulator

Side view of canals & river and their bed & waterlevel profiles:
Cross section $A = A(x, t) = A(h)$ with depth $h = h(x, t)$ above bottom $b = b(x)$, e.g. $A = w_r h$, velocity $u(x, t)$:

$$\partial_t A + \partial_x (Au) = F$$

$$\partial_t u + u \partial_x u + g \partial_x h = -g (\partial_x b + C_m^2 u |u| / R^{4/3})$$  \(2\)

- hydraulic radius $R(h) = \frac{\text{wet area}}{\text{wetted perimeter}} = \frac{w_r h}{2h + w_r}$
- Manning friction coefficient $C_m \in [0.01, 0.15]$
- **Kinematic model** for $u > 0$, upwind information speed $dQ(A)/dA > 0$ for flux $Q(A) = Au$ and inflow $A(h(0, t))$:

$$u = R^{2/3} \sqrt{-\partial_x b / C_m}$$

$$\partial_t A + \partial_x (AR^{2/3} \sqrt{-\partial_x b / C_m}) = F$$

or

$$\partial_t (w_r h) + \partial_x (w_r hR(h)^{2/3} \sqrt{-\partial_x b / C_m}) = F.$$  \(3\)
River Flow

- Kinematic model or nonlinear conservation law in $h$ for $u > 0$ with $A = w_r h$:

$$ \partial_t (w_r h) + \partial_x (w_r h R(h)^{2/3} \sqrt{-\partial_x b/C_m}) = 0 $$

- Upwind information speed $dQ(A)/dA > 0$ for flux $Q(A) = A u$ and inflow $A(h(0, t))$:

- $F$: influx water from moor $x = L_v \approx 2.038$, $y = 0$

$$ Q_m(h_m) = (1 - \gamma_m) m_{por} \sigma_e \omega_v g \alpha \frac{1}{2} \partial_y (h^2)|_{y=0} $$ (4)

- $F$ inflows: canal section 1 water at $x = L_{1c}$, inflow reservoir water at $x = L_{res}$. 
Groundwater Flow

- **Depth-averaged ground water model** with level \( h_m(y, t) \) from Barenblatt (2002) in cell of width \( w_v \), e.g. \( w_v = 0.095 \text{m} \).

- **Nonlinear diffusion equation** for ground water level \( h_m(y, t) \) for \( y \in [0, L_y = 0.932 \text{m}] \):

\[
\partial_t (w_v h_m) - \alpha g \partial_y (w_v h \partial_y h) = \frac{w_v R}{m_{por} \sigma_e} \quad (5)
\]

- **Rainfall** \( R(y, t) \), via wider funnel, into Hele-Shaw cell

- **Porosity** \( m_{por} \in [0.1, 0.3] \), fraction \( \sigma_e \in [0.5, 1] \) pores filled with water

- \( \alpha = k/(\nu m_{por} \sigma_e) \) with permeability \( k = 10^{-8} \text{m}^2 \) and viscosity \( \nu = 10^{-6} \text{m}^2/\text{s} \)
Groundwater Flow

- **Nonlinear diffusion equation** in ground water level $h_m(y, t)$ for $y \in [0, L_y]$: 

$$ \partial_t (w_v h_m) - \alpha g \partial_y (w_v h \partial_y h) = \frac{w_v R}{m_{por} \sigma_e} $$  \hspace{1cm} (6)

- **Boundary conditions**: no flux/wall at $y = L_y$
- Time-dependent canal level $h_{3c}(t)$ at $y = 0$.
- **In summary**: porous cell represents sub-catchment to conceptualise/visualise groundwater flow.
Canals 2 & 3

Canal with 3 sections; canal 2,3 of width $w_c$ and depth $h_{2c,3c}(t)$:

- section 2 for $x \in [L_{3c}, L_{2c}]$, e.g. $L_{3c} = 1.724m$:
- section 3 for $x \in [0, L_{2c}]$, e.g. $L_{2c} = 3.608m$:

$$w_c(L_{2c} - L_{3c}) \frac{d h_{2c}}{d t} = \gamma_m m_{por} \omega \alpha g \frac{1}{2} \partial_y (h^2)|_{y=0} - Q_{2c}$$ \hspace{1cm} (7)

- Canal 3: berm at $z = 0.06m$ & bottom at $z = 0.04m$
- Canal 2: berm at $z = 0.04m$ & bottom at $z = 0.02m$
- Weirs outflow into canal 2, 1 based on Bernoulli & flow criticality at $x = L_{3c,2c}$, e.g. for subcritical downstream

$$V_c = \sqrt{gh_c} \quad \text{and} \quad h_c = \left(\frac{2}{3}\right)(h_{2c} - P_{2w})$$
$$gh_{2c} + \frac{1}{2}V_{2c}^2 = g(h_c + P_{2w}) + \frac{1}{2}V_c^2 = \frac{3}{2}gh_c + gP_{2w}$$

s.t. $Q_{2c} = h_c V_c = \sqrt{gh_c^{3/2}} = C_f \sqrt{g \max(h_{2c} - P_{2w}, 0)^{3/2}}$. 

\textit{Wetropolis}

\textbf{Onno Bokhove}

\textbf{Introduction}

\textbf{Rain in future?}

\textbf{Extreme precipitation & flooding?}

\textbf{Predict heavy precipitation & floods?}

\textbf{Mitigate flooding?}

\textbf{Wetropolis Simulator}

\textbf{Discussion}
Canal 1

- **Section 1** for $x \in [L_{2c}, L_{1c}]$, e.g. $L_{1c} = 3.858m$, width $w_c$ and depth $h_{1c}(t)$:

$$w_c(L_{1c} - L_{2c}) \frac{dh_{1c}}{dt} = Q_{2c} - Q_{1c}$$

$$Q_{1c} = h_c V_c = \sqrt{gh_c^{3/2}} = C_f \sqrt{g \max(h_{1c} - P_{1w}, 0)^{3/2}}$$

- **Canal 1** has berm at $z = 0.021m$ and its bottom at $z = 0.001m$

- **Weir** at $x = L_{1c}$ when flow into river subcritical, i.e. sufficient drop from canal 1 to river level.
Reservoir with Weir

- **Reservoir** with length $L_{yy} = 0.293\text{m}$, width $w_{res} = 0.123\text{m}$, level $h_{res}(t)$:

$$\frac{dh_{res}}{dt} = R_{res}(t) - \frac{Q_w}{(L_{res}w_{res})}$$

$$Q_w = C_f \sqrt{g w_{res} \max(h_{res} - P_{wr}, 0)^{3/2}}$$

- **Weir** located at $x = L_{res} = 0.932\text{m}$, where water flows into the river.
Discretise rainfall events (risk):

- **Rainfall** is discretized. In our Simulator, we discretise rainfall further into two sets of categories: location & rain amount.
- Our **Wetropolis Day** $wd$ is $TUs$.
- **Rain location** has 4 categories: rain in reservoir, moor & reservoir, moor, or no rain in the catchment.
- **Rain amount** has 4 categories per location $(1, 2, 4, 9)r_0/wd$, gauged such that there is no flooding for $(1, 2, 4)r_0/wd$ rainfall, with limited flooding for $(8, 9)r_0/wd$ rain in one location & **flooding** for $18r_0/wd$ rain in moor & reservoir.
Rainfall

- The probabilities for rain amount are “binomially” distributed physically using a half/stepped 8-pins Galton board: $\frac{3}{16}, \frac{7}{16}, \frac{5}{16}, \frac{1}{16}$ (risk).

- The probabilities for rain location are determined using another 8-pins Galton board as $\frac{3}{16}, \frac{7}{16}, \frac{5}{16}, \frac{1}{16}$.

- There are therefore five rainfall outcomes possible per Time Unit with probabilities: no rain at $\frac{1}{16}$

- The rain amount per $wd$ will be determined by gauging, such that there is no flooding with rain amounts $(1, 2, 4)r_0$, moderate flooding possible when $(8, 9)r_0$ & massive flooding for $18r_0$. 

Rainfall

- Table of rainfall amount/TU versus rain location:

<table>
<thead>
<tr>
<th></th>
<th>$r_0$</th>
<th>$2r_0$</th>
<th>$4r_0$</th>
<th>$9r_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>reservoir</td>
<td>9</td>
<td>21</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>both</td>
<td>21</td>
<td>49</td>
<td>35</td>
<td>7</td>
</tr>
<tr>
<td>moor</td>
<td>15</td>
<td>35</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>no rain</td>
<td>3</td>
<td>7</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

- Accumulated rainfall amounts (divided by Time Units) in catchment: $0 : 0.0625 = 1/16$, $r_0 : 0.0938 = 24/256$, $2r_0 : 0.3008 = 77/256$, $4r_0 : 0.3477 = 89/256$, $8r_0 : 0.1367$, $9r_0 : 8/256 = 0.0469$, $18r_0 : 7/256 = 0.0273$. 
Rainfall

Two draws from asymmetric Galton board:

Figure: Predictions 1D synthetic model 100 random $wd = 10s$ rainfall episodes using draws from Galton board: a) Evolution water levels in 3 canals & reservoir. b) River levels upstream & downstream. c) Snapshot groundwater level in moor. d) Rainfall amounts $(1, 2, 4, 8, 9, 18)r_0$ with 2 extremes (average 2.73), coinciding with 2 flood peaks in c). Rain falls in reservoir, both, moor or not.
Sample Simulation 1

At \( t = 0 \): canals, moor and reservoir empty. \( wd = 10s, r_0 = 0.18 \) l/\( wd \)
At $t = 0$: canals, moor and reservoir empty. $wd = 10s$, $r_0 = 0.18l/wd$
At $t = 0$: canals, moor and reservoir empty. *Simulation.*
8. Discussion

- Ronstructed 2-day rainfall induced *Boxing Day floods*:

- Redesign Wetropolis with deterministic location generator (chaotic, fluid dynamical) instead of probabilistic one, e.g. hydraulic & chaotic seesaw (Prof Chris Jones & Church).
Discussion

- **0(1) 1D mathematical model** led us to **Wetropolis**: experience extreme rainfall & flooding events.
- **To do**: control & data assimilation: natural measures, river & canal levels & rainfall.
- **More advanced**: 2D shallow-water modelling, 3D Richard’s equation for groundwater, ensemble Kalman DA, homogenisation of streets/sewers . . .
- **Invitations**: Boxing Day flood exhibit (Leeds Industrial Museum Dec. 8 4-6pm) & etc.
- **The End.**