Modifications to an idealised data assimilation scheme for research in convective-scale satellite data assimilation

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1. Question

Satellites provide a fundamental source of observations in data assimilation (DA) with new instruments added to the observing system every year. The 'modRSW' model is an idealised model which has shown suitability for DA research[1, 2, 3]. Can we utilise such a model to conduct idealised satellite DA?

2. DA with 'modRSW'

The 'modRSW' model is based on the 1D modified (single-layer) shallow water system of equations:

$$h_t + (hu)_x = 0, \qquad (1a)$$

$$(hu)_t + \left(hu^2 + P(h)\right)_x + hc_0^2 r_x = \frac{1}{\mathrm{Ro}} hv, \qquad (1b)$$

$$(hv)_t + (huv)_x = -\frac{1}{\mathrm{Ro}} hu, \qquad (1c)$$

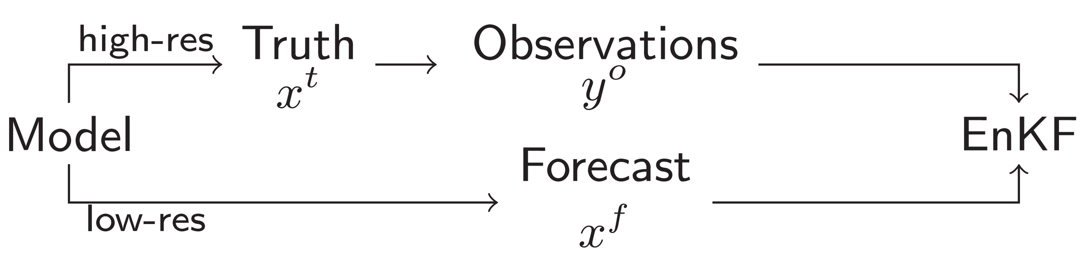
$$(hr)_t + (hur)_x + h\beta u_x + \alpha\, hr = 0, \qquad (1d)$$

Convection and precipitation are triggered as h overcomes the thresholds H_c and H_r, altering the dynamics of the model such that:

$$P = \begin{cases} \frac{h^2}{2\mathrm{Fr}^2} & \text{if } h < H_c, \\ \frac{H_c^2}{2\mathrm{Fr}^2} & \text{if } h \geq H_c, \end{cases} \qquad \beta = \begin{cases} \tilde{\beta} & \text{if } h \geq H_r, u_x < 0, \\ 0 & \text{otherwise.} \end{cases}$$

h	fluid depth	P	effective pressure
u	horizontal velocity	α	rain removal
v	meridional velocity	β	rain generation
r	rain	c_0^2	convection suppression
Ro	Rossby number	Fr	Froude number

DA is performed via twin-experiments setting:



DA scheme: **Ensemble Kalman Filter**.

Observations: interpolated directly at equally-spaced, fixed locations in space for all variables.

3. Limitations

Satellite DA requirements	modRSW DA
Non-linear observation operator	No (linear)
Spatially varying observations	No (static)
Generation of radiance observations	Yes (*)
Vertical structure	No (single-layer)

(*) **Scaling issue**: Hydrostatic equilibrium + ideal gas law would give the expression for temperature T:

$$T = \frac{gH}{R} h$$

with R specific gas constant for dry air and g gravity acceleration. Dynamics of 'modRSW' model dependent on Froude number $\mathrm{Fr} = \frac{U}{\sqrt{gH}}$, with U a typical scale velocity. In the supercritical regime (i.e. $\mathrm{Fr} = 1.1$, $U = 20\,\mathrm{m/s}$), essential for downstream convection propagation, $T \sim \mathcal{O}(10^0)\mathrm{K}$.

4. Modifications

- From an isopycnal to an **isentropic** model.
- From a single-layer to a **1.5-layer** model.
- A revised observing system: spatially-varying horizontally-averaged radiance observations.
- A **new non-linear observation operator** $\mathcal{H}$.

References

[1] Kent, T. (2016): *An idealised fluid model of Numerical Weather Prediction: dynamics and data assimilation*, PhD Thesis.

[2] Kent, T., Bokhove, O., Tobias, S. (2017): *Dynamics of an idealized fluid model for investigating convective-scale data assimilation*, Tellus A: Dynamic Meteorology and Oceanography, 69(1), 1369332.

[3] Kent, T., Inverarity, G., Cantarello, L., Tobias, S., Bokhove, O., (2019). *Idealised forecast-assimilation experiments and their relevance for convective-scale Numerical Weather Prediction*, EGU Abstract, EGU2019-8318.

5. The isentropic 1.5-layer model

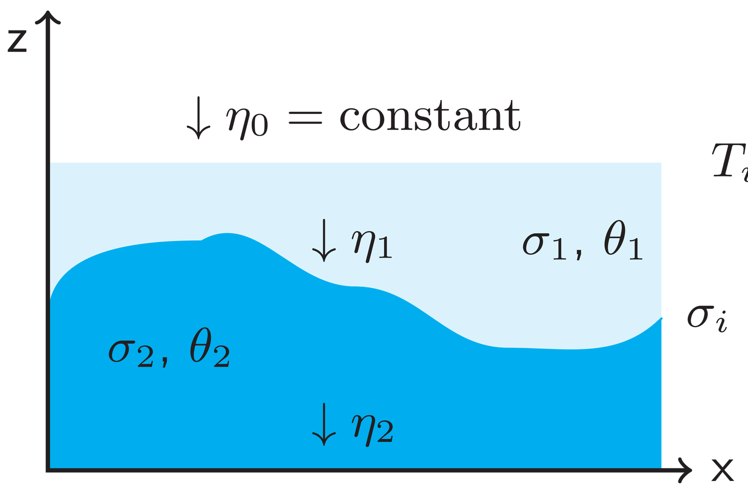
The new model has two layers at constant potential temperature (with $\theta_1 > \theta_2$). The layer on the top is inactive ($\mathbf{u}_1 = 0$) and capped by a rigid lid. The system is described solely by the equations for the layer underneath. W.r.t. the 'modRSW' model, the fluid depth h is replaced by the pseudo-density variable σ_2 and a function $\mathcal{M}(\sigma)$ plays the role of the effective pressure $P(h)$. $\eta_i = p_i/p_{ref}$ is the non-dimensional pressure within the i-th layer with p_{ref} a reference value. η_2 can be diagnosed from σ_2. At this stage, topography is not included.

$$(\sigma_2)_t + (\sigma_2 u_2)_x = 0, \qquad (2a)$$

$$(\sigma_2 u_2)_t + \left(\sigma_2 u_2^2 + \mathcal{M}(\sigma_2)\right)_x + \sigma_2 c_0^2 r_x = \frac{1}{\mathrm{Ro}} \sigma_2 v_2, \qquad (2b)$$

$$(\sigma_2 v_2)_t + (\sigma_2 u_2 v_2)_x = -\frac{1}{\mathrm{Ro}} \sigma_2 u_2, \qquad (2c)$$

$$(\sigma_2 r)_t + (\sigma_2 u_2 r)_x + \beta\sigma_2 (u_2)_x + \alpha\sigma_2 r = 0. \qquad (2d)$$



$$T_i = \theta_i \eta_i^\kappa, \; i = 1, 2 \qquad \kappa = \frac{R}{c_p} \qquad \sigma_i = \frac{p_{ref}}{g}(\eta_i - \eta_{i-1})$$

The functioning of the model is the same as described in §2. The generation of satellite observations is described in §6, whereas §7 summarises the properties of the data assimilation scheme.

Model feature	modRSW	new model
Convection threshold	H_c (fluid height)	σ_c (pseudo-density)
Rain threshold	H_r (fluid height)	σ_r (pseudo-density)
Effective pressure	$P(h)$	$\mathcal{M}(\sigma_2(\eta_2))$

6. Idealised satellite observations

The focus is on **sounding observations** from **polar-orbiting satellites**.

Radiative transfer scheme Synthetic observations of radiance B are generated from the true model trajectory using the Rayleigh-Jeans law (valid for wavelengths of $\lambda > 50\,\mu m$ at $T \simeq 300\,\mathrm{K}$, i.e. infrared/microwave approximation):

$$B = 2\frac{k_B c}{\lambda^4} T = 2\frac{k_B c}{\lambda^4}\theta(\eta_2^t)^\kappa \longrightarrow B' = \frac{B}{B_0} = (\eta_2^t)^\kappa, \qquad \text{with } B_0 = \frac{2k_B c}{\lambda^4}\theta,$$

in which k_B is the Boltzmann constant and c the speed velocity.

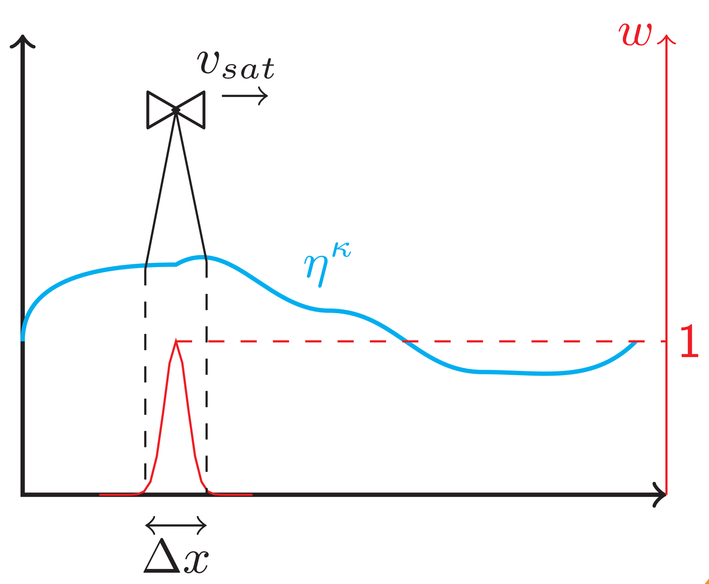
Spatially varying observations Since the position of a satellite varies in time observing different portions of the earth, our observations shall do the same. 1D approximation: our satellite observations move at velocity v_{sat} along a periodic domain of length L. Its position at a specific time t will be:

$$x_{sat} = v_{sat} \cdot t \mod L.$$

Horizontally-averaged observations To mimic the satellite's field of view (FOV), a weighted mean is applied to a Δx interval of values of the true radiance $(\eta^t)^\kappa(x)$ centered on x_{sat}, in order to generate a single horizontally-averaged observation:

$$y^o(x_{sat}) = \int_{x_{sat}-\Delta x/2}^{x_{sat}+\Delta x/2} w(x)(\eta_2^t)^\kappa(x)\, \mathrm{d}x.$$

The weights $w(x)$ are values of a Gaussian function centered on x_{sat} with $\Delta x/2 \simeq 3\sigma$, with σ being the standard deviation of the Gaussian function.



7. DA with new setup

DA setup: **Ensemble Kalman Filter**.

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left(\mathbf{y}^o - \mathcal{H}(\mathbf{x}^f)\right).$$

Observations are split into satellite (y^o_{sat}, see §6) and ground observations ($\mathbf{y}^o_{grn}$), for u, v and r, derived from the true model trajectory at a fixed set of k locations $x_1, ..., x_k$ along the domain:

Model	Obs.	Characteristics
σ_2	$(\eta_2)^\kappa$	Satellite, moving, spatially-averaged
u	u	Ground, static, point-wise
v	v	Ground, static, point-wise
r	r	Ground, static, point-wise

The new observation operator $\mathcal{H}$ applied to the model state $\mathbf{x}^f = (\boldsymbol{\sigma}^f, \mathbf{u}^f, \mathbf{v}^f, \mathbf{r}^f)^T$ gives:

$$\mathcal{H}(\mathbf{x}^f) = \begin{pmatrix} y^f(x_{sat}) \\ \mathbf{y}^f_{grn} \end{pmatrix} = \begin{pmatrix} (\eta_2^f)^\kappa(x_{sat}) \\ \mathbf{y}^f_{grn} \end{pmatrix},$$

where $\mathbf{y}^f_{grn}$ contains the values of $\mathbf{u}^f$, $\mathbf{v}^f$ and $\mathbf{r}^f$ interpolated at locations $x_1, ..., x_k$. Given $\mathcal{H}$, we can compute its Jacobian $\mathbf{H}$ as:

$$\mathbf{H} \simeq \partial_{\mathbf{x}^f}\mathcal{H} = \left(\partial_{\boldsymbol{\sigma}}\mathcal{H}, \partial_{\mathbf{u}}\mathcal{H}, \partial_{\mathbf{v}}\mathcal{H}, \partial_{\mathbf{r}}\mathcal{H}\right).$$

This is used to calculate the Kalman gain matrix $\mathbf{K}$:

$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T\left(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R}\right)^{-1}.$$

8. Conclusions

- In order to conduct satellite DA, the isopycnal single-layer 'modRSW' model has been modified to an isentropic one-and-a-half-layer version.
- The new model presents larger vertical structure and offers the possibility of modeling some fundamental aspects of satellite DA.
- The new configuration includes a non-linear observation operator.

9. Future work

- Modify the model in order to include topography.
- Given the two-layer configuration, explore the possibility of defining weighting functions and using a multi-channel approach in assimilating radiance.
- Find the best strategy to define clouds.
- Explore alternative radiation schemes.

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