# Leeds Computability Days 2022
# University of Leeds School of Mathematics
# Leeds, UK
# 30 May – 01 June 2022

## Zoom information

- Meeting ID: `834 6305 0417`
- Passcode: `LT1@z8`
- Link:
  `https://universityofleeds.zoom.us/j/83463050417?pwd=STR6aVJpRUZLRzZpblNiRXEwc1ZxUT09`

## Schedule

**ALL TIMES ARE UK TIMES!**
The UK is on British Summer Time: UTC/GMT +1.

**Monday, 30 May 2022**

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|---|---|
| Before 10:00am: | Coffee available! |
| 10:00am – 10:30am: | Alexander Melnikov (Victoria University of Wellington)<br>*Computable Polish groups* |
| 10:30am – 11:00am: | Morning break! |
| 11:00am – 12:00pm: | Valentino Delle Rose (University of Siena)<br>*Classical results and new developments on logical depth of infinite binary sequences* |
| 12:00pm – 2:00pm: | Lunch! |
| 2:00pm – 3:00pm: | Manlio Valenti (University of Wisconsin–Madison)<br>*New operators on Weihrauch degrees and their applications to separation results* |
| 3:00pm – 3:30pm: | Afternoon break! |
| 3:30pm – 4:00pm: | Volodya Shavrukov<br>*Applications of single sky to r.e. sets* |
| 4:10pm – 4:40pm: | Steffen Lempp (University of Wisconsin–Madison)<br>*Maximal towers and ultrafilter bases in computability* |

**Tuesday, 31 May 2022**

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|---|---|
| Before 10:00am: | Coffee available! |
| 10:00am – 10:30am: | Wei Wang (Sun Yat-sen University)<br>$\mathsf{P}\Sigma_1$ *and combinatorial principles* |
| 10:30am – 11:00am: | Morning break! |
| 11:00am – 12:00pm: | Katarzyna Kowalik (University of Warsaw)<br>*Some Ramsey-theoretic principles in reverse mathematics over a weak base theory* |
| 12:00pm – 2:00pm: | Lunch! |
| 2:00pm – 3:00pm: | Tomasz Steifer (Pontificia Universidad Católica de Chile and Polish Academy of Sciences)<br>*Effective ergodic theory* |
| 3:00pm – 3:30pm: | Afternoon break! |
| 3:30pm – 4:00pm: | Vasco Brattka (Universität der Bundeswehr München)<br>*On the complexity of computing Gödel numbers* |
| 4:10pm – 4:40pm: | Mariya Soskova (University of Wisconsin–Madison)<br>*One point extensions of antichains in the local structure of the enumeration degrees* |

**Wednesday, 01 June 2022**

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|---|---|
| Before 10:00am: | Coffee available! |
| 10:00am – 10:30am: | Kenshi Miyabe (Meiji University)<br>*Generality of computable measures* |
| 10:30am – 11:00am: | Morning break! |
| 11:00am – 12:00pm: | Maribel Fernández (King's College London)<br>*Hierarchical higher-order port-graph rewriting as a modelling tool* |
| 12:00pm – 2:00pm: | Lunch! |
| 2:00pm – 2:30pm: | Dariusz Kalociński (Polish Academy of Sciences)<br>*Degree spectra and recovering the successor* |
| 2:40pm – 3:10pm: | Marta Fiori Carones (Sobolev Institute of Mathematics)<br>*When is unbounded search necessary?* |
| 3:10pm – 3:40pm: | Afternoon break! |
| 3:40pm – 4:10pm: | Ludovic Patey (CNRS)<br>*Partition genericity and pigeonhole basis theorems* |
| 4:20pm – 4:50pm: | Rachael Alvir (Notre Dame)<br>*Finitely $\alpha$-generated structures* |

## Abstracts (alphabetical by author)

**Rachael Alvir** (Notre Dame)

*Finitely $\alpha$-generated structures*

Scott's isomorphism theorem states that every structure can be described up to isomorphism among countable structures by a single sentence $\phi$ of $L_{\omega_1,\omega}$. Here, $L_{\omega_1,\omega}$ is the logic obtained by closing regular finitary first-order logic under countable conjunctions and disjunctions. The least syntactic complexity of a Scott sentence of the structure—called the *Scott complexity*—tells us a host of information about the structure, in the sense that there are several equivalent conditions to having a certain complexity. In particular, if a structure $A$ has a $\Sigma_{\alpha+1}$ Scott sentence, this is equivalent to a certain tuple "controlling" the automorphism orbits of the other tuples in the structure. Roughly speaking, we say that such structures are *finitely $\alpha$-generated* by such a tuple. Thinking about such structures in this way, we can lift several theorems about Scott sentences previously known only to hold for the class of finitely generated structures. We give additional examples of finitely $\alpha$-generated structures which are not finitely generated: the divisible abelian groups of finite non-zero rank. Along the way, we also exhibit new methods of computing Scott complexity.

**Vasco Brattka** (Universität der Bundeswehr München)

*On the complexity of computing Gödel numbers*

Given a computable sequence of natural numbers, it is a natural task to find a Gödel number of a program that generates this sequence. It is easy to see that this problem is neither continuous nor computable. In algorithmic learning theory this problem is well studied from several perspectives and one question studied there is for which sequences this problem is at least learnable in the limit. In contrast to this, we study the problem on all computable sequences and we classify the Weihrauch and Borel complexities of it. For this purpose we can import some techniques from learning theory. As a benchmark for the classification we use closed and compact choice principles and their jumps on natural numbers and we argue why they correspond to induction and boundedness principles known from reverse mathematics.

**Valentino Delle Rose** (University of Siena)

*Classical results and new developments on logical depth of infinite binary sequences*

How can we measure the "value" of some piece of information? And how can we distinguish between useful and useless information? Concerning finite binary strings, several notions have been proposed motivated by these questions, including Koppel's *sophistication*, Gell-Mann and Lloyd's *effective complexity*, and Bennett's *logical depth*.

Logical depth has been also successfully extended to the case of infinite binary sequences, resulting in a novel approach to investigate the interplay between the "internal organization" of sequences and their computational power and usefulness. According to Bennett's intuition, a *deep* sequence, i.e. a sequence containing useful information, should be one for which, the more time we have to compress the prefixes of such sequence, the better we can actually compress them: the prototypical example of a deep sequence is the halting problem $\emptyset'$. On the other hand, as we would expect, both computable sequences (containing only completely predictable information) and sufficiently random ones (encoding absolutely unpredictable information) are shallow, namely not deep.

In recent years logical depth has received renewed attention: for example, new results exploring the boundaries between deep and shallow sequences, or giving new evidence of the correlation between depth and computational power of sequences, have been discovered. Moreover, several variants of Bennett's notion have been proposed, with the aim of capturing the intuitive idea of "useful information" in different frameworks.

In this talk we will review the state of art in this research field, which still appears very promising, and of which a general understanding is still far from being achieved.

**Maribel Fernández** (King's College London)

*Hierarchical higher-order port-graph rewriting as a modelling tool*

Graph rewriting systems can be used to specify computation and more generally to specify and analyse the dynamics of complex systems. They provide visual, intuitive models that are directly executable. In this talk, we will describe the use of strategic port graph rewriting as a visual modelling tool and its implementation in PORGY. In the first part of the talk we will present hierarchical higher-order port graphs (HOPs) and HOP-rewriting. In the second part of the talk, we will describe PORGY (a port-graph rewriting tool) and give examples of application: the specification of a lambda-calculus evaluator and a simple model of rational negligence in financial markets. This is joint work with members of the PORGY team.

**Marta Fiori Carones** (Sobolev Institute of Mathematics)

*When is unbounded search necessary?*

Reverse mathematics allows one to formally answer questions like whether a certain theorem from ordinary mathematics implies another one. Since one works over the base theory $\mathsf{RCA}_0$, reverse mathematics allows one to prove that theorem $A$ implies theorem $B$, or not, provided that computable operations can be carried out. We want to consider the same type of questions, but restricting our tools to primitive recursive operations. To this end we need to consider a weaker base theory, which encapsulates the idea of primitive recursiveness. $\mathsf{PRA}^2$, introduced by Kohlenbach, serves our purposes. In fact, it is a second order extension of the first order theory $\mathsf{PRA}$ in the two sorted language with variables for natural numbers and for functions. Axioms of $\mathsf{PRA}^2$ extend those of $\mathsf{PRA}$ with defining equations for all primitive recursive functionals of type 2 (i.e. functions with functions as argument), which allows one to speak about e.g. analysis, infinite combinatorics, and algebra. Taking $\mathsf{PRA}^2$ as the base theory thus allows one to study whether a statement (possibly computably true) is also primitively recursively true, and if not, which is its strength. During the talk some examples are presented. This work is joint with Nikolay Bazhenov, Jiayi Liu, and Alexander Melnikov.

**Dariusz Kalociński** (Polish Academy of Sciences)

*Degree spectra and recovering the successor*

The degree spectrum of a computable relation $R$ on a computable structure consists of all Turing degrees of the images of the relation across all computable copies of the structure. Investigations of the degree spectra of computable relations $R$ on the structure $(\omega, <)$ (natural numbers with standard order) have revealed spectra such as the trivial one, the set of all c.e. degrees or the set of all $\Delta_2$ degrees. In the talk, I will focus on the spectrum of all c.e. degrees, realized, among others, by the successor. A typical way of showing that a computable relation $R$ exhibits this spectrum involves recovering the successor from $R$: we show that for each computable copy $\mathcal{A}$ of $(\omega, <)$, the image of the successor in $\mathcal{A}$ is computable relative to the image of $R$ in $\mathcal{A}$. I will ponder on the question, and a partial answer to it, which goes in the other direction: does having all c.e. degrees as a spectrum of $R$ make the successor recoverable from $R$? This work is joint with Nikolay Bazhenov.

**Katarzyna Kowalik** (University of Warsaw)

*Some Ramsey-theoretic principles in reverse mathematics over a weak base theory*

We present results on the reverse mathematics of Ramsey's theorem for $n$-tuples and $k$-colors, $n, k \geq 2$, and some of their consequences (CAC, ADS, $\mathrm{CRT}^2_2$, COH) over the weak base theory $\mathrm{RCA}^*_0$. This theory is obtained from the usual axiomatic theory formalizing "computable mathematics," $\mathrm{RCA}_0$, by replacing the axiom of mathematical induction for computably enumerable properties with mathematical induction only for computable properties.

We give a complete axiomatization of the first order consequences of $\mathrm{RCA}^*_0 + \mathrm{RT}^n_k$ for $n \geq 3$ and respective partial results for $\mathrm{RT}^2_2$. In particular, $\mathrm{RT}^n_k$ is $\Pi^0_3$-conservative over $\mathrm{RCA}^*_0$ for all $n, k$ and so are consequences of $\mathrm{RT}^2_2$ such as CAC, ADS, and $\mathrm{CRT}^2_2$.

To obtain these results, we in particular have to determine the amount of induction needed to prove the classical computability-theoretic lower bounds on the complexity of solutions for $\mathrm{RT}^n_k$. It turns out that $\Sigma_\ell$ induction is required to prove that there exists a $\Delta_\ell$ 2-coloring of pairs without a $\Delta_\ell$ infinite homogeneous set. As a special case of this, it is consistent with $\mathrm{RCA}^*_0$ that $\mathrm{RT}^n_k$, $n, k \geq 2$, as well as CAC, ADS, and $\mathrm{CRT}^2_2$ are satisfied by the family of computable sets. We show that this does not hold for COH and, as a consequence, $\mathrm{RT}^2_2$ does not imply COH over $\mathrm{RCA}^*_0$.

This work is joint with Marta Fiori Carones, Leszek Kołodziejczyk, and Keita Yokoyama.

[1] Marta Fiori Carones, Leszek A. Kołodziejczyk, and Katarzyna W. Kowalik. *Weaker cousins of Ramsey's theorem over a weak base theory.* Annals of Pure and Applied Logic, 172(10), 2021, article 103028.

[2] Leszek A. Kołodziejczyk, Katarzyna W. Kowalik, and Keita Yokoyama. *How strong is Ramsey's theorem if infinity can be weak?* Submitted, 2021, Available at arXiv:2011.02550.

**Steffen Lempp** (University of Wisconsin–Madison)

*Maximal towers and ultrafilter bases in computability*

I will report on joint work with J. Miller, Nies and M. Soskova on a connection between computability and set theory: Set theory has defined a number of cardinal characteristics between $\aleph_1$ and the continuum, many of which are summarized in the Cichoń diagram. Lately, many of these notions have been carried over to the computability-theoretic setting, replacing arbitrary subsets of, or functions on, $\omega$ by computable sets and functions. In place of cardinality, the various notions from the Cichoń diagram are now expressed by Turing degrees and can be viewed as mass problems under Medvedev and Muchnik reducibility. I will focus on notions involving domination of functions and maximal almost disjoint families of sets.

**Alexander Melnikov** (Victoria University of Wellington)

*Computable Polish groups*

I will talk about several recent works that either study computable Polish groups or use computable Polish groups to answer questions seemingly unrelated to groups. I will also suggest and compare several potential answers to the following fundamental question: "What is the 'right' definition of a computable Polish group?"

**Kenshi Miyabe** (Meiji University)

*Generality of computable measures*

Suppose that we are sequentially given a random sequence with respect to an unknown measure on Cantor space. We try to predict the conditional measure for the next bit from the given initial segments. The theory of universal induction says that every optimal c.e. semi-measure performs pretty well for this task, but the convergence rate is challenging to analyze.

We say that a computable measure is more general than another if it dominates the other. We prove that roughly speaking, domination means better prediction. At the same time, the sum of prediction errors of a sufficiently general measure is always ML-random. Thus, we can determine the decreasing speed of the remaining sum of errors up to a multiplicative constant. Furthermore, we restrict the class of unknown measures and consider properties of general computable predictions.

**Ludovic Patey** (CNRS)

*Partition genericity and pigeonhole basis theorems*

There exist two notions of typicality in computability theory, namely, genericity and randomness. In this presentation, we introduce a new notion of genericity, called partition genericity, which is at the intersection of these two notions of typicality, and show that many basis theorems apply to partition genericity. More precisely, we prove that every co-hyperimmune set and every Kurtz random is partition generic, and that every partition generic set admits weak infinite subsets. This work is joint with Benoit Monin.

**Volodya Shavrukov**

*Applications of single sky to r.e. sets*

Restricting the functions allowed into ultrapowers $\mathbb{N}[u]$ of the standard model of arithmetic to total recursive functions, one obtains what is known as *recursive ultrapowers*. An ultrafilter $u$ in the algebra of recursive sets rather than a full ultrafilter on $\omega$ suffices for the construction. Recursive ultrapowers are exactly those models of true $\forall\exists$ arithmetic that are finitely generated with respect to total recursive functions.

When all nonstandard elements of $\mathbb{N}[u]$ find themselves at an at most total recursive distance from one another, we say that $u$ is *single-sky*.

We identify some instances of single-sky recursive ultrafilters, and present new results on r-maximal, hyperhypersimple, and D-maximal sets obtained with the help of single sky. For example, the r.e. $Q_1$-degrees below any nowhere simple D-maximal set form a countable atomless Boolean algebra.

**Mariya Soskova** (University of Wisconsin–Madison)

*One point extensions of antichains in the local structure of the enumeration degrees*

We would like to characterize the complexity of the theory of various degree structures and their fragments. For the Turing degrees and its local substructure of the $\Delta^0_2$ degrees, we know exactly at what level of quantifier complexity decidability breaks down: the two quantifier theory is decidable, while the three quantifier theory is not. For the c.e. degrees, the enumeration degrees, and the local structure of the $\Sigma^0_2$ enumeration degrees, there is a gap in our knowledge: we know that the one quantifier theory is decidable and that the three quantifier theory is not decidable, but in each case we do not know what happens at level two. The decidability of the two quantifier theory can be phrased structurally as follows: given any partial order $P$ and finitely many extensions of $P$, say $Q_1, Q_2, \ldots, Q_k$, decide whether every embedding of $P$ in the considered degree structure extends to an embedding of at least one of the $Q_i$. The case when $k = 1$ is known as the extension of embeddings problem, and it is decidable for each of the structures. In this talk I will describe our partial progress towards a renewed attack of a further sub-problem for the local substructure of the enumeration degrees: given an antichain $P$ and finitely many one point extensions of $P$, say $Q_1, Q_2, \ldots, Q_k$, placing a new element only below some of the members of the antichain $P$, decide whether every embedding of $P$ extends to an embedding of one of the $Q_i$. A central structural feature at the heart of this problem is the existence of an Ahmad pair: a pair of incomparable $\Sigma^0_2$ degrees $a$ and $b$ such that every degree $c$ strictly below $a$ is also below $b$. The renewed interest in this problem was sparked by the surprising discovery: the non-existence of an Ahmad triple. We showed that there is no triple of enumeration degrees $a, b, c$ such that each pair $a, b$ and $b, c$ is an Ahmad pair. This is joint work in progress with Goh, Lempp, and Ng.

**Tomasz Steifer** (Pontificia Universidad Católica de Chile and Polish Academy of Sciences)

*Effective ergodic theory*

The effectivization research program proposes to reformulate almost sure statements into corresponding statements about algorithmically random points. Usually, standard proofs are already constructive enough and the effectivization is straightforward. However, for many results from ergodic theory, such as the Birkhoff ergodic theorem or the Shannon-Mcmillan-Breiman theorem, effectivization proved much more challenging. The talk is aimed as a survey of both positive and negative results in effective ergodic theory, including several ergodic theorems, elements of coding theory, and ergodic decomposition.

**Manlio Valenti** (University of Wisconsin–Madison)

*New operators on Weihrauch degrees and their applications to separation results*

In computability theory, a natural strategy to prove that $A \not\leq_T B$ is to prove the existence of a set $C$ such that $C \leq_T A$ but $C \not\leq_T B$. Of course, proving the existence of such a $C$ is often a very hard task. Such a strategy, however, is not exclusive of Turing reducibility: the same reasoning can be applied to any partial order, or even quasi-order.

In this talk, we focus on the applications of this idea in the context of TTE and Weihrauch reducibility. In particular, Weihrauch reducibility is a quasi-order on partial multi-valued functions between represented spaces. Such objects can be quite diverse and complicated to work with. A natural idea, suggested for the first time by Dzhafarov, Solomon, and Yokoyama [1], is to look at the "strongest computational problem that is Weihrauch-below $f$." This led to the definition of the *first-order part* ${}^1f$. In a similar fashion, in joint work with Jun Le Goh and Arno Pauly [2], we introduced the notion of the *deterministic part* of a problem $f$, capturing the "strongest single-valued function that is Weihrauch-below $f$."

While the formal definition of the first-order part and its degree are relatively new, many results in the literature are, in fact, implicitly studying the first-order part of some problems. Obtaining a precise characterization of the first-order part and the deterministic part of a given problem can be challenging, but it proved to be a very useful tool, especially when comparing principles that are (relatively) high in the Weihrauch hierarchy.

In order to effectively work with these tools, in joint work with Giovanni Soldà [3], we studied the first-order part operator from a more algebraic perspective, and explored its relations with several other operators already defined in the literature. In particular, to better describe the connections between the first-order part and the infinite parallelization, we introduced a new degree-theoretic operator, called the *unbounded finite parallelization*, capturing the idea of using a problem a finite number of times, but without having to commit in advance on how large such number has to be. We will then show how the obtained results can be used to easily characterize the first-order part of many known problems.


[1] Damir D. Dzhafarov, Reed Solomon, and Keita Yokoyama. *On the first-order parts of Weihrauch degrees.* In preparation.

[2] Jun Le Goh, Arno Pauly, and Manlio Valenti. *Finding descending sequences through ill-founded linear orders.* The Journal of Symbolic Logic, 86(2):817–854, 2021.

[3] Giovanni Soldà and Manlio Valenti. *Algebraic properties of the first-order part of a problem.* Submitted.


**Wei Wang** (Sun Yat-sen University)

$\mathsf{P}\Sigma_1$ *and combinatorial principles*

In recent years, $\mathsf{P}\Sigma_1$ has played interesting roles in the reverse mathematics of Ramseyan combinatorial principles. In this talk, I will try to demonstrate how we work with $\mathsf{P}\Sigma_1$ via some joint work with Chong and Yang.