Workshop on Mirror Symmetry and Cluster algebras
University of Leeds, 11-13 September 2013

ABSTRACTS OF TALKS

Karin Baur
Dimer models with boundary and cluster categories associated to Grassmannians

A dimer model can be defined as a quiver with an embedding of it into a compact oriented surface, giving rise to a tiling of the surface. Such dimer models can also be considered in the case of a surface with boundary. In particular, we are interested in dimer models arising from alternating strand diagrams (Postnikov, 2006). To such a diagram D we associate a quiver $Q(D)$ with a natural potential W_D. The quiver $Q(D)$ embedded into a disk is a dimer model with boundary. We then show that the algebra associated to the Grassmannian by B. Jensen, A. King and X. Su can be realized as an idempotent subalgebra of the algebra of $Q(D)$ (under relations from the potential W_D). This is joint work with A. King (Bath) and R. Marsh (Leeds).

Raf Bocklandt
Mutatis Mutandis

We discuss the phenomenon of mutation and how it relates to mirror symmetry of (punctured) surfaces for different levels in the construction of the mirror correspondence. At least 3 different types of mutation occur in this context and they correspond to three different ways of defining 'cluster'-type objects in the Fukaya category or its equivalent category of singularities.

Alessandro Chiodo
Applications of quantum theories for Landau-Ginzburg models

Landau-Ginzburg models are singularities of the form $f : X \to B$ arising as mirror partners of several Fano manifolds. More precisely, we consider mirror symmetry as an identification of two local systems. The first local system is classical in algebraic geometry and encodes the cohomology of the fibres of $f : X \to B$. The second local system embodies quantum periods of the Fano manifold and arises from Gromov-Witten theory. Fan-Jarvis-Ruan and Polishchuk-Vaintrob set up analogue quantum theories also for Landau-Ginzburg models and allow us to cast mirror symmetry in a global, more flexible, framework where some well known obstacles in Gromov-Witten theory, such as non-convexity or non-concavity, become more approachable. As an example I will describe recent work by Jérémy Guéré proving a mirror theorem relating Landau-Ginzburg models without concavity.

Tom Coates
Mirror Symmetry, Fano Manifolds, and Mutations

I describe recent progress in constructing Laurent polynomials which correspond under mirror symmetry to 3-dimensional Fano manifolds, and in understanding the "mutations" which relate different Laurent polynomial mirrors for the same Fano manifold. This is joint work with Akhtar, Corti, Galkin, Golyshev, Kasprzyk, and Prince.

Joseph Grant
Braid groups and quiver mutation

I will report on some joint work with Robert Marsh which is currently in progress. First I will talk about how quiver mutation can give us new presentations of braid groups. Then I will talk about some of the underlying homological algebra that explains this connection.

Alastair King
Grassmannian cluster categories

I will describe a new categorification of the cluster algebra structure on the homogeneous coordinate rings of Grassmannians.

Yankı Lekili
Equivariant Fukaya Categories and Representation Theory

Building on Seidel-Solomon's fundamental work, we define the notion of a $\mathfrak{g}$-equivariant Lagrangian brane in a symplectic manifold M if $\mathfrak{g} \subset SH^1(M)$ is a sub-Lie algebra of symplectic cohomology of M. This allows us to construct a mirror theory to Bott-Borel-Weil theory on the A-side. We will make our construction completely explicit in the case of sl_2 and comment on generalizations to arbitrary semisimple Lie algebras. This is a joint work (in progress) with James Pascaleff.

Travis Mandel
Theta Functions on Cluster Varieties

One of the original motivations for defining cluster algebras was the goal of better understanding Lusztig's dual canonical bases. With this in mind, Fock and Goncharov defined cluster A and X varieties, and predicted that the integral tropical points on one would parameterize a basis of global functions on the other. Meanwhile, Gross, Hacking and Keel (GHK) were using counts of "tropical holomorphic discs" to construct canonical theta functions on the mirror to a log Calabi-Yau variety. After realizing that cluster varieties are very close to being log Calabi-Yau, GHK and Kontsevich used their methods to construct the desired canonical bases. I will explain this construction.

Clelia Pech
Landau-Ginzburg models for Lagrangian Grassmannians.

According to mirror symmetry, the mirror of a Fano manifold should be a Landau-Ginzburg model, which looks like a (non-compact) manifold endowed with a complex-valued potential function. Recently, R. Marsh and K. Rietsch have found a new expression for the Landau-Ginzburg model of Grassmannians in terms of Plücker coordinates, which they have used to find flat sections of the A-model connection. In this talk, I will present joint work in progress with K. Rietsch on a partial extension of these results to Lagrangian Grassmannians (Grassmannians of maximal isotropic subspaces in a symplectic vector space). In particular, I will give the main ideas of how to obtain an explicit formula for the Landau-Ginzburg model in term of generalised Plücker coordinates, using representation theory. If time allows, I will explain how these results extend to other cominuscule homogeneous spaces.

Konstanze Rietsch
On mirror symmetry for Grassmannians

(joint work with Robert Marsh) We will describe a new way to write down a Landau-Ginzburg (LG) model for an arbitrary complex Grassmannian X in terms of Plücker coordinates on a dual Grassmannian. We will relate this description to previously conjectured formulas. The main result is that the Dubrovin connection of X appears in the Gauss-Manin system associated to this LG-model. The proof makes heavy use of the cluster algebra structure on the Grassmannian and associated combinatorics.

Helge Ruddat
Mirror symmetry and critical loci in dual Landau-Ginzburg models

I explain how a mirror duality of Landau-Ginzburg models relates to other mirror constructions from the Strominger-Yau-Zaslow point of view and discuss recent results on the mirror duality of the geometries appearing as critical loci of such models. In particular, I will introduce perverse curves for the quintic threefold and its mirror dual.

Ed Segal
Mixed braid group actions from B-brane monodromy

Seidel and Thomas famously found a braid group action on the derived category of the resolution of a type-A surface singularity. I'll describe a related construction of a whole system of higher-dimensional varieties, indexed by partitions, whose derived categories carry actions of the corresponding 'mixed' braid groups. Heuristically, these actions arise from monodromy in the FI-parameter space associated to some particular gauged linear sigma models. This is joint work with Will Donovan.