

Lecture Notes on Cluster Algebras, by R. J. Marsh

List of errata

1. Page 8, Section 1.3, line 2 should read:

$$x_n x_{n+r} = \begin{cases} x_{n+1}x_{n+r-1} + x_{n+2}x_{n+r-2} + \cdots x_{n+\frac{r}{2}}^2 & \text{if } r \text{ is even;} \\ x_{n+1}x_{n+r-1} + x_{n+2}x_{n+r-2} + \cdots x_{n+\frac{r-1}{2}}x_{n+\frac{r+1}{2}} & \text{if } r \text{ is odd,} \end{cases}$$

2. Page 8, Section 1.3, line 5 should read:

$$x_n x_{n+4} = x_{n+1}x_{n+3} + x_{n+2}^2.$$

3. Page 11, Definition 2.1.3 line 1 should read: “Fix k in I .”

4. Page 11. Equation (2.1) should read:

$$x_k x'_k = \prod_{\substack{j=1 \\ b_{jk} > 0}}^m x_j^{b_{jk}} + \prod_{\substack{j=1 \\ b_{jk} < 0}}^m x_j^{-b_{jk}}.$$

5. Page 12, Example 2.1.8. Line 1 should read: “Let $n = 1$, $m = 3$.”

6. Page 14, Definition 2.3.2(a), line 2 should read: “i.e. if there are a arrows from i to k and b arrows from k to j , we add ab arrows from i to j .”

7. Page 16, Remark 2.4.1, line 5 should read: “a map $d : Q_0 \rightarrow \mathbb{N}_{>0}$ such that $v(\alpha)_1 d(i) = v(\alpha)_2 d(j)$ whenever $\alpha : i \rightarrow j$ is an arrow in Q .”

8. Page 19, Section 3.1, line 9 should read:

$$M_j(t) = p_j(t) \prod_{i \in I} x_i^{\alpha_i}$$

Thus $M_j(t)$ is a monomial in the x_i , $i \in I$, while $M_j(\mathbf{x}(t))$ is a monomial in the $x_i(t)$, $i \in I$. This requires additional corrections in the rest of Chapter 3, detailed in the following points.

9. Page 19, Equation (3.2) should read:

$$x_j(t)x_j(t') = M_j(\mathbf{x}(t)) + M_j(\mathbf{x}(t'))$$

10. Pages 19–30. $x_i(t)$ should read x_i throughout (for any i), *except* for the following:

- Page 19, Section 3.1, line 7.
- Page 19, Equations (3.1) and (3.2),
- Section 3.2, lines 4–11.

11. Page 20, Statement (E4). Delete the following: “ x_k represents $x_k(t_1) = x_k(t_2)$ ”, and “(with $x_j = x_j(t_2) = x_j(t_3)$)”

12. Page 21, Remark 3.1.2(c), line 6 should read:

$$M_0 = (M_k(t_2) + M_k(t_3))|_{x_j=0},$$

13. Page 23, Proof of Lemma 3.3.1, lines 4-5. Delete: “Here $x_i = x_i(t) = x_i(t')$ for all $i \in I$, $i \neq j$.”

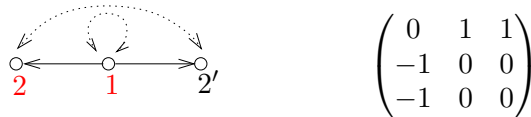
14. Page 23, Proof of Lemma 3.3.1, line 11. Delete: “(where $x_i = x_i(t_1) = x_i(t_2)$).”

15. Page 23, proof of Lemma 3.3.1, line 10 should read: “To see (E2), suppose that $t_1 \xrightarrow{j} t_2$ is an edge in \mathbb{T}_n ,”

16. Page 24, line -3. Delete: “for $i \neq k$, we write $x_i = x_i(t_2) = x_i(t_3)$,”
17. Page 25, line 12 should read: “If $b_{ik}b_{kj} > 0$, the exponent”
18. Page 26, line 8 should read: “If $b_{kj}(t_2) > 0$ then $b_{jk}(t_2) < 0$, so $b_{jk}(t_3) > 0$, so x_j appears in the second term.”
19. Page 31, Section 4.1, line 4 should read: “It is *positive definite* if $(\alpha, \alpha) > 0$ for all **nonzero** $\alpha \in V$.”
20. Page 31, Section 4.1, line 13 should read: “i.e. $(\varphi(\alpha), \varphi(\beta)) = (\alpha, \beta)$ for all $\alpha, \beta \in V$.”
21. Page 31, Lemma 4.1.1, line 3 should read: “is the subspace orthogonal to V' .”
22. Page 35, Lemma 4.2.6, line 1 should read: “If $\Phi \cong \Phi'$, then $W_\Phi \cong W_{\Phi'}$.”
23. Page 36, Theorem 4.4.1, line 1 should read: “Let $W = W_\Phi$ be **the** finite reflection group...”
24. Page 46, line -5, should read:

$$B = \left\langle \sigma_1 \dots \sigma_n \mid \begin{array}{l} \sigma_i \sigma_j \sigma_i = \sigma_j \sigma_i \sigma_j, |i - j| = 1 \\ \sigma_i \sigma_j = \sigma_j \sigma_i, |i - j| > 1 \end{array} \right\rangle$$

25. Page 48, Proposition 4.11.3. The equation in (a) should read: “ $\sum_{i=1}^n e_i = nh/2 = |\Phi^+|$ ”. The equation in (c) should read “ $|W| = \prod_{i=1}^n d_i$ ”.
26. Page 54, Figure 5.3 should be:



27. Page 54, line 11 after Lemma 5.2.1 should read: “ F_4 from E_6 ”
28. Page 58, line 10 should read: “in type A_n , if $|i - j| = 1$ then $s_i(-\alpha_i) = -\alpha_i - \alpha_j$.”
29. Page 64, Section 6.2, line 2 should read:

$$\Phi|_{I \setminus J} = \left\{ \sum_{i=1}^n d_i \alpha_i : d_i = 0 \text{ for all } i \in J \right\}$$

30. Page 68, line 10 should read:

$$\mathcal{P}([1, 4]) \cup \mathcal{P}(\{1, 6\}) \cup \mathcal{P}([3, 5]) \setminus \{\emptyset\},$$

31. Page 78, Section 7.3, line -7 should read: “for positive integers i, j such that $i < j \leq n/2$ ”
32. Page 78, Theorem 7.3.1, line 1 should read: “Fix positive integers i, j, k such that $i < j \leq n/2$.”
33. Page 78, Theorem 7.3.1, line 4 should read: “is Laurent in the first n terms,”
34. Page 90, Section 9.1, line 3 should read:

$$T(V) = \mathbb{C} \oplus V \oplus (V \otimes V) \oplus \dots$$

35. Page 91, line 4 should read (deleting second ‘if’): “An element x of $\bigwedge^k(V)$ is said to be *decomposable* if it is of the form”
36. Page 91, Equation (9.1) should read:

$$\sum_{r=0}^k (-1)^r p_{i_1, i_2, \dots, i_{k-1}, j_r} p_{j_0, \dots, \hat{j}_r, \dots, j_k} = 0,$$

37. Page 92, line 1, “where the sum is taken over all tuples satisfying” should read: “**where**”.
38. Page 106, Reference 96. The author is **A. Hubery**.

Contributors to this list: Bing Duan, Lisa Lamberti, Amit Shah and Yang Yang. Further corrections welcome. Many thanks!
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REFERENCES

- [1] R. J. Marsh, Lecture notes on cluster algebras. Zurich Lectures in Advanced Mathematics. European Mathematical Society (EMS), Zürich, 2013. ii+117 pp. ISBN: 978-3-03719-130-9