

MATH 0111 Elementary Differential Calculus, Homework 1

To be handed in on **Thursday 11 October**

SHOW YOUR WORKING. DO NOT USE YOUR CALCULATOR

1. For each of the following straight lines, write down its gradient and intercept with the y -axis.

a. $y = 2 + 4x$, b. $x - y = 5$, c. $y = 6 - 3x$,
d. $7x + 9y + 1 = 0$, e. $5y - 8x = 2$, f. $-x + 2y = 3$.

2. Find the equations of the lines joining the following pairs of points.

a. $(3, 3)$, $(1, 5)$, b. $(-1, 2)$, $(0, 6)$, c. $(4, -1)$, $(4, 4)$ (*draw it!*),
d. $(1, 7)$, $(2, 9)$, e. $(3, 1)$, $(7, 1)$.

3. Find the equations of the lines with given gradients passing through the given points.

a. 3, $(1, 4)$, b. $-\frac{1}{3}$, $(6, -1)$, c. 7, $(-1, 2)$,
d. $\frac{1}{4}$, $(0, 2)$, e. 10, $(0, 0)$.

4. For each of the lines in Question 1 find the gradient of lines perpendicular to it.
5. Find the equation of the line parallel to the line through $(1, 3)$ and $(3, 7)$, that passes through $(1, 0)$.
6. Find the equation of the line passing through the midpoint of the line joining $(1, 1)$ and $(3, 5)$, that is perpendicular to it. (First find the mid-point!)

PRACTICE EXERCISES, NOT TO BE HANDED IN

1. For each of the following straight lines, write down its gradient and intercept with the y -axis.

a. $y = 3 + 5x$, b. $y = 4 - 2x$, c. $x - y = 1$,
d. $3x + 4y + 5 = 0$, e. $2y - 5x = 7$, f. $5x - y = 2$.

2. Find the equations of the lines joining the following pairs of points.

a. $(2, 3)$, $(1, 5)$, b. $(0, 2)$ $(-1, 6)$, c. $(-3, -1)$, $(2, 4)$,
d. $(2, 5)$, $(2, 9)$ (*draw it!*), e. $(1, 3)$, $(7, 3)$.

3. Find the equations of the lines with given gradients passing through the given points.

a. 3, $(1, 7)$, b. $-\frac{1}{4}$, $(2, -1)$, c. 3, $(-5, 2)$,
d. $\frac{1}{2}$, $(1, -3)$, e. -2 , $(0, 0)$.

4. For each of the lines in Question 1 find the gradient of lines perpendicular to it.
5. Find the equation of the line parallel to the line through $(2, 3)$ and $(1, 5)$, that passes through $(2, 5)$.
6. Find the equation of the line passing through the midpoint of the line joining $(2, 4)$ and $(4, 10)$, that is perpendicular to it. (First find the mid-point!)

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MATH 0111 Elementary Differential Calculus, Homework 2

To be handed in on **Thursday 18 October**

SHOW YOUR WORKING

1. (a) Convert to radians: 240° , 18° , 720° . (b) Convert to degrees: $\pi/12$, $4\pi/5$, $9\pi/4$.

2. In each of the following questions, the triangle ABC is a right-angled triangle (drawn below, but not to scale) with the right angle at B. Given the information provided, find all the missing sides and angles. You need not use a calculator: leave expressions like $\sin 50^\circ$ in your answer.

(a) angle $A = 35^\circ$ and $AC = 13\text{cm}$; (b) angle $A = 3\pi/8$ and $BC = 5\text{cm}$.

3. Find all solutions for θ in the given range:

(a) $\cos \theta = \sqrt{3}/2$, $0 \leq \theta \leq 4\pi$;

(b) $\sin \theta = -\frac{1}{2}$, $-\pi \leq \theta \leq \pi$; (c) $\tan \theta = -1$, $0 \leq \theta \leq 2\pi$.

PRACTICE EXERCISES, NOT TO BE HANDED IN

1. (a) Convert to radians: 120° , 225° , 330° . (b) Convert to degrees: $\pi/8$, $3\pi/4$, $5\pi/3$.

2. In each of the following questions, the triangle ABC is a right-angled triangle (drawn below, but not to scale) with the right angle at B. Given the information provided, find all the missing sides and angles. You need not use a calculator: leave expressions like $\sin 50^\circ$ in your answer.

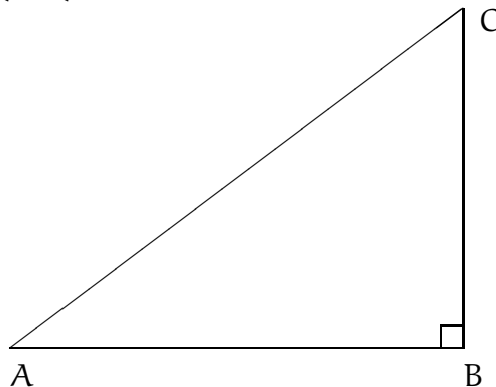
(a) angle $A = 50^\circ$ and $AC = 11\text{cm}$; (b) angle $A = \pi/8$ and $BC = 3\text{cm}$.

3. Find all solutions for θ in the given range:

(a) $\sin \theta = \sqrt{3}/2$, $0 \leq \theta \leq 3\pi$;

(b) $\cos \theta = 1$, $-\pi \leq \theta \leq \pi$;

(c) $\tan \theta = -1/\sqrt{3}$, $-\pi \leq \theta \leq 2\pi$.



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MATH 0111 Elementary Differential Calculus, Homework 3

To be handed in on **Thursday 25 October**

SHOW YOUR WORKING. DO NOT USE YOUR CALCULATOR

Simplify the following, expressing your answer in index form:

1. $a^3 \times a^2 \times a^5$,
2. $(a \times a^6)/a^4$,
3. $a^2 \times a^5 \times a^{-2}$,
4. $\frac{(b^{1/2})^4 \times b^{3/2}}{b^{5/2}}$,
5. $\frac{(ab)^4 \times a^3 \times b^6}{b^4 \times a^3}$,
6. $\sqrt{a^8 b^{12}}$,
7. $\sqrt[3]{a^6 b^{15}}$,
8. $(a^3/b^4)^{-2}$,
9. $\frac{(a^8 y^4)^{1/4} \times (y^4)^{1/2}}{(x^2 y^8)^{1/4} \times x^{-1}}$,
10. $(a^5 \times b^{-2})^0$.

Evaluate the following, leaving your answers as whole numbers or fractions:

11. $(3/4)^4$,
12. $(64/27)^{1/3}$,
13. $(64/27)^{-1/3}$,
14. $16^{3/4}$,
15. $125^{-1/3}$,
16. $6^6/4^3$,
17. $(\sqrt[3]{64})^{-1}$,
18. $\sqrt{4^4}$,
19. $4^{50}/8^{33}$,
20. $(\sqrt{13})^6$,
21. $2001^{100}/2001^{99}$.

22. Calculate $\frac{(ab)^2}{b^{1/2}}$ when $a = 2$ and $b = 9$.

PRACTICE EXERCISES, NOT TO BE HANDED IN

Simplify the following, expressing your answer in index form:

1. $\frac{b^3 \times b^5}{b^7}$,
2. $\frac{2^3 \times 4^2 \times 3^6}{2 \times 4^3 \times 3^5 \times 5}$,
3. $\frac{2^3 a^3 b^7}{2^2 a^4 b^2}$,
4. $(x^2)^{-3/2} \times (y^9)^{1/3}$,
5. $(a^2 \times d^4 \times (ac)^3)/(cd)^5$,
6. $\sqrt[3]{a^9 b^{15}}$,
7. $\frac{(2xy)^3 \times (y^4)^{1/2}}{(x^8 y^4)^{1/4}}$,
8. $((x^{1/2}y)^4 \times x^2)/(x^2 y^2)^2$.

Evaluate the following, leaving your answers as whole numbers or fractions:

9. $(3/6)^2$,
10. $(81/16)^{1/4}$,
11. $8^{-5/3}$,
12. $(\sqrt[4]{16})^{-5}$,
13. $\sqrt[3]{6}/36^{1/6}$,
14. $(30^9/200^3)^{1/3}$,
15. $(\sqrt[3]{17})^6$,
16. $18^{10}/36^8$.

17. Calculate $a^2 b^3 c^{-1/3}$ when $a = 3$, $b = 2$ and $c = 27$.

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MATH 0111 Elementary Differential Calculus, Homework 4

To be handed in on **Thursday 1 November**

SHOW YOUR WORKING. DO NOT USE YOUR CALCULATOR

Multiply out the following expressions:

$$1. (x - 4z)(2x - 3z^2), \quad 2. (y - x)^3, \quad 3. (x^2 - 2y)(x^2 + 2y).$$

Express the following as fractions with a single denominator:

$$4. \frac{2}{a-1} + \frac{3}{a+5}, \quad 5. \frac{y}{x+2} + \frac{x}{y-4}.$$

Factorise the following expressions:

$$\begin{array}{lll} 6. x^2 + 7x + 6, & 7. x^2 - 9x + 20, & 8. z^2 - 36, \\ 9. y^2 + y - 42, & 10. x^4 - 16x^2, & 11. 15 + 8x + x^2, \\ 12. 2x^3 - 4x^2 - 16x, & 13. 5y^2 - 20. & \end{array}$$

Solve the following equations.

$$\begin{array}{lll} 14. x^2 - 10x + 21 = 0, & 15. x^2 - 14x + 49 = 0, & 16. a^2 + 4a = 5, \\ 17. z^2 + 10z + 16 = 0, & 18. x^3 - 2x^2 + x = 0, & 19. 2z^2 - 12z + 18 = 0. \end{array}$$

PRACTICE EXERCISES, NOT TO BE HANDED IN

Multiply out the following expressions:

$$1. (5x + y)(x - 2y), \quad 2. (x + y^2)(x - y^2), \quad 3. (x + y)(x - y)^2.$$

Express the following as fractions with a single denominator:

$$4. \frac{1}{x+3} + \frac{1}{x-1}, \quad 5. \frac{9a}{a-2} - \frac{2(a-9)}{a+3}.$$

Factorise the following expressions:

$$\begin{array}{lll} 6. x^2 + 5x - 6, & 7. y^2 - 10y + 21, & 8. x^2 - 10x + 25, \\ 9. z^2 - 81, & 10. 3x^2 + 18x - 48, & 11. x^4 + 9x^3 + 14x^2. \end{array}$$

Solve the following equations.

$$\begin{array}{lll} 12. y^2 - 9y = 0, & 13. x^2 + 12x + 36 = 0, & 14. z^2 + 7z - 8 = 0, \\ 15. x^2 - 8x + 15 = 0, & 16. 4x^2 - 24x + 20 = 0, & 17. 3x^3 - 3x^2 - 90x = 0. \end{array}$$

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MATH 0111 Elementary Differential Calculus, Homework 5

To be handed in on **Thursday 8 November**

SHOW YOUR WORKING

Differentiate the following functions of x .

1. x^6 ,

2. x^{60} ,

3. $2x^{20}$,

4. x^{-4} ,

5. $3x^{-2}$,

6. $2x^{-12}$,

7. $x^{9/2}$,

8. $\frac{1}{4}x$,

9. $4x^{-1/2}$,

10. $6x^{1/3}$,

11. $x^2 + 5x + 8$,

12. $x^3 + 4x^2 - 3x + 1$,

13. $2x^4 + 3x^{-2} + x^{1/2}$,

14. $x^3(x + 2)$,

15. $\sqrt[4]{x}$,

16. $\frac{4x^4 + x^2 + 2}{x^2}$

(hint: this is the same as $4x^2 + 1 + 2x^{-2}$),

17. $-2x^2\sqrt{x}$

(hint: this is the same as $-2x^{5/2}$).

PRACTICE EXERCISES, NOT TO BE HANDED IN

Differentiate the following functions of x .

1. x^5 ,

2. x^{100} ,

3. $5x^{100}$,

4. x^{-2} ,

5. $-\frac{1}{3}x^{-3}$,

6. $4x^{-17}$,

7. $x^{5/2}$,

8. $5x^{2/3}$,

9. $2x^{-1/2}$,

10. $x^3 + 3x + 1$,

11. $5x^4 - 2x^2 - 4x + 2$,

12. $3x^3 + \frac{1}{2}x^2 + 2x^{1/2}$,

13. $x^3(2 + x^{-1})$,

14. $3x^2 + 2x^{-1} + 4x^{-1/2}$,

15. $\frac{3x^4 + 6x^2 + 1}{x}$

(hint: this is the same as $3x^3 + 6x + x^{-1}$),

16. $x^2\sqrt{x}$

(hint: this is the same as $x^{5/2}$).

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MATH 0111 Elementary Differential Calculus, Homework 6

To be handed in on **Thursday 15 November**

SHOW YOUR WORKING

Differentiate the following functions of x .

- | | |
|---|--|
| 1. $(4x + 1)^3$, | 2. $(x^3 + 1)^5$, |
| 3. $(2x + 1)^{1/2}$, | 4. $(3x^2 + 1)^{-1}$, |
| 5. $(x^3 + 1)^2 + (x^4 + 1)^4$, | 6. $(3x^3 + 1)^{-1/3} + (x^2 - 1)^2$, |
| 7. $\frac{1}{x^4 + 5x - 2}$, | 8. $x^6(x^2 - 1)$, |
| 9. $(1 + x)^4(1 - x^2)$, | 10. $x^3(x + 1)^{1/2}$, |
| 11. $(2x + 1)^5(3x + 2)^{1/2}$, | 12. $\frac{x^3 + 7}{7x^2 - 1}$, |
| 13. $\left(\frac{x - 1}{2x - 1}\right)^{1/2}$. | |

PRACTICE EXERCISES, NOT TO BE HANDED IN

Differentiate the following functions of x .

- | | |
|--|---|
| 1. $(2x^2 + 1)^5$, | 2. $(3x + 1)^4$, |
| 3. $(3x + 2)^{1/2}$, | 4. $(3x^2 + 4)^5$, |
| 5. $(x^2 + 1)^{1/2} + (x^3 + 1)^{1/2}$, | 6. $(2x^3 + 1)^{-3/2} + (2x + 1)^{1/2}$, |
| 7. $\frac{1}{2x^3 + 3x^2 - 1}$, | 8. $x^4(x + 1)^5$, |
| 9. $(1 - x)^3(1 + x^2)$, | 10. $(x^2 + 3x + 2)\sqrt{x}$, |
| 11. $(3x^2 + 5)^{1/2}(x + 1)^{3/2}$, | 12. $\frac{2x^3 + 2}{2x^2 + 4}$, |
| 13. $\left(\frac{x + 1}{x - 1}\right)^{1/2}$. | |

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MATH 0111 Elementary Differential Calculus, Homework 7

To be handed in on **Thursday 22 November**

SHOW YOUR WORKING

For each of the following functions do the following:

- Find the turning points, and identify whether they are local maxima, local minima, or points of inflexion.
- Draw a rough sketch of the graph.
- Find the equation of the tangent line at the point where $x = 1$.

1. $y = 2x^2 - 8x + 1,$

2. $y = (x + 3)(7 - x),$

3. $y = x^3 - 3x^2 - 9x + 1,$

4. $y = x^7,$

5. $y = x^4 - 4x.$

PRACTICE EXERCISES, NOT TO BE HANDED IN

Do as above for:

1. $y = 3 + 4x - 4x^2,$

2. $y = (3x + 1)(4 - x),$

3. $y = x^3 - 12x + 1,$

4. $y = x^3 - \frac{9}{2}x^2 + 6x + \frac{1}{2},$

5. $y = x^4 - \frac{8}{3}x^3.$

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MATH 0111 Elementary Differential Calculus, Homework 8

To be handed in on **Thursday 29 November**

SHOW YOUR WORKING

- Find (without using a calculator) the values of $\sin 5\pi/12$, $\cos 5\pi/12$ and $\tan 5\pi/12$, using the formulas for $\sin(\theta + \phi)$ and $\cos(\theta + \phi)$ and the fact that $5\pi/12 = \pi/6 + \pi/4$.
- Solve (for θ) the equation $2 \cos^2 \theta = 2 - \sin \theta$, $0 \leq \theta \leq 2\pi$ (remember that $\sin^2 \theta + \cos^2 \theta = 1$).
- Suppose that $0 < \theta < \pi/2$ and $\tan \theta = 3$. Find $\sin \theta$, $\cos \theta$ and $\sin 2\theta$.
- Sketch the graph of $y = \cos 3x$ for $0 \leq x \leq \pi$. Make a table of the values of y when x is a multiple of $\pi/6$.
- Differentiate the following with respect to x .

$$\begin{array}{lll} \text{(a)} \quad y = \cos^4 x, & \text{(b)} \quad y = \sin(3x^2 + 4), & \text{(c)} \quad y = \frac{1}{\sin^2 x}, \\ \text{(d)} \quad y = \frac{\sin x}{x}, & \text{(e)} \quad y = \sin^2 x + \cos^2 x, & \text{(f)} \quad y = x \tan 2x. \end{array}$$

PRACTICE EXERCISES, NOT TO BE HANDED IN

- Find (without using a calculator) the values of $\sin 7\pi/12$, $\cos 7\pi/12$ and $\tan 7\pi/12$, using the formulas for $\sin(\theta + \phi)$ and $\cos(\theta + \phi)$ and the fact that $7\pi/12 = \pi/3 + \pi/4$.
- Show that if $-\pi < \theta < \pi$ then $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta$.
- Suppose that $0 < \theta < \pi/2$ and $\tan \theta = 4/3$. Find $\sin \theta$, $\cos \theta$ and $\sin 2\theta$.
- Sketch the curve $y = \sin 2x$ for $0 \leq x \leq 2\pi$. Make a table of the values of y when x is a multiple of $\pi/4$.
- Differentiate the following with respect to x .

$$\begin{array}{lll} \text{(a)} \quad y = \sin^3 x - \cos 2x, & \text{(b)} \quad y = \sin\left(\frac{\pi}{2} - x^2\right), & \text{(c)} \quad y = \cos(x^2 - x^{3/2}), \\ \text{(d)} \quad y = x^2 \tan x, & \text{(e)} \quad y = \sin(x^2 + 1)^{1/2}, & \text{(f)} \quad y = \sin(9x^2). \end{array}$$

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MATH 0111 Elementary Differential Calculus, Homework 9

To be handed in on **Thursday 6 December**

SHOW YOUR WORKING

1. Find (a) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$, (b) $\arccos(0)$, (c) $\arctan(-\sqrt{3})$.

2. Differentiate the following with respect to x .

(a) $y = \frac{\cos x}{x^2 + 4}$,	(b) $y = \arcsin(3x - 1)$,
(c) $y = \tan(x^4) + \arctan\left(\frac{1}{x}\right)$,	(d) $y = (4x - 7) \arccos(1 - x^2)$.

3. Find the following.

(a) $\log_3 27$,	(b) $\log_2 \sqrt{8}$,	(c) $\log_{16} 2$,
(d) $\log_{10} 0.01$,	(e) $\log_7 1$,	(f) $\log_{2002} 2002$.

PRACTICE EXERCISES, NOT TO BE HANDED IN

1. Find (a) $\arcsin\left(\frac{1}{2}\right)$, (b) $\arccos\left(\frac{1}{\sqrt{2}}\right)$, (c) $\arctan(-1)$.

2. Differentiate the following with respect to x .

(a) $y = \frac{\sin x}{x^2}$,	(b) $y = \sin((x^2 + 1)^{1/2}) + \arctan(x^2 + 1)$,
(c) $y = \arcsin(x^3)$,	(d) $y = x^2 \arccos(2x + 1)$.

3. Find the following.

(a) $\log_{10} 10\,000$,	(b) $\log_2 256$,	(c) $\log_3\left(\frac{1}{81}\right)$,
(d) $\log_{81} 3$,	(e) $\log_5 5$.	

MATH 0111 Elementary Differential Calculus, Homework 10

1. Differentiate the following with respect to x .

(a) $\ln(2x^4 + 1)$,	(b) $\ln(x + 2)^{3/4}$,	(c) $\tan(e^x)$,
(d) $x^4 \ln(3x)$,	(e) $e^{\sin x + \cos x}$,	(f) $\ln \arcsin x$,
(g) $e^{2x} \sin x$,	(h) $e^{4 \ln x}$.	

2. In each of the following cases, express $\frac{dy}{dx}$ in terms of x and y .

(a) $xy - x + 2y = 1$, (b) $x^2 + xy = y^3$, (c) $x^2y^3 = 2x - y$.

3. Find the tangent to the curve $x^2 + xy + y^2 = 7$ at the point $(2, 1)$.

4. Find the tangent to the curve $x \ln y = x^2 - y^2$ at the point $(1, 1)$.

Good manners of homework submissions

- Clearly write *your name* (exactly as it is known to the University) on the top of the first page of your work.
- Put your *student number* along with your name.
- If you submit more than one page please *staple all of them together*.
- Avoid usage of plastic folders.