

Elementary Differential and Integral Calculus

FORMULA SHEET

Exponents

$$x^a \cdot x^b = x^{a+b}, \quad a^x \cdot b^x = (ab)^x, \quad (x^a)^b = x^{ab}, \quad x^0 = 1.$$

Logarithms

$$\ln xy = \ln x + \ln y, \quad \ln x^a = a \ln x, \quad \ln 1 = 0, \quad e^{\ln x} = x, \quad \ln e^y = y, \\ a^x = e^{x \ln a}.$$

Trigonometry

$$\cos 0 = \sin \frac{\pi}{2} = 1, \quad \sin 0 = \cos \frac{\pi}{2} = 0, \\ \cos^2 \theta + \sin^2 \theta = 1, \quad \cos(-\theta) = \cos \theta, \quad \sin(-\theta) = -\sin \theta, \\ \cos(A + B) = \cos A \cos B - \sin A \sin B, \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta, \\ \sin(A + B) = \sin A \cos B + \cos A \sin B, \quad \sin 2\theta = 2 \sin \theta \cos \theta, \\ \tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad 1 + \tan^2 \theta = \sec^2 \theta.$$

Inverse Functions

$$y = \sin^{-1} x \text{ means } x = \sin y \text{ and } -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}. \\ y = \cos^{-1} x \text{ means } x = \cos y \text{ and } 0 \leq y \leq \pi. \\ y = \tan^{-1} x \text{ means } x = \tan y \text{ and } -\frac{\pi}{2} < y < \frac{\pi}{2}. \\ y = x^{1/n} \text{ means } x = y^n. \quad y = \ln x \text{ means } x = e^y.$$

Alternative Notation

$$\arcsin x = \sin^{-1} x, \quad \arccos x = \cos^{-1} x, \quad \arctan x = \tan^{-1} x, \quad \log_e x = \ln x.$$

$$\text{Note: } \sin^{-1} x \neq (\sin x)^{-1}, \quad \cos^{-1} x \neq (\cos x)^{-1}, \quad \tan^{-1} x \neq (\tan x)^{-1}.$$

$$\text{However: } \sin^2 x = (\sin x)^2, \quad \cos^2 x = (\cos x)^2, \quad \tan^2 x = (\tan x)^2.$$

Lines

The line $y = mx + c$ has slope m .

The line through (x_1, y_1) with slope m has equation $y - y_1 = m(x - x_1)$.

The line through (x_1, y_1) and (x_2, y_2) has slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ and equation $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$.

The line $y = mx + c$ is perpendicular to the line $y = m'x + c'$ if $mm' = -1$.

Circles

The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

The circle with centre (a, b) and radius r is given by $(x - a)^2 + (y - b)^2 = r^2$.

Triangles

In a triangle ABC:

$$\text{(Sine Rule)} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}; \quad \text{(Cosine Rule)} \quad a^2 = b^2 + c^2 - 2bc \cos A.$$

Pascal's Triangle

$(x + y)^2 = x^2 + 2xy + y^2$, $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and so on.

The coefficients in $(x + y)^n$ form the n th row of Pascal's triangle:

$$\begin{array}{ccccccc}
& & & & & & 1 \\
& & & & & & 1 & 1 \\
& & & & & 1 & 2 & 1 \\
& & & 1 & 3 & 3 & 1 \\
& 1 & 4 & 6 & 4 & 1 \\
& \dots & \dots & \dots & \dots & \dots & \dots
\end{array}$$

and so on.

Quadratics

If $ax^2 + bx + c = 0$, with $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Calculus

If $y = u + v$ then $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$. If $y = uv$ then $\frac{dy}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$.

If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \left\{ \frac{du}{dx}v - u\frac{dv}{dx} \right\} / v^2$.

$\int (u + v) dx = \int u dx + \int v dx$. $\int u \frac{dv}{dx} dx = uv - \int \frac{du}{dx} v dx$.

If y is a function of u where u is a function of x , then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} \quad \text{and} \quad \int y \frac{du}{dx} dx = \int y du.$$

Standard Derivatives and Integrals

If $y = x^a$ then $\frac{dy}{dx} = ax^{a-1}$; and $\int x^a dx = \frac{x^{a+1}}{a+1} + \text{constant}$ ($a \neq -1$).

If $y = \sin x$ then $\frac{dy}{dx} = \cos x$; and $\int \sin x dx = -\cos x + \text{constant}$.

If $y = \cos x$ then $\frac{dy}{dx} = -\sin x$; and $\int \cos x dx = \sin x + \text{constant}$.

If $y = \tan x$ then $\frac{dy}{dx} = \sec^2 x$; and $\int \tan x dx = \ln|\sec x| + \text{constant}$.

If $y = e^x$ then $\frac{dy}{dx} = e^x$; and $\int e^x dx = e^x + \text{constant}$.

If $y = \ln x$ then $\frac{dy}{dx} = \frac{1}{x}$; and $\int \frac{1}{x} dx = \ln|x| + \text{constant}$.

If $y = \sin^{-1} x$ then $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$; and $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + \text{constant}$.

If $y = \cos^{-1} x$ then $\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$.

If $y = \tan^{-1} x$ then $\frac{dy}{dx} = \frac{1}{1+x^2}$; and $\int \frac{1}{1+x^2} dx = \tan^{-1} x + \text{constant}$.