**Elementary Differential and Integral Calculus**

**FORMULA SHEET**

**Exponents**

$x^a \cdot x^b = x^{a+b}, \quad a^x \cdot b^x = (ab)^x, \quad (x^a)^b = x^{ab}, \quad x^0 = 1.$

**Logarithms**

$\ln xy = \ln x + \ln y, \quad \ln x^a = a \ln x, \quad \ln 1 = 0, \quad e^{\ln x} = x, \quad \ln e^y = y,$
$a^x = e^{x \ln a}.$

**Trigonometry**

$\cos 0 = \sin \frac{\pi}{2} = 1, \qquad \sin 0 = \cos \frac{\pi}{2} = 0,$
$\cos^2 \theta + \sin^2 \theta = 1, \qquad \cos(-\theta) = \cos \theta, \qquad \sin(-\theta) = -\sin \theta,$
$\cos(A + B) = \cos A \cos B - \sin A \sin B, \qquad \cos 2\theta = \cos^2 \theta - \sin^2 \theta,$
$\sin(A + B) = \sin A \cos B + \cos A \sin B, \qquad \sin 2\theta = 2 \sin \theta \cos \theta,$
$\tan \theta = \dfrac{\sin \theta}{\cos \theta}, \qquad \sec \theta = \dfrac{1}{\cos \theta}, \qquad 1 + \tan^2 \theta = \sec^2 \theta.$

**Inverse Functions**

$y = \sin^{-1} x$ means $x = \sin y$ and $-\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$.
$y = \cos^{-1} x$ means $x = \cos y$ and $0 \leqslant y \leqslant \pi$.
$y = \tan^{-1} x$ means $x = \tan y$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$.
$y = x^{1/n}$ means $x = y^n$. $\qquad y = \ln x$ means $x = e^y$.

**Alternative Notation**

$\arcsin x = \sin^{-1} x, \quad \arccos x = \cos^{-1} x, \quad \arctan x = \tan^{-1} x, \quad \log_e x = \ln x.$
Note: $\sin^{-1} x \neq (\sin x)^{-1}, \quad \cos^{-1} x \neq (\cos x)^{-1}, \quad \tan^{-1} x \neq (\tan x)^{-1}.$
However: $\sin^2 x = (\sin x)^2, \quad \cos^2 x = (\cos x)^2, \quad \tan^2 x = (\tan x)^2.$

**Lines**

The line $y = mx + c$ has slope $m$.
The line through $(x_1, y_1)$ with slope $m$ has equation $y - y_1 = m(x - x_1)$.
The line through $(x_1, y_1)$ and $(x_2, y_2)$ has slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ and equation $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$.
The line $y = mx + c$ is perpendicular to the line $y = m'x + c'$ if $mm' = -1$.

**Circles**

The distance between $(x_1, y_1)$ and $(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.
The circle with centre $(a, b)$ and radius $r$ is given by $(x - a)^2 + (y - b)^2 = r^2$.

**Triangles**

In a triangle $ABC$:

$$\text{(Sine Rule)} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}; \qquad \text{(Cosine Rule)} \quad a^2 = b^2 + c^2 - 2bc \cos A.$$

**Pascal's Triangle**

$(x+y)^2 = x^2 + 2xy + y^2, \quad (x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ and so on.

The coefficients in $(x+y)^n$ form the $n$th row of Pascal's triangle:

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & 3 & & 1 \\
1 & 4 & & 6 & & 4 & 1 \\
\multicolumn{7}{c}{\cdots\cdots\cdots\cdots}
\end{array}
$$

and so on.

**Quadratics**

If $ax^2 + bx + c = 0$, with $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Calculus**

If $y = u + v$ then $\dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$. If $y = uv$ then $\dfrac{dy}{dx} = \dfrac{du}{dx}v + u\dfrac{dv}{dx}$.

If $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \left\{\dfrac{du}{dx}v - u\dfrac{dv}{dx}\right\} \Big/ v^2$.

$\displaystyle\int (u+v)\,dx = \int u\,dx + \int v\,dx. \qquad \int u\frac{dv}{dx}\,dx = uv - \int \frac{du}{dx}v\,dx.$

If $y$ is a function of $u$ where $u$ is a function of $x$, then

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} \quad \text{and} \quad \int y\frac{du}{dx}\,dx = \int y\,du.$$

**Standard Derivatives and Integrals**

If $y = x^a$ then $\dfrac{dy}{dx} = a\,x^{a-1}$; and $\displaystyle\int x^a\,dx = \frac{x^{a+1}}{a+1} + \text{ constant} \quad (a \neq -1)$.

If $y = \sin x$ then $\dfrac{dy}{dx} = \cos x$; and $\displaystyle\int \sin x\,dx = -\cos x + \text{ constant}$.

If $y = \cos x$ then $\dfrac{dy}{dx} = -\sin x$; and $\displaystyle\int \cos x\,dx = \sin x + \text{ constant}$.

If $y = \tan x$ then $\dfrac{dy}{dx} = \sec^2 x$; and $\displaystyle\int \tan x\,dx = \ln|\sec x| + \text{ constant}$.

If $y = e^x$ then $\dfrac{dy}{dx} = e^x$; and $\displaystyle\int e^x\,dx = e^x + \text{ constant}$.

If $y = \ln x$ then $\dfrac{dy}{dx} = \dfrac{1}{x}$; and $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + \text{ constant}$.

If $y = \sin^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$; and $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + \text{ constant}$.

If $y = \cos^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-x^2}}$.

If $y = \tan^{-1} x$ then $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$; and $\displaystyle\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + \text{ constant}$.