Review of 'The Poincaré Conjecture: In search of the shape of the Universe by Donal O'Shea'

Before I begin I should first confess that many years ago I considered writing a 'popular' account of the Poincaré Conjecture (PC). Whilst browsing in the library as an undergraduate I had discovered *The Shape of Space* by Weeks, [9], a book I recommend to my own undergraduates since it takes a reader as far as Thurston's Geometric Conjecture and introduces the basics of the PC without using any really difficult mathematics. As a postgraduate the Mathematical Intelligencer article by Steve Smale introduced more of the story to me. Subsequently I learned about Poincaré's life and the obsession amongst mathematicians that his conjecture caused and thought it merited a popularized account. This was before Perelman announced his proof and refused to publish it in a journal, before he refused to claim the $1 million dollar Clay prize and before he refused the Fields medal. And before Yau began legal proceedings against the New Yorker over an alleged slanderous article. The story just gets better and better.

An outline of this story was described in a chapter in Devlin's book on the Clay Institute's Millennium Problems, [2], but certainly warranted a full book. Of course, once Perelman had posted his papers I knew I would competing with a number of people to produce that book and as I did not have time to enter the race I waited for others. First out of the blocks is O'Shea's book under review here closely followed by George Szpiro's account, [7].

O'Shea's approach is similar to Weeks', he utilizes the idea of the shape of space to introduce the concepts surrounding the Conjecture. Since his intended audience is non-mathematicians he concentrates on relating stories and educating the reader regarding geometry. Mathematicians wanting a more detailed treatment should see [1] or [3].

To view the story as the history of the search for the shape of the universe is pedagogically sound (as Week's book showed) and this perspective does give a context for the solution but it is perhaps misleading. Certainly I do not believe that those on the front line of the assault on the the PC had any real interest in applications to cosmology. Even with the proof of the PC no-one can say anything meaningful about the the shape of the universe. That would first require that cosmologists demonstrate that the universe is simply connected - which is difficult - and that it is a 3-manifold - which is really difficult after Einstein melded space and time into a 4-dimensional space.

Having said that the book is well worth reading and recommending to mathematics students and to non-mathematical friends and family. To begin with, however, I shall cover a serious defect. The PC is about topology and geometry, extremely visual topics, and so any book should contain many illustrations, as indeed this one does. Unfortunately, they are of extremely poor quality. For example, the reproduction of maps in chapter 2 is done in such a low resolution that sharpness is lost, the maps look out of focus and the text within them is rendered unreadable. Worse still throughout the book illustrations of the concepts are highly pixelated and are reminiscent of the computer graphics from video games in the 1980s. How a serious publishing house can publish this without embarrassment is beyond me. It would seem that, paradoxically, the proliferation of graphic packages for creating pictures has led to a reduction of quality of graphics in books.

Furthermore, some key concepts would benefit from illustrations. For example, on pages 83-86 the concept of generalizing Euclidean space to higher dimensions benefits from pictures (in this writer's experience with students). Similarly, the concept of the fundamental group (page 131) is made so much clearer through diagrams (again from experience).

Though one could argue that the cosmology applications are not especially relevant, they do provide the author with a central theme and a hook on which to hang anecdotes and stories. Thus the book progresses from the Greeks, Euclid, Pythagoras, and Eratosthenes, through the non-Euclidean revolution, through Gauss and Riemann to Poincaré. The last quarter of the main text relates the hundred years following the conjecture up to Perelman's papers and his refusal of a Fields Medal at the Madrid ICM in 2006. The more recent part of the story involving Sylvia Nasar and David Gruber's New Yorker article, [4], and S.-T. Yau's subsequent legal action are mentioned only briefly. Along the way, interesting diversions into the destruction of German superiority in mathematics and the rise of the USA are discussed. Again not strictly relevant but they put the evolution of the ideas into context. One particular exchange involving a battle for advantage between the German Klein and the French Poincaré brings to life the fight within mathematics for precedent and demonstrates that mathematicians are not merely disinterested parties working together towards a common goal for the greater good of mankind.

Steve Smale solving the higher dimensional version of the PC on the beaches of Rio de Janeiro is one of my favourite part of the PC story but it is dispatched here in a few sentences in a single paragraph. Similarly Rourke and Rego's failed attack on the problem is not elaborated upon. Gary Taubes' article [8] brings to life the story of how Rourke sinned against the mathematics community and contacted the press before subjecting the proof to peer scrutiny. His trip to Berkley to defend his ultimately fallacious proof and how was subsequently run out of town by Kirby's graduate student attack dogs is explained in detail. (Despite being one of its many victims Rourke has remained involved with the PC and has an interesting essay on the Poincaré conjecture on his website, [5].)

Turning to the books many strengths, the explanations of the mathematics are well done and not overly technical. Those that know O'Shea will recognize his lively personality and sense of humour; he employs the gruesome analogy of a misanthropic surgeon carrying out surgery on intestines to exemplify Dehn Surgery. O'Shea had intended his book for an intelligent general reader and has provided helpful notes for the more mathematically minded. This means that students can benefit from reading this book twice. Once as an undergraduate and later as a postgraduate when the more advanced concepts such as homology are available to them. In addition to these notes, the reader has the benefit of a Glossary of Terms, a Glossary of Names, a Timeline, a Bibliography, Further Reading as well as the usual index.

The story of the Poincaré Conjecture is not finished and further surprises could be in store. The Clay Institute has yet to distribute the $1 million bounty that was placed on the problem's solution. Perelman has clearly stated that he does not need nor want the money. The rules allow the prize to be shared amongst those who took us on the road to the solution and since Perelman was completing the program of Hamilton, who was following a suggestion of the late great Jim Eells, there are many interesting possibilities for the final destination of the money.

With the story unfinished the book will need to be updated and should get a well deserved second edition (hopefully with the diagrams done properly). This is not the book I would have written - I don't intend that as a criticism - I will certainly recommend it to students and non-mathematicians as an enjoyable, well-written book.

References

[1] http://www.claymath.org/millennium/Poincare_Conjecture/perelman+expositions.php.

[2] Keith Devlin, The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time, 2005.

[3] John Morgan and Gang Tian, *Ricci Flow and the Poincaré Conjecture*, arXiv:math/0607607v2 and to be published by AMS.

[4] Sylvia Nasar and David Gruber, Manifold destiny, The New Yorker, August 28, 2006, 4457.

[5] Colin Rourke, http://www.maths.warwick.ac.uk/∼cpr/

[6] Steve Smale, *What really happened on the beaches of Rio*, Mathematical Intelligencer, 12 (1990), no. 2, 44–51.

[7] George Szpiro, *Poincare's Prize: The Hundred-year Quest to Solve One of Math's Greatest Puzzles*, 2007.

[8] Gary Taubes, What happens when hubris meets nemesis, Discover 8, July 1987, 66-77.

[9] Jeffrey Weeks, *The Shape of Space*, 1985.

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