‘Curves and Singularities’ by Bruce and Giblin

Many years ago, at the tail end of the Eighties, when Margaret Thatcher was still in power in Britain and the Communists ruled most of Eastern Europe, I studied for an undergraduate Mathematics degree. In those days students did not have the wide choice of courses they have today and so I was happy to have an option during my final year to take ‘Curves and Catastrophes.’ It wasn’t a core module and only four of my fellow students took it with me. The recommended textbook was ‘Curves and Singularities’ by J.W. Bruce and P.J. Giblin, the lecturer being the first author.

The difference between the title of the course and that of the book is worth noting. There is a wide gulf between Singularity Theory and Catastrophe Theory. The former is a respectable descendent of Newton’s Calculus. The latter is a discredited fad, popular in the 1960s and 70s. Books and newspaper articles claimed it was a revolution to rival calculus. It promised to predict discontinuous phenomena from continuous inputs – for example, predicting the capsize of a ship, or explaining how liquids become gases. This would, or so it was hoped, be better than standard calculus at predicting the future – ‘an intellectual revolution’ proclaimed the back cover of one book. This promised revolution never came, of course. Unsupported assumptions are made in the applications, claimed the critics. It’s all been done before in so-and-so’s work, they said. The allure of the theory began to wane and hence few today would call themselves a Catastrophe Theorist without blushes. But at the end of the 80s the name was still useful for attracting undergraduate students and hence its use in the title of a course that was really about singularities. Singularity Theory is far more respectable – the name was adopted, in part, to distance its practitioners from the wilder claims of the Catastrophe Theorists, such as the ability to predict prison riots, cure anorexia, etc. In Bruce and Giblin’s book respectability comes from applying Singularity Theory to the ancient study of curves and surfaces – a core subject within mathematics.

The book starts with some basic differential geometry, leads on to the Serret-Frenet formulas, and even defines manifolds (in the decent way – i.e., avoiding the ugly axiomatic method and instead going for the description as an immersion into a space). It progresses to envelopes. Envelopes are great – though at the time I didn’t realize. They can be defined in many ways but these insufficiently illuminate the idea: you draw a family of curves and your eye sees a curve that seems to bound these – a curve that is not actually drawn. For example, if you draw the normals to a parabola, you see a curve with a cusp. Another example is seen on the cover of the book – with many circles making an apple shape.

In the book, along with examples such as parallel and evolute curves, the two most important aspects of Singularity Theory – transversality and unfolding – are explained. Both ideas were emphasized by René Thom, one of the important originators of the Catastrophe Theory revolution.

The majority of us do not get to participate in a revolution – we just get to stand and stare. Studying the book I had the opportunity to stare as communism in Eastern Europe spectacularly collapsed. I stared as thousands massed in Prague’s Wenceslas Square to demand the end to communism. I watched as the destruction of the Berlin Wall turned into a dance party. Hundreds of wretched Trabant cars, the potent symbol of the failure of communism, rolled through the freshly redundant checkpoints and into West Berlin. All this formed a backdrop to my work routine: finish exercises from the book and then watch the late night news with my flatmates to witness the latest regime to fall. All of us hoping that it would not end with a massacre as we had seen the year before in China.

Like all revolutions there is the victory party and then the hangover from the night before. Catastrophe Theory was no exception and suffered a huge backlash when its initial promise did not produce the goods (a bit like communism). Many today will criticize Thom's work on Catastrophe Theory as a dead end that will be forgotten. They will also say his reputation is assured because of his Fields Medal winning early work on cobordism characteristic classes. My own thoughts are that one day we will realize how much Thom was ahead of his time. Forget how it all ended, with Thom retreating into philosophical ramblings along with the sterile French school of Derrida, his ideas swept from the battlefield by what he saw as Grothendieck's crushing technical superiority. True, Grothendieck school was victorious – no book on algebraic geometry can ignore his influence, but while Grothendieck's ideas were hard to follow because he was creating incredibly difficult mathematics, Thom's ideas were harder because he was unable to clearly communicate them, even with an avatar in the form of Christopher Zeeman. (Thom claimed that he couldn't put what he called 'his daydreams' into rigourous mathematics.) And so Catastrophe Theory faded to the proverbial footnote in history. Eventually, it was swept from university syllabuses like communism was swept away from Eastern Europe and was replaced by the new fad, the new revolution – Chaos Theory.

And what of Bruce and Giblin's book? Well, I am about to begin teaching second year students about curves and surfaces – it will be one of my recommended texts. I always suggest it to anyone who is interested in Singularity Theory, particularly PhD students. It contains most of the important ideas and as a bonus teaches some useful stuff about the differential geometry of curves. It is easy to follow and has plenty of exercises. What more could you want from a book?

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