

Analytic Methods for PDEs

Examples 2: quasilinear PDEs & system of PDEs

1. Solve the equation

$$x(u + y^2) \frac{\partial u}{\partial x} - y(u + x^2) \frac{\partial u}{\partial y} = (x^2 - y^2)u$$

subject to the condition that

$$u = 1 \quad \text{when} \quad x + y = 0.$$

2. Solve the equation

$$(y - u) \frac{\partial u}{\partial x} + (u - x) \frac{\partial u}{\partial y} = x - y$$

in $x > 0$, $y > 0$, subject to the condition that

$$u = 0 \quad \text{when} \quad xy = 1.$$

3. Solve the system of equations

$$\begin{aligned} \frac{\partial u_1}{\partial t} + 3 \frac{\partial u_1}{\partial x} - 6 \frac{\partial u_2}{\partial x} &= 0, \\ \frac{\partial u_2}{\partial t} + \frac{\partial u_1}{\partial x} - 4 \frac{\partial u_2}{\partial x} &= 0, \end{aligned}$$

subject to the initial condition

$$u_1 = \sin x, \quad u_2 = \cos x \quad \text{when} \quad t = 0.$$