

Analytic Methods for PDEs

Examples 1: linear & semilinear equations

1. The function $u(x, y)$ satisfies the linear first-order equation

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2xy$$

in $y > 0$, subject to

$$u = x^2 + 1 \quad \text{on} \quad y = 1, \quad 0 < x < 1.$$

In what region of the (x, y) -plane can u be fully determined? Find its value in this region.

2. Find the general solution to the equation

$$x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = (x + y)u$$

in $x > 0, y > 0$.

Hence, find the solution that satisfies $u = y^2$ on $x = 1$ ($y > 0$).

3. The function $u(x, y)$ satisfies the first-order partial-differential equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^2$$

in $y > 0$, subject to

$$u = x^2 + 1 \quad \text{on} \quad y = 0, \quad 0 \leq x \leq 1.$$

Indicate, on a suitable diagram, in what region of the (x, y) -plane u can be fully determined and find the solution in this region.