Global Properties of Magnetorotational Instabilities and Non-Linear Dynamo in Accretion Discs

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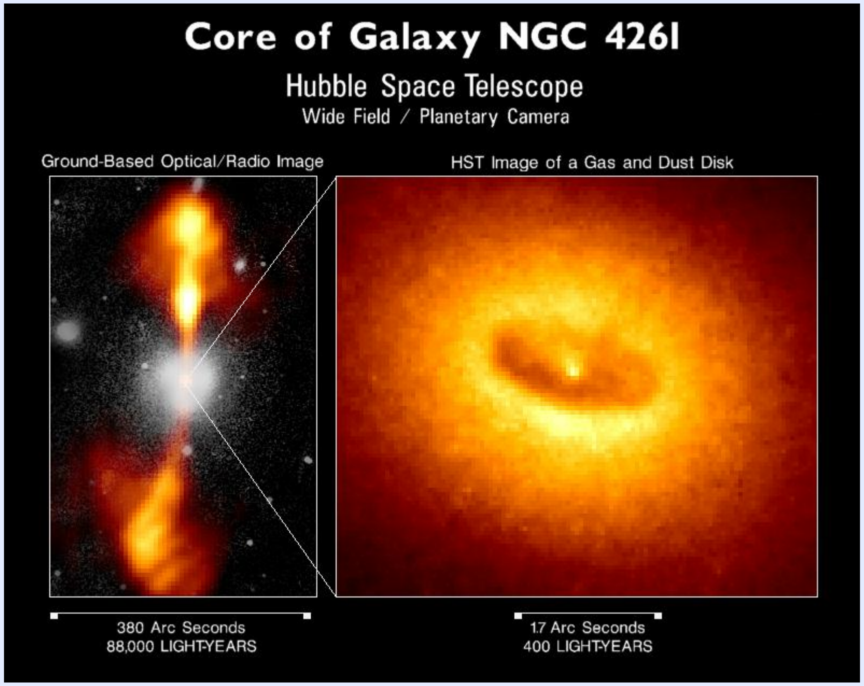
Accretion Discs



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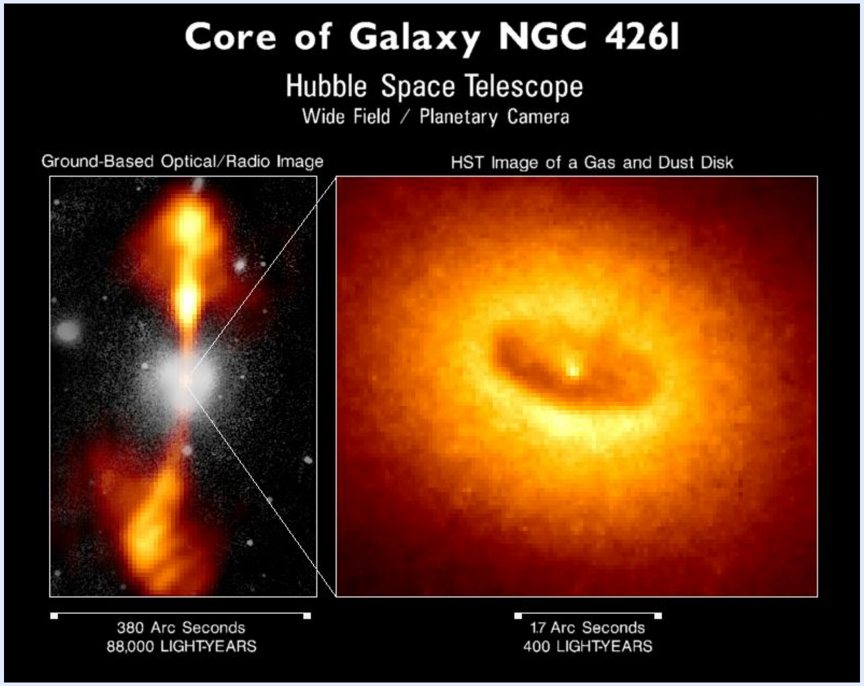

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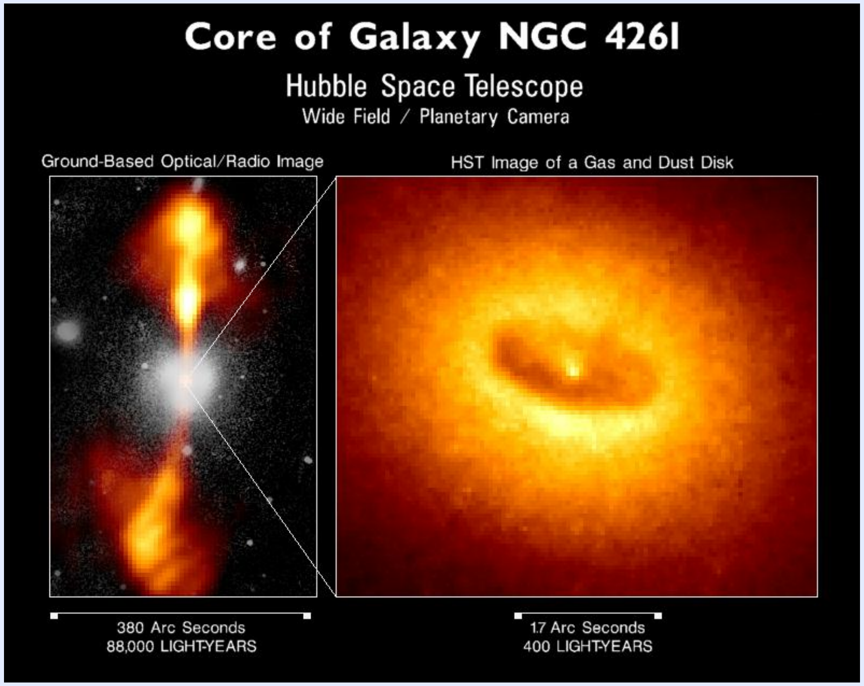

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Magnetic field $\equiv$ link between accretion and ejection

- MRI drives turbulence $\Rightarrow$ outward angular momentum transport
- Magneto-centrifugal ejection and B_φ-collimation

Origin of the large scale magnetic field $\langle \mathbf{B} \rangle$?

MRI & Non-Linear Dynamo

Magnetorotational Instability:

- Weakly magnetized ($\beta \gg 1$) and differential rotating flows unstable if $\mathrm{d}\Omega/\mathrm{d}r < 0$
- Differential rotation $\Rightarrow$ Free energy & MRI extremely powerful $\sigma \sim r|\mathrm{d}\Omega/\mathrm{d}r|$

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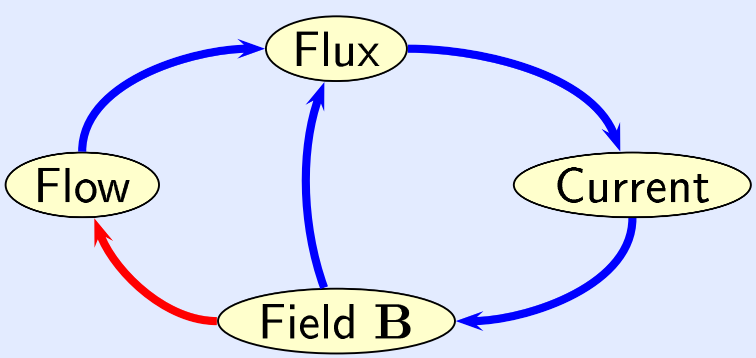

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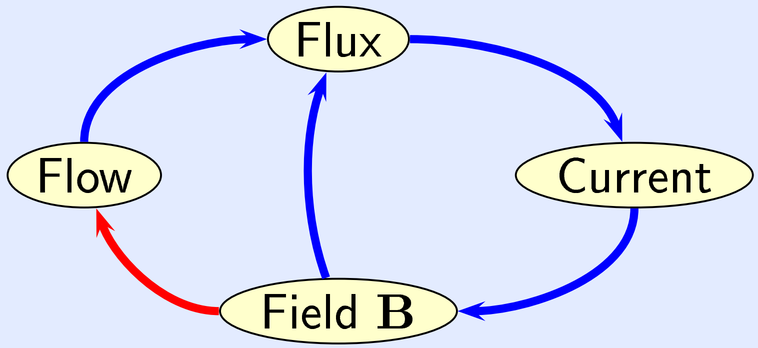

Main studies:

- Local radially (shearing sheet): vorticity gradient not taken into account
- Global: no accretion of material in the basic state
- Rely on numerical dissipation
- No strong conclusion about $\langle \mathbf{B} \rangle$

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NL evolution equations:

$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{U} = -\nabla\Phi - \nabla\Pi + \mathbf{B}\cdot\nabla\mathbf{B} + \nu\Delta\mathbf{U}$$

$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{U} + \eta\Delta\mathbf{B}$$

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Axisymmetric basic state:

$$\Phi = -GM_\star/r \Rightarrow U_\varphi = \sqrt{GM_\star/r}$$

$$U_r = -3\nu/2r \Rightarrow P = C - 9\nu/8r^2$$

$$U_z = 0$$

$$\mathbf{B} = B_o\,\mathbf{e}_z \quad \text{or} \quad \mathbf{B} = B_o/r\,\mathbf{e}_\varphi$$

Numerical Development

Three steps: Linear, Non-Linear bi-dimensional then tri-dimensional

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Boundary Value problem

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NL regime & turbulence:

- Semi-spectral: Fourier in φ & z, compact finite differencies in r
- Semi-implicit in time
- Physical dissipation dominated
- Flux and stream functions in 2D
- High resolution $\Rightarrow$ parallel coding

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No assumption about the external plasma or the connection with the central object

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Accretion compatible BCs:

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Vertical $(\pm\, H)$: $\begin{cases} U_z = 0 & \\ \partial_z U_\varphi = 0 & \\ \partial_z U_z = 0 & \\ B_r = 0 & \partial_z B_r = 0 \\ B_\varphi = 0 & \partial_z B_\varphi = 0 \\ \partial_z B_z = 0 & B_z = 0 \end{cases}$

Linear Instabilities

Generalized eigenvalue problem:

- Linearization of the evolution equations
- Guess the normal modes: $\underline{\mathcal{K}}(\mathbf{r},t) = \underline{\kappa}(r)\ \exp(\sigma t + im\,\varphi + ik\,z)$

10th order linear system: $\sigma\ \underline{\underline{\mathcal{I}}}(r)\ \underline{\kappa}(r) = \underline{\underline{\mathcal{L}}}(r)\ \underline{\kappa}(r)$

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We find two different instabilites:

- Magnetorotational Instability
- Faster growing non-axisymmetric mode localized next to the boundaries

Growth Rates

Parameters : $GM_\star \equiv 1, r_1 \equiv 1, r_2 = 2, B_{zo} = 0.1, \nu = \eta = 3 \times 10^{-3}, m = 1$

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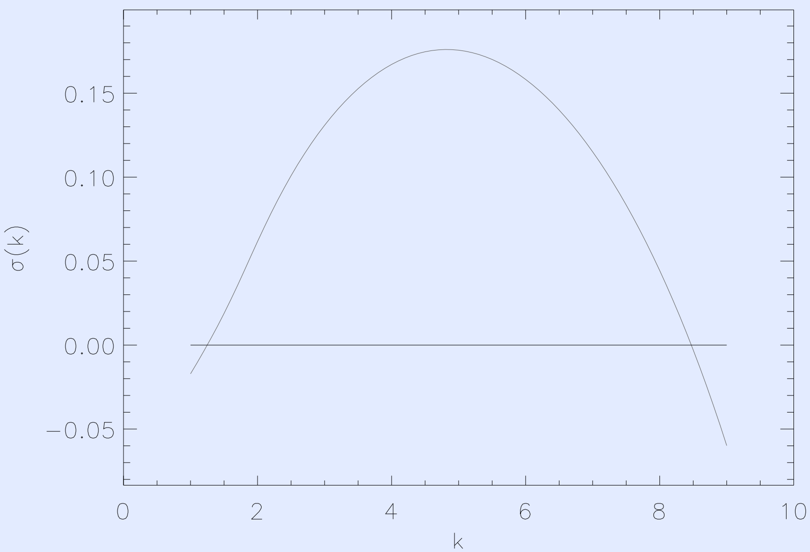

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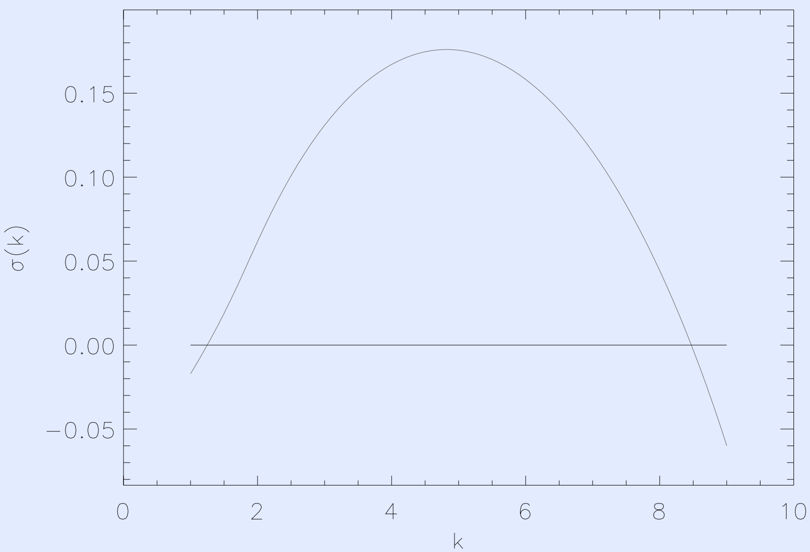

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Fast growing mode:

- One mode on each boundary: the outer decays as r_2 increases but the inner is not sensitive to r_2
- σ significantly increases with Bo

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- Development of the nonlinear codes