Global MRI in Keplerian and Quasi-Keplerian Accretion Discs

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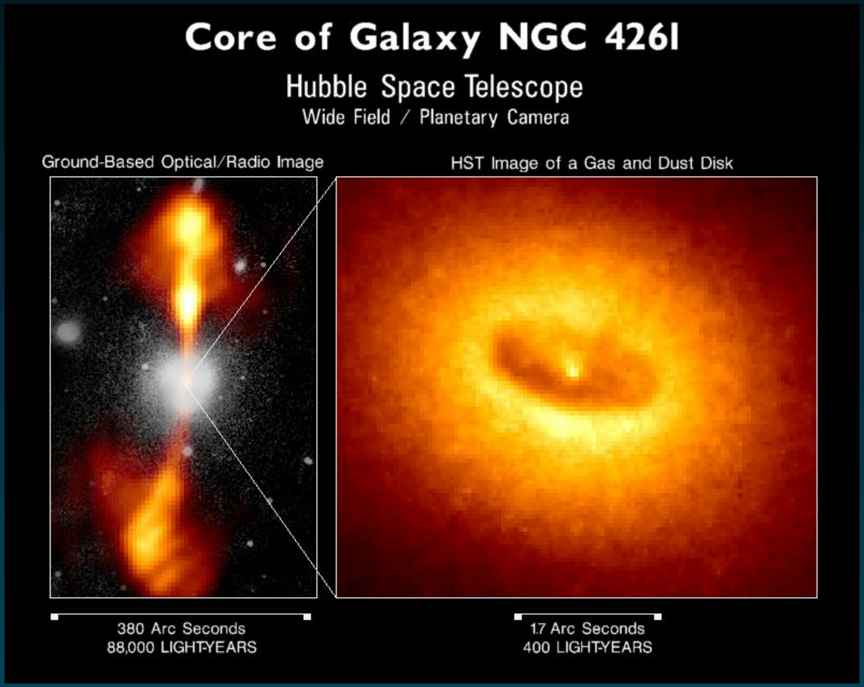
London Mathematical Society: Astrophysical & Geophysical Fluid Mechanics, Cambridge — 24th January 2003

Accretion Discs



Accretion of material occurs in different galactic or extra-galatic environments like AGNs or YSOs

Which mechanisms to explain:

- the turbulent transport of angular momentum
- the existence of the well-collimated jets

Magnetic field $\equiv$ link between accretion and ejection

- MRI drives turbulence $\Rightarrow$ outward angular momentum transport
- Magneto-centrifugal ejection and B_φ-collimation

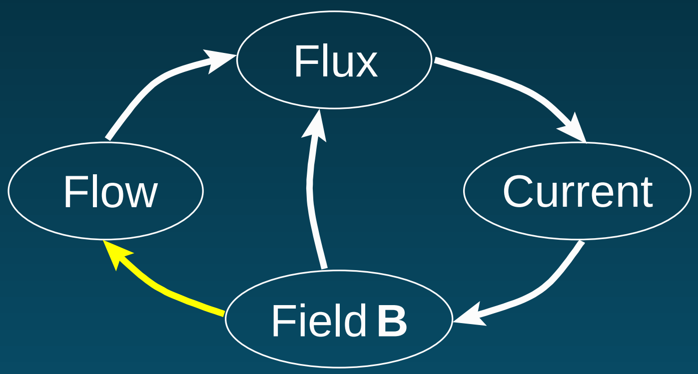
Origin of the large scale magnetic field $\langle \mathbf{B} \rangle$?

MagnetoRotational Instability & Dynamo in Accretion Discs

MagnetoRotational Instability:

- Weakly magnetized ($\beta \gg 1$) and differential rotating flows unstable if $\mathrm{d}\Omega/\mathrm{d}r < 0$
- Free energy $\equiv$ differential rotation $\Rightarrow$ MRI extremely powerful $\gamma \sim r|\mathrm{d}\Omega/\mathrm{d}r|$

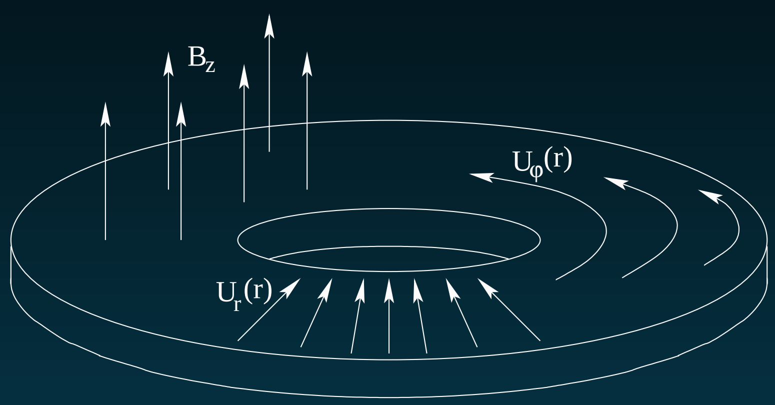
Nonlinear dynamo:



Main studies:

- Local radially (shearing sheet): vorticity gradient not taken into account
- Global: no accretion of material in the basic state
- Rely on numerical dissipation
- No strong conclusion about $\langle \mathbf{B} \rangle$

Global Dissipative Study



NL evolution equations:

$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{U} = -\nabla\Phi - \nabla\Pi + \mathbf{B}\cdot\nabla\mathbf{B} + \nu\Delta\mathbf{U}$$

$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{U} + \eta\Delta\mathbf{B}$$

$$\nabla\cdot\mathbf{B} = \nabla\cdot\mathbf{U} = 0$$

Boundary conditions:

- Two BCs only for ideal discs
- Ten BCs for dissipative discs

Generalized pressure:

$\mathcal{P} = \Phi + \Pi$
- Does not evolve explicitly
- Can vanish $\Rightarrow$ rotation not supported

Differential rotation sustained by the BCs which may either enforce the rotation or constrain the pressure variations

Basic State & Boundary Conditions

Disc equilibrium: Axisymmetric and Z-invariant, $U_z = 0$, $B_r = B_\varphi = 0$

$$\left.\begin{aligned} U_{r\,o} &= -\frac{3\,\nu}{2\,r} \\ U_{\varphi\,o} &= \frac{\zeta}{r^{1/2}} \\ \Pi_o &= \delta - \frac{9}{8}\frac{\nu^2}{r^2} + \frac{GM_* - \zeta^2}{r} \\ B_{Z\,o} &= B_0 \end{aligned}\right\} \quad \begin{aligned} &\zeta^2 = GM_* \ \text{(Keplerian), or} \\ &\zeta^2 = GM_* - \nu^2|\alpha| \ \text{(Sub-Keplerian)} \end{aligned}$$

No BCs on U_r or B_r

$$\left| \begin{aligned} &\partial_r U_\varphi = -U_{\varphi\,o}/2r \\ &\partial_r U_z = 0 \\ &\partial_r (r B_\varphi) = 0 \\ &\partial_r B_z = 0 \end{aligned} \right.$$

and

$$\left| \begin{aligned} &U_\varphi = U_{\varphi\,o} \\ &\text{or} \\ &\Pi = \Pi_o \end{aligned} \right.$$

to drive the shearing flow

Linearized Problem

Linear evolution equations:

- Normal modes:
$$\underline{\mathcal{K}}(\mathbf{r}, t) = \underline{\kappa}(r)\ \exp(\sigma t + im\ \varphi + ik\ z)$$
- 10th order linear system:
$$\sigma\ \underline{\underline{\mathcal{I}}}(r)\ \underline{\kappa}(r) = \underline{\underline{\mathcal{L}}}(r)\ \underline{\kappa}(r)$$
- Π evolves on much shorter time scales:
$$\nabla \cdot \mathbf{U} = 0 \Longleftrightarrow \mathcal{I}_\pi = 0$$

$\Longrightarrow$ Solved numerically:

- inverse iteration
- shooting (double checking)

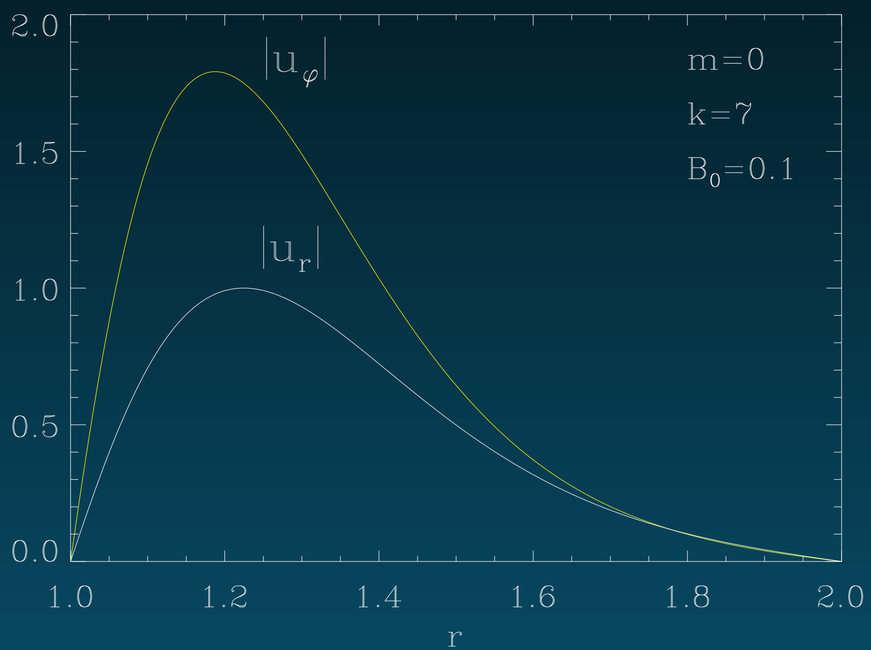
Linear boundary conditions:

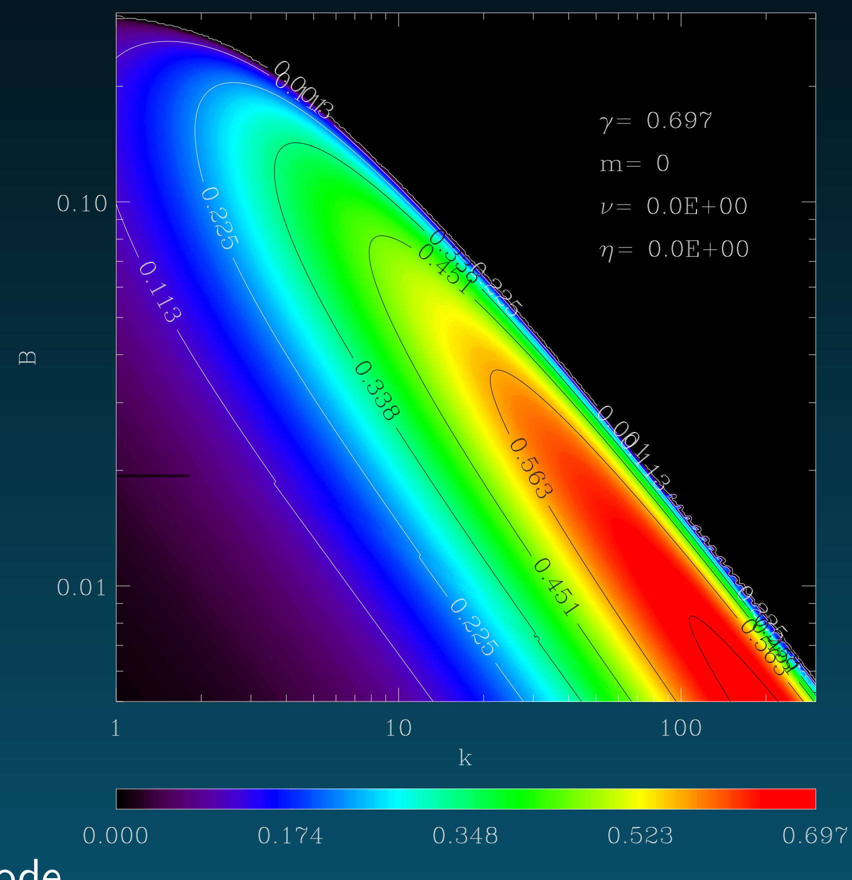
$$\left|\begin{array}{l} \mathrm{d}_r u_\varphi = 0 \\ \mathrm{d}_r u_z = 0 \\ \mathrm{d}_r (r b_\varphi) = 0 \\ \mathrm{d}_r b_z = 0 \end{array}\right. \quad \text{and} \quad \left|\begin{array}{c} u_\varphi = 0 \\ \text{or} \\ \pi = 0 \end{array}\right.$$

but forcing U_φ itself seems more reliable at first to sustain differential rotation

Ideal MRI Modes

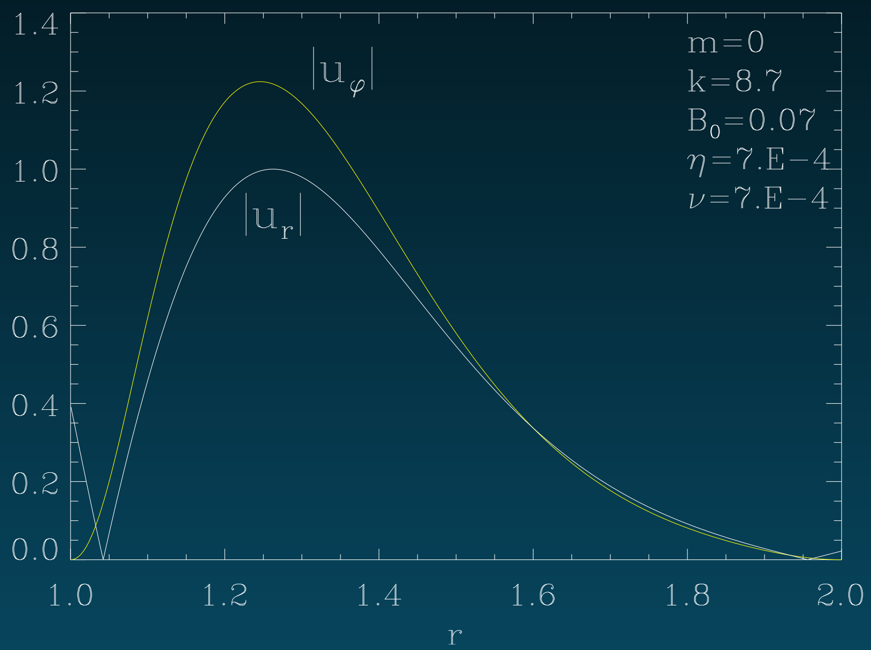
No inflow in the basic state

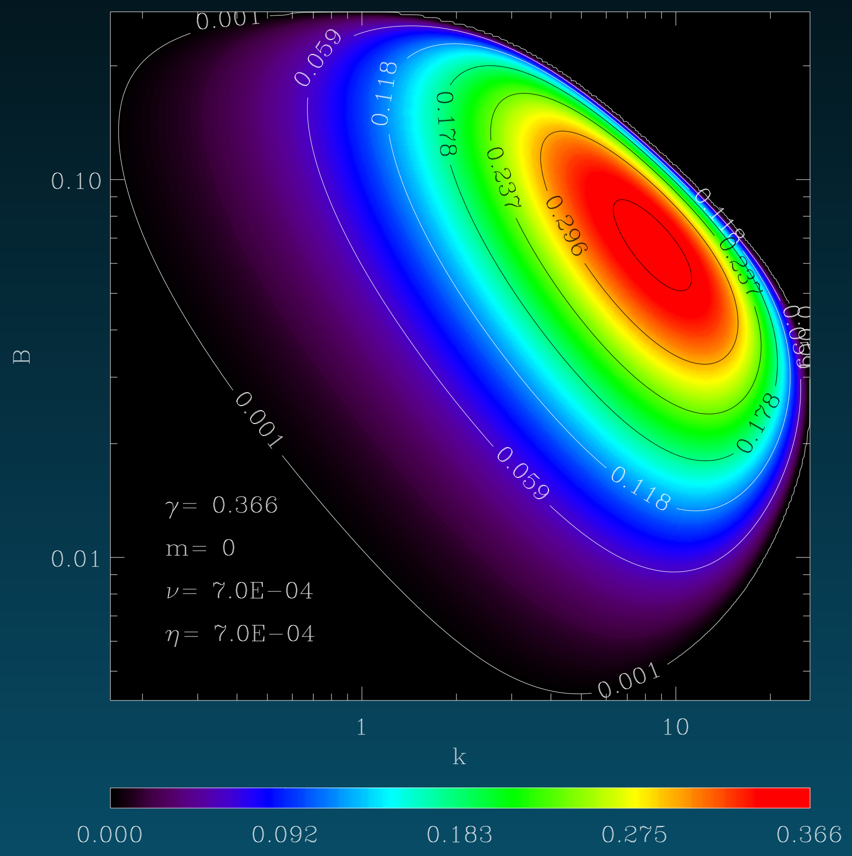




- $m = 0$ is the most unstable global mode
- Quenching by the magnetic tension
- Saturation: $\gamma_{\mathsf{max}} \longrightarrow r_1/2\ |\mathrm{d}\Omega/\mathrm{d}r|_{r_1}$

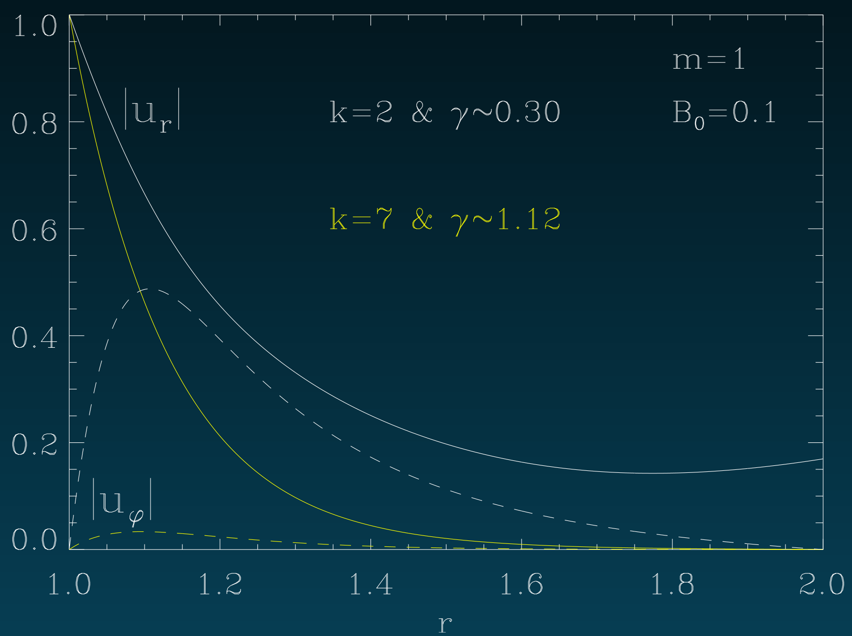
Dissipative MRI Modes

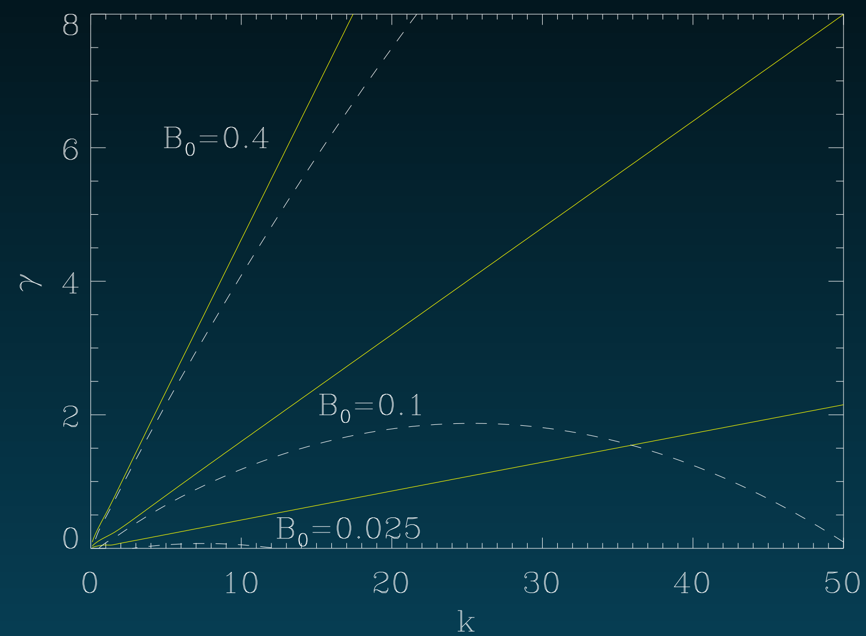




- Modes slightly modified by inflow & dissipation
- Growth rates globally reduced
- Damping of the small-scale modes

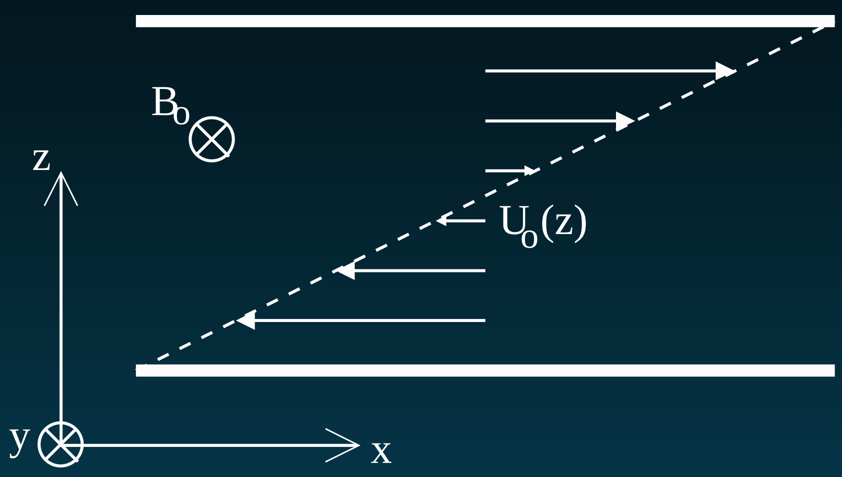
Wall Modes





- Wall modes solutions of the linear system too
- γ too large for the free energy available and large range of k unstable
- Ideal case: γ scales linearly with k and increases rapidly with B_0
- Significant flux of energy through the boundaries to feed these modes
- Inflow, curvature and Coriolis force non crucial

Cartesian Linear Shearing Flow



Incompressible, non dissipative basic state:

$$\rho_0 = 1$$
$$\mathbf{U}_0 = z\,\mathbf{e}_x, \quad z \in [-z_0, +z_0]$$
$$\mathbf{B}_0 = B_0\,\mathbf{e}_y$$

2nd order system of linear ODEs:

$$\chi\mathcal{H}\,u_x = -U_0'\mathcal{H}\,u_z - ik_x\,\pi$$
$$\chi\mathcal{H}\,u_y = -ik_y\,\pi$$
$$\chi\mathcal{H}\,u_z = -\pi'$$
$$0 = ik_x\,u_x + ik_y\,u_y + u_z'$$

where,

$$\underline{\mathcal{K}}(\mathbf{x},t) = \underline{\kappa}(z)\ \exp(\sigma t + ik_x\,x + ik_y\,y)$$

$$\omega_a = kB_0$$
$$\chi = \sigma + ik_xU_0$$
$$\mathcal{H} = \left(1 + \omega_a^2/\chi^2\right)$$

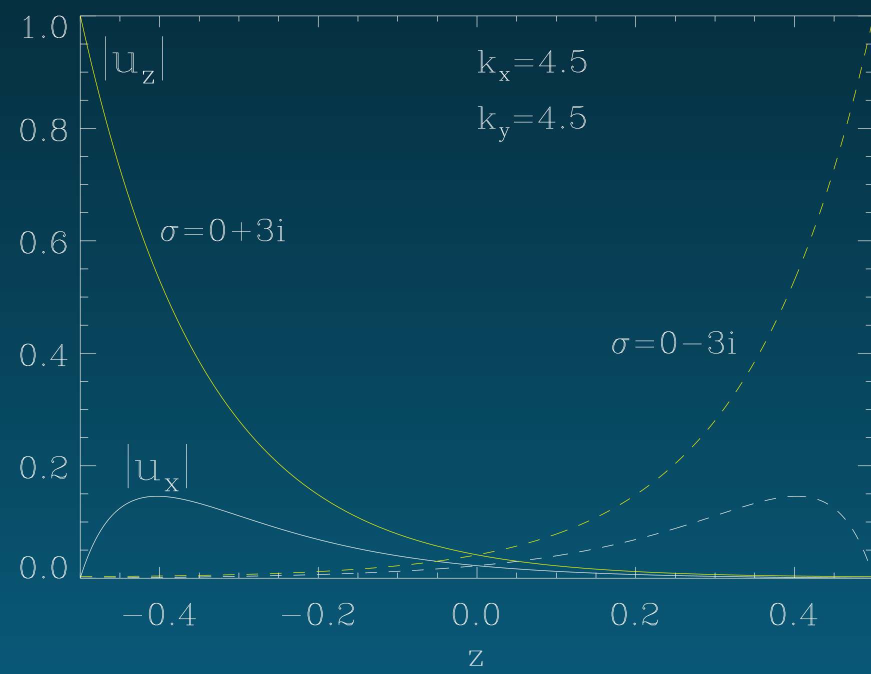
Cartesian Wall Modes: HD Limit

Hydrodynamic limit: $\omega_a = 0$ and $\mathcal{H} = 1$

HD modes are solutions of $$\chi \left[u_z'' - \left(k^2 + \frac{\chi''}{\chi} \right) u_z \right] = 0$$

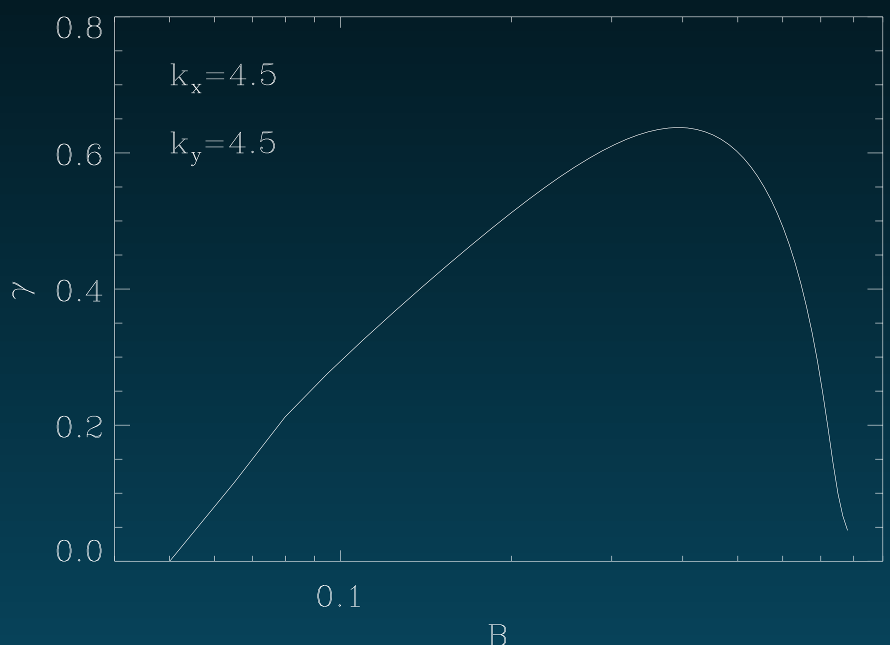
Linear shear $\Rightarrow \chi'' = 0$ and $u_z = c_- \exp(-kz) + c_+ \exp(kz)$

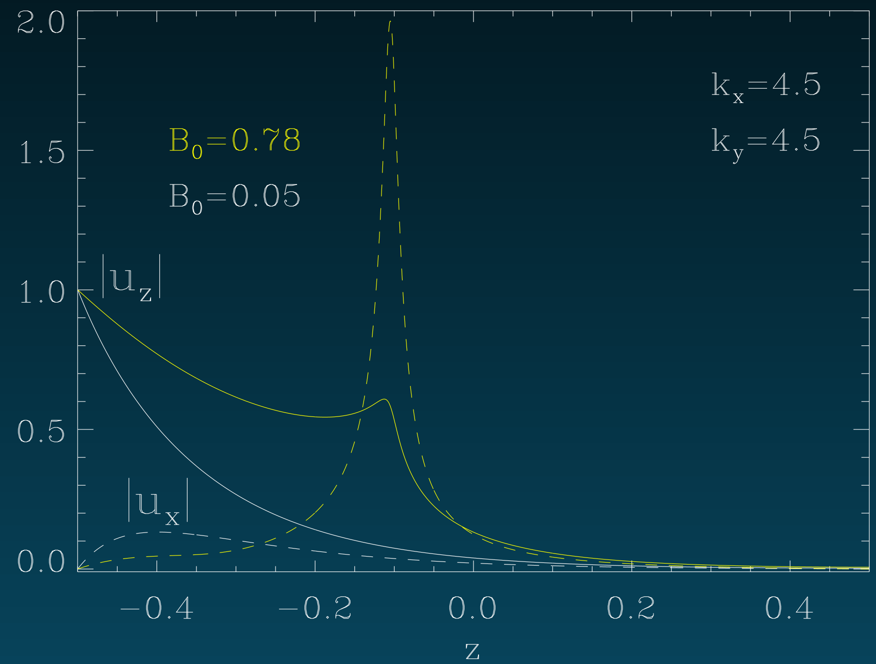
- No discrete mode with the BCs $u_z = 0$, only a continuum of stable modes
- Neutral wall modes solutions with BCs $u_x = 0$



Cartesian Wall Modes in MHD

Magnetic field destabilizes the wall modes





$$u_z'' - 2\frac{\omega_a^2}{\chi^2+\omega_a^2}\frac{\chi'}{\chi}u_z' - \left[k^2 + \frac{\chi''}{\chi} - 2\frac{\omega_a^2}{\chi^2+\omega_a^2}\left(\frac{\chi'}{\chi}\right)^2\right]u_z = 0$$

Singularity when $\gamma = 0$ and $\omega = -k_x U_0 \pm \omega_a$

Origin of the Instability

Analyse on the boundaries:

$$\gamma^2 = \frac{(\mathcal{S}^2\, U_0')^2}{(\mathcal{S}^2\, U_0')^2 + \omega_a^2\, k^2\, k_x^2} \left[(\,U_0')^2\, \frac{\mathcal{K}^2}{k_x^2} + \omega_a^2\, k^2\, \mathcal{L}^2 - \omega_a^2 \right]$$

where $\mathcal{S}^2 = |\pi''|/|\pi|$, $\mathcal{K} = |\pi|'/|\pi|$, $\mathcal{L}^2 = |\pi|\,|\pi|''/|\pi''|^2$ and $k_x = k_y$

B_0 and U_0' both non zero at either of the boundaries $\Rightarrow$ wall modes unstable

Mechanism:

$$u_z \xrightarrow{k_y\, B_0} b_z \xrightarrow{U_0'} b_x \xrightarrow{k_y\, B_0} T_x = k_y\, B_0\, b_x$$

In the vicinity of the boundary $u_x \sim 0 \Rightarrow$ energy from the outside required to balance T_x

Wall Modes Treatment in Accretion Discs

- Wall modes are solutions of incompressible shearing flows when rigid BCs are relaxed
- B_0 makes them linearly unstable if $u_\varphi = 0$ on the boundaries

Forcing the differential rotation of the boundaries impossible unless Ω_0' or B_0 locally zero

or

BCs on the pressure to keep it low in agreement with quasi-Keplerian accretion discs

Basic State:

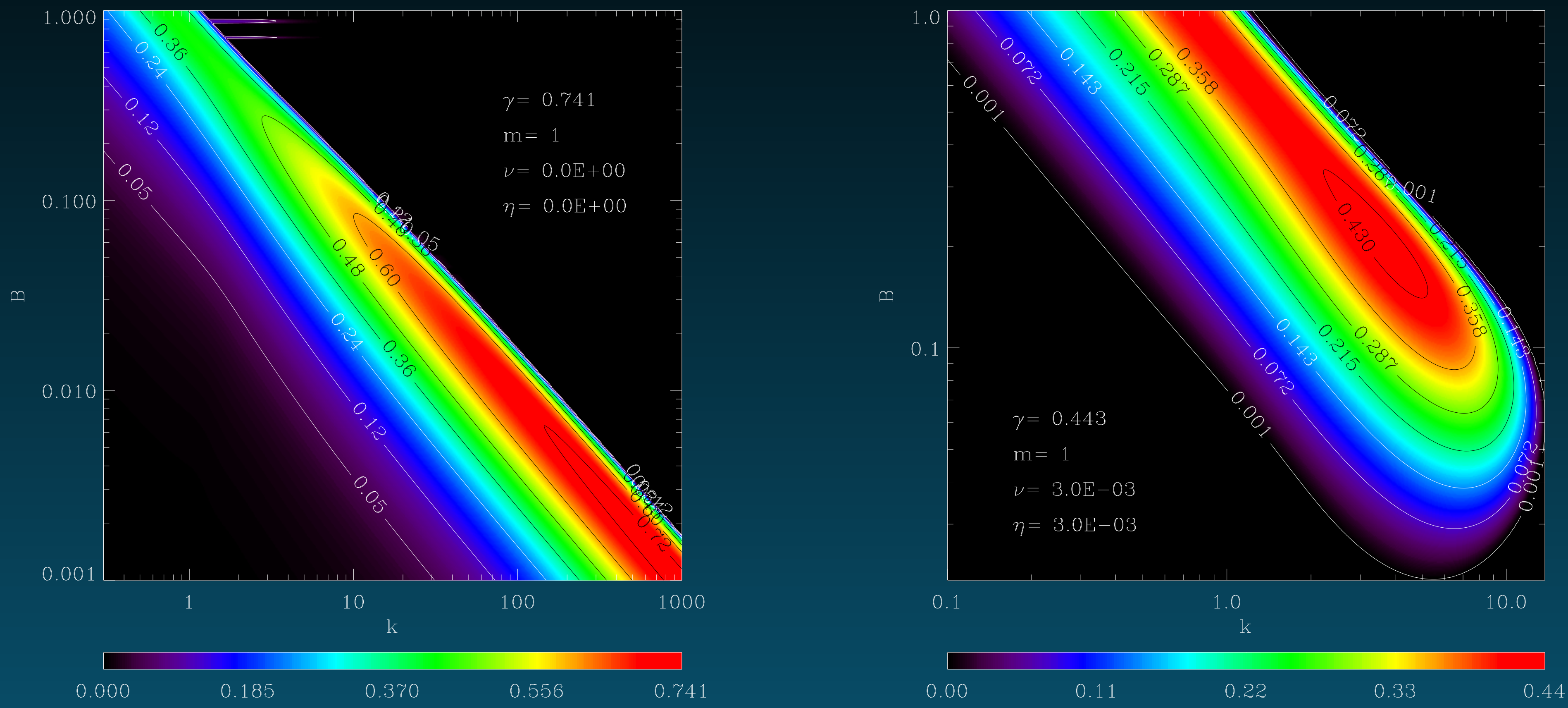
$$\Pi_o(r_1) = \Pi_o(r_2) = \delta + \frac{9\,\nu}{8\,r_1 r_2}$$

$$\Pi_o = \delta + \frac{9}{8}\frac{\nu}{r_1 r_2}\left(\frac{r_1 + r_2}{r} - \frac{r_1\,r_2}{r^2}\right)$$

$$U_{\varphi_o} = \sqrt{\frac{1}{r}\left(GM_* - \frac{9\,\nu}{8}\,\frac{r_1 + r_2}{r_1\,r_2}\right)}$$

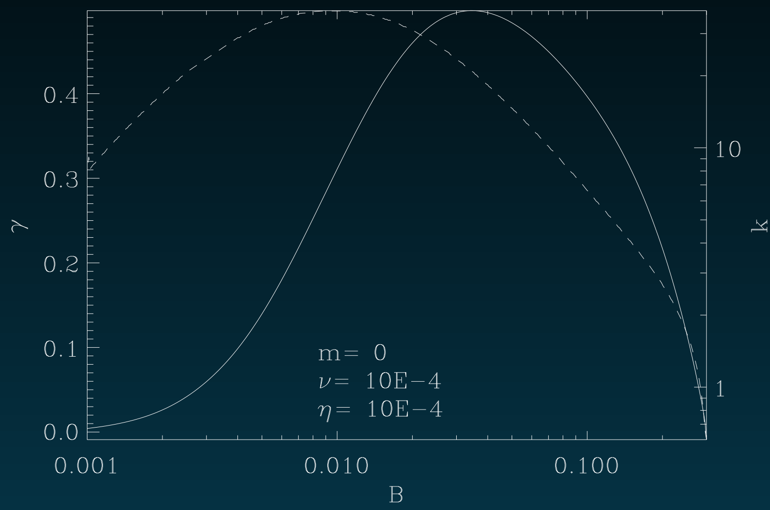
BCs $\Pi = \Pi_o$:
external pressure does not work

Boundary Conditions on the Total Pressure



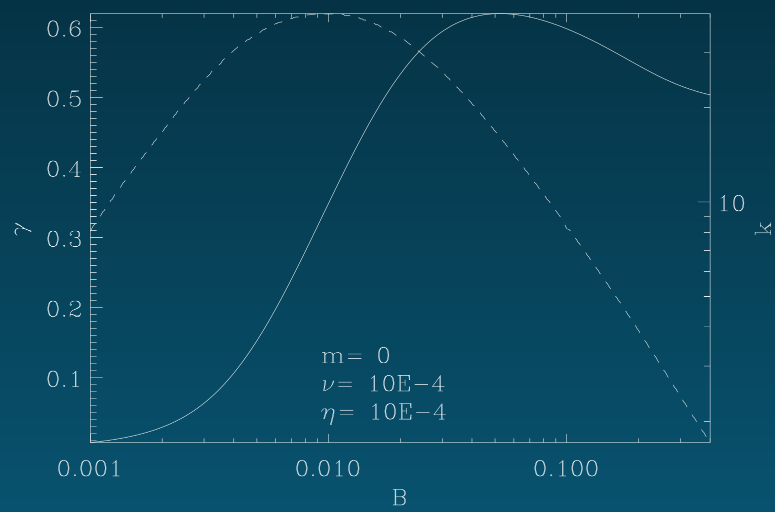
- Body modes still unstable and wall modes properties now consistent with MRI
- Larger range of unstable m for the wall modes

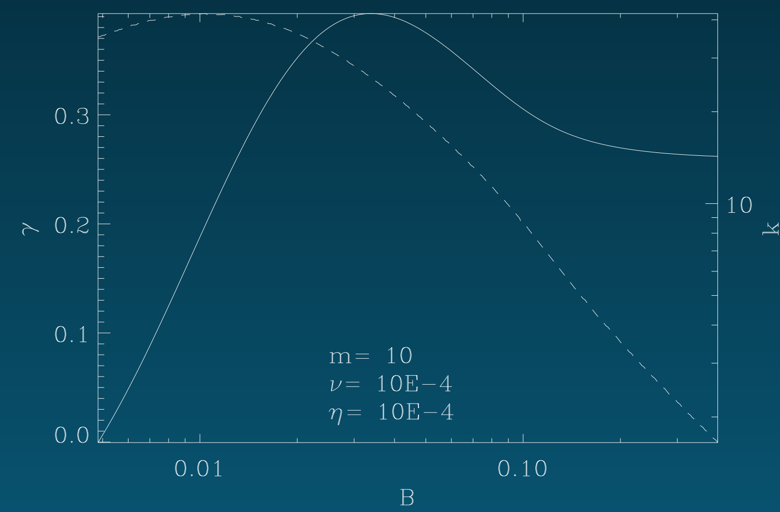
Modes in Slim Discs



Disc thickness constrains $\min(k) \Rightarrow$ low ν & η

$$H/R = \begin{cases} 1/4 \\ 1/10 \end{cases} \Rightarrow k_{\text{min}} = \begin{cases} 12.6 \\ 31.4 \end{cases}$$





Conclusions & Future Work

Linear study

- Slightly sub-keplerian basic state with inflow
- No assumption of the behavior of U_r and B_r on the boundaries
- Accurate description of the global dissipative MRI modes

Fully nonlinear study

- Will rely on high performance computations
- Parallel, 2D & 3D
- Systematic parameter space survey
- Toward a comparison of dynamical properties with a reduced MRI approach