

Introduction to the Pressure-Driven Instabilities in Astrophysical Jets

Evvy Kersalé,
AMSTA — University of Leeds

Guy Pelletier
Pierre-Yves Longaretti
LAOG, Grenoble, France

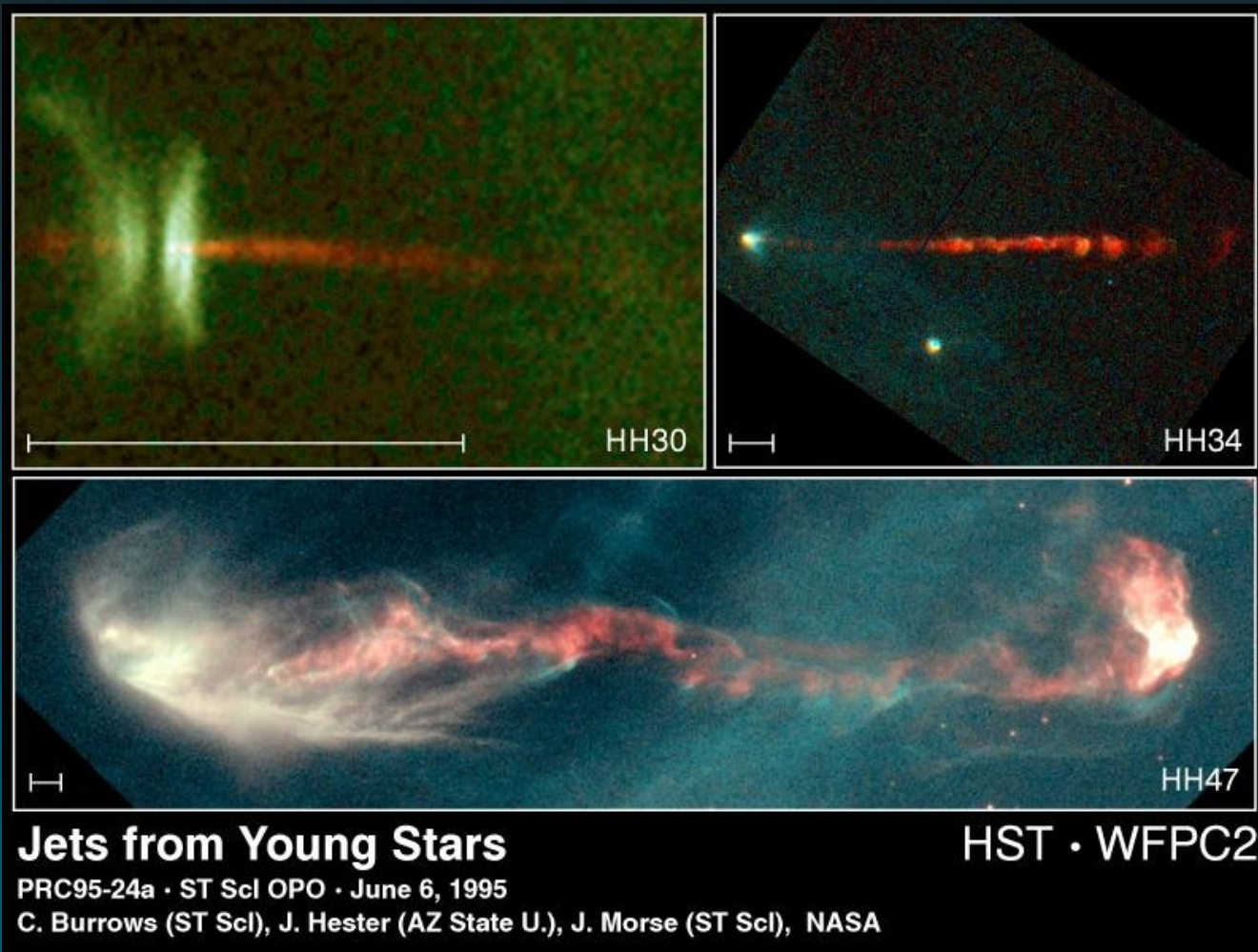
Ejection of Matter in Astrophysical Objects

Ejections of matter occur in many astrophysical environments

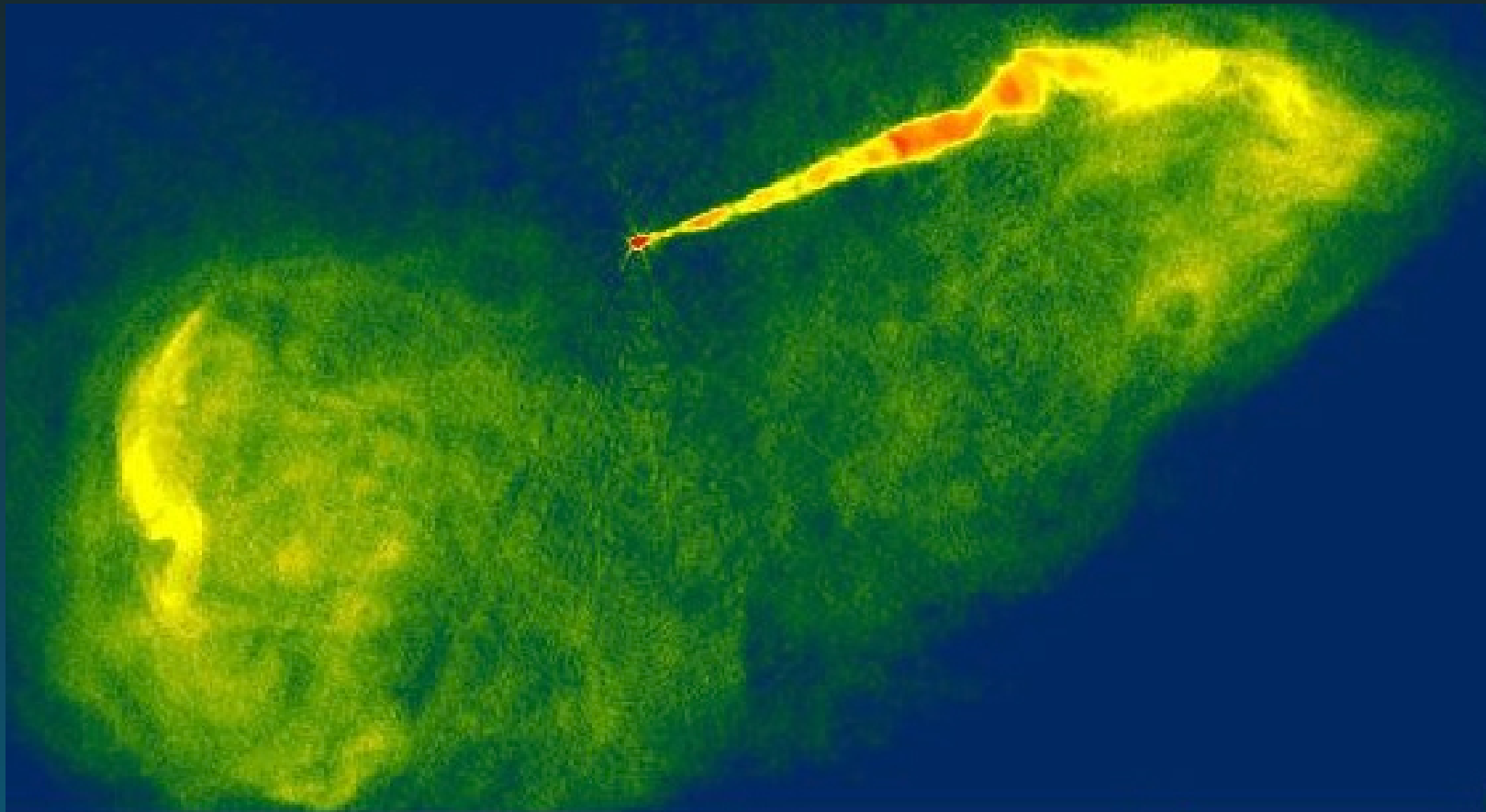
- Objects :
 - ★ Galactic : YSOs, stellar winds, CMEs, supernovæ, microquasars
 - ★ Extra-galactic : AGNs, GRBs
- Multiple :
 - ★ Sources of matter and ejection mechanisms
 - ★ Scales in space, time and energy

Stationary, highly collimated and non relativistic jets : YSOs, AGNs

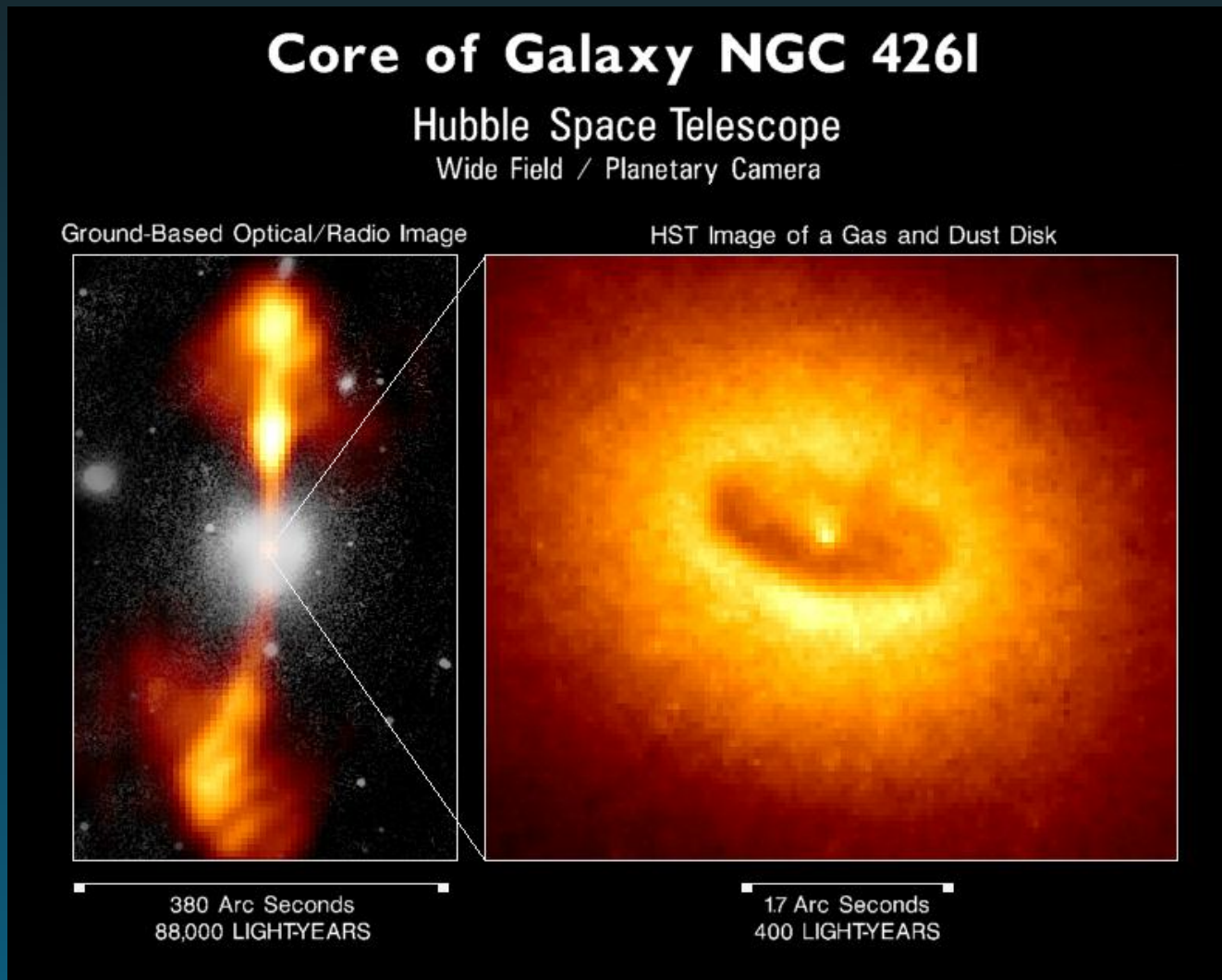
Jets in YSOs



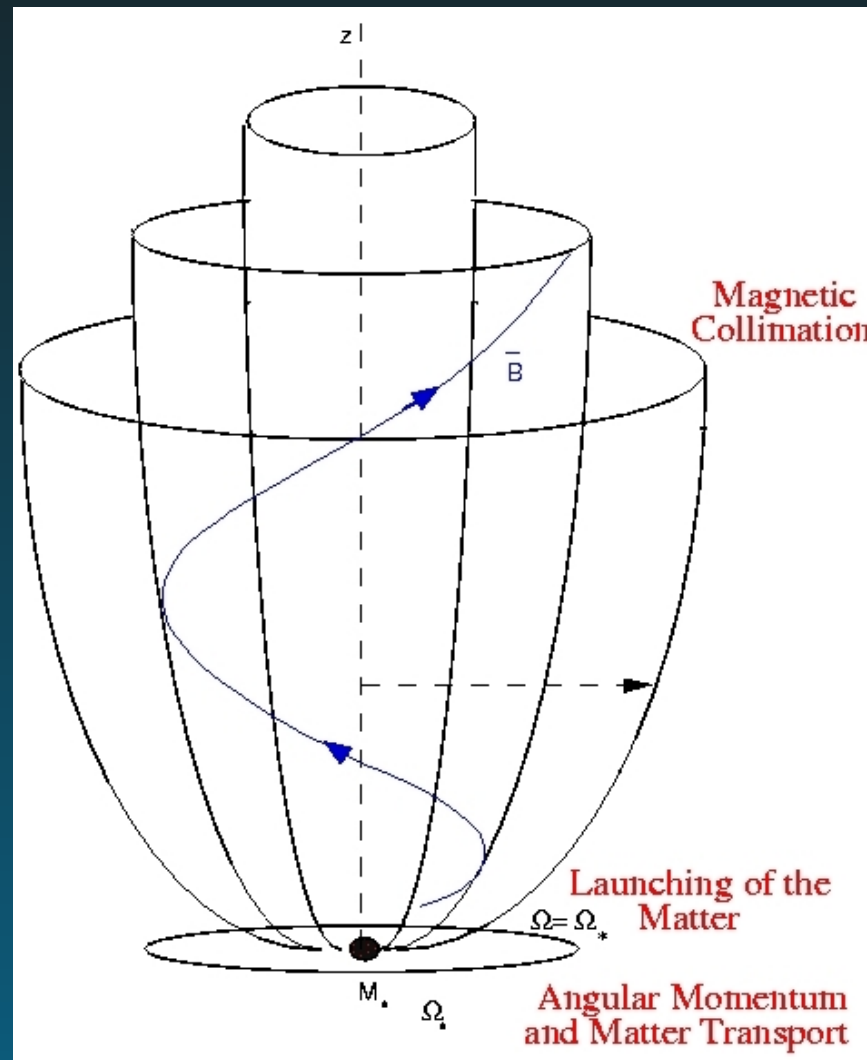
Jets in AGNs



Accretion-Ejection in AGNs



Magnetized Accretion-Ejection Structures



Motivations for this Work

- Source of energy in the interiors of jets ?
- Variations of the luminosity in jets : avalanche process ?
- Magnetized discs with $\beta = 1$: which mechanism is involved ?
- Sporadic ejection of matter : avalanche process ?

Magneto-Hydrodynamic

Plasma { completely ionized
magnetic field
non resistive

Continuity : $\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \vec{v}$

Induction : $\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$

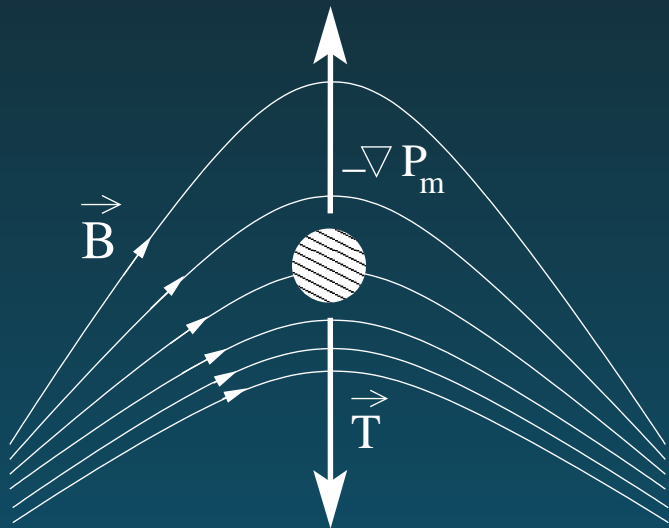
State : $P \propto \rho^\gamma$

Momentum : $\frac{d\rho \vec{v}}{dt} = -\nabla P + \vec{j} \times \vec{B} + \vec{\mathcal{F}}$

The magnetic field is frozen in the plasma

Characteristic scales

Laplace force : $\vec{j} \times \vec{B} = -\nabla P_m + \vec{T}$



- Pressure : $P_m = \frac{B^2}{2\mu}$

- Tension : $\vec{T} = \frac{1}{\mu} \vec{B} \cdot \nabla \vec{B}$

Set of characteristic scales :

$\vec{\kappa}_\rho$: density — $\vec{\kappa}_b$: magnetic field — $\vec{\kappa}_c$: curvature

Energy Principle

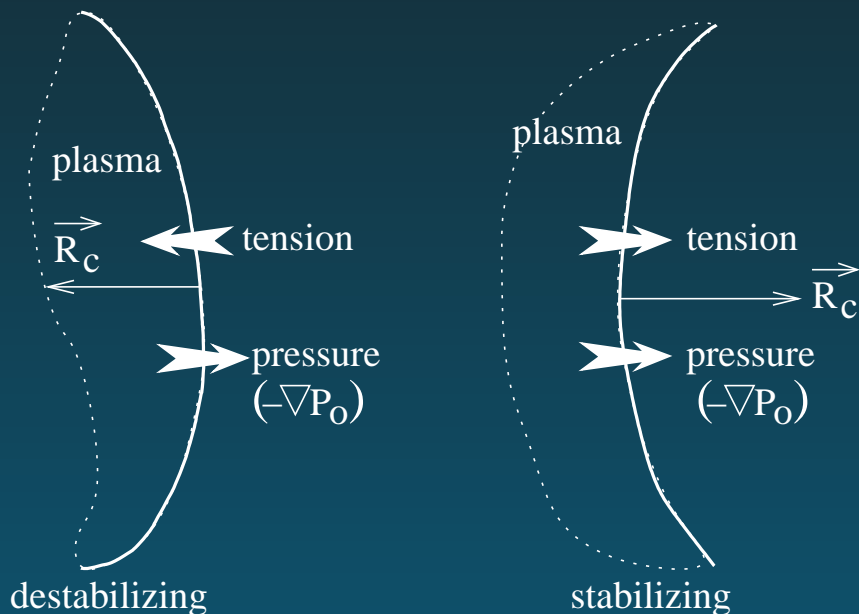
- Growth of an infinitesimal perturbation \Rightarrow **instability**
- Conversion of potential energy in kinetic energy : $\delta W < 0$

$$\delta W_F = \frac{1}{2} \int_P d^3\vec{r} \left[\underbrace{\frac{|\delta\vec{B}_\perp|^2}{\mu}}_{\text{Alfvén}} + \underbrace{\frac{B_o}{2\mu} |\nabla \cdot \vec{\xi}_\perp + 2 \vec{\xi}_\perp \cdot \vec{\mathcal{K}}_c|^2}_{\text{magnetosonic}} + \underbrace{\gamma P_o |\nabla \cdot \vec{\xi}|^2}_{\text{sonic}} \right. \\ \left. - \underbrace{2 \left(\vec{\xi}_\perp \cdot \nabla P_o \right) \left(\vec{\mathcal{K}}_c \cdot \vec{\xi}_\perp^* \right)}_{\text{interchange - ballooning}} - \underbrace{j_{o//} \left(\vec{\xi}_\perp^* \times \vec{e}_{//} \right) \cdot \delta\vec{B}_\perp}_{\text{kink}} \right]$$

MHD instabilities : the magnetic field increases

Non-Resistive Linear MHD Instabilities

Source $\left\{ \begin{array}{l} \text{Parallel Current } (L_o k_{\perp} \sim 1) \\ \text{Thermal pressure } (L_o k_{\perp} \gg 1) \end{array} \right. \begin{array}{l} \implies \text{Kink (works in the force-free limit)} \\ \implies \text{Interchange et Ballooning} \end{array}$



Pressure Instability

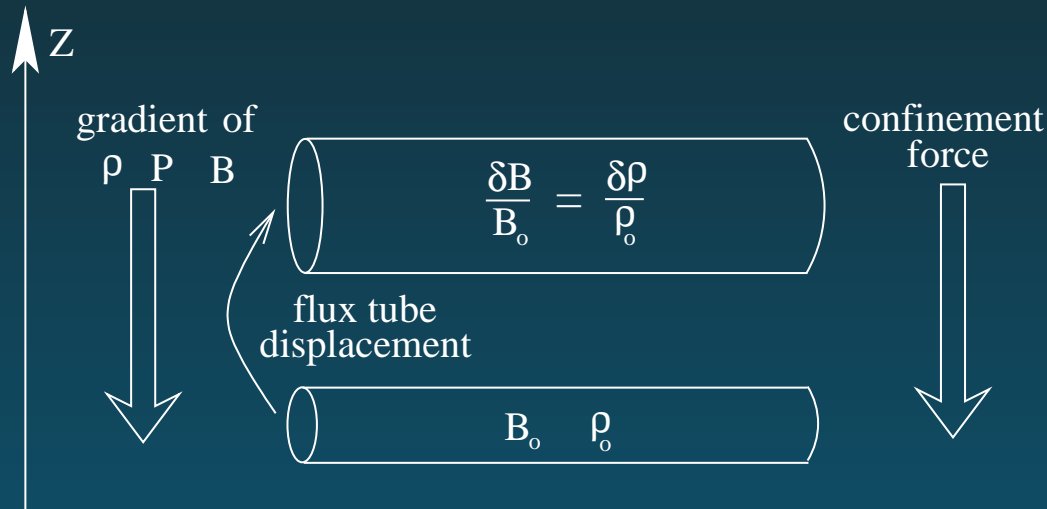
$$\vec{\mathcal{K}}_c \cdot \nabla P_0 < 0 \implies \text{stabilizing}$$

$$\vec{\mathcal{K}}_c \cdot \nabla P_0 > 0 \implies \text{destabilizing}$$

Interchange Process

Interchange of two fluid particles \Rightarrow decrease in potential energy \Rightarrow instability

Archetype : Rayleigh-Taylor instability



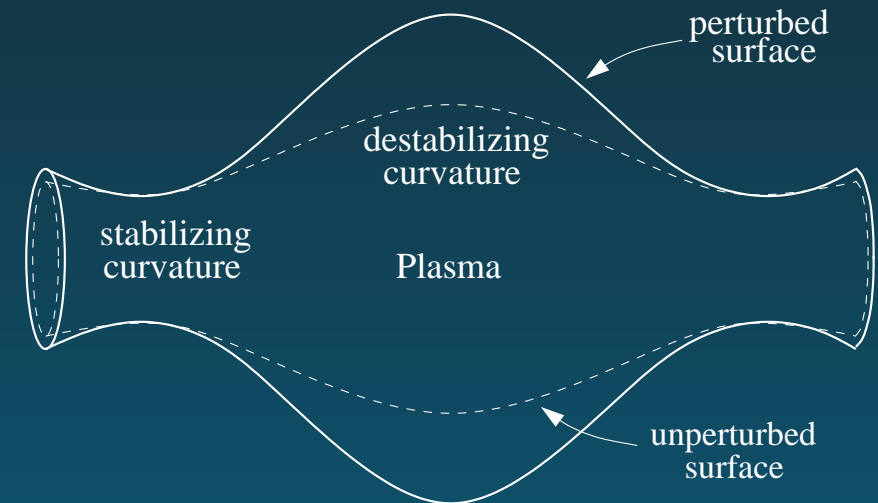
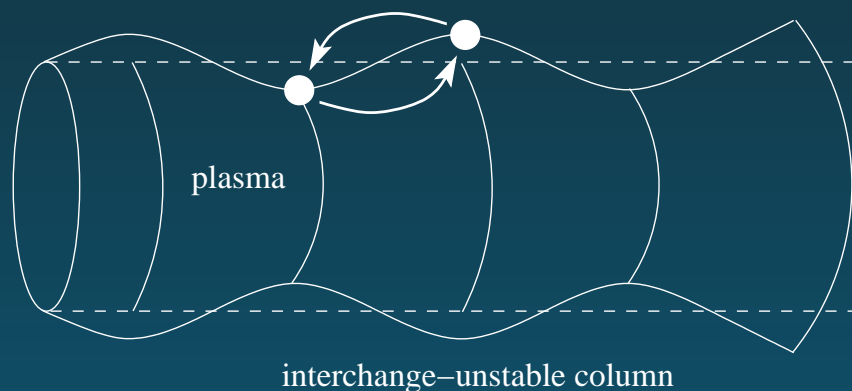
- Gravitational confinement \Rightarrow Rayleigh-Taylor, convection
- In the presence of magnetic field \Rightarrow Parker modes, magnetic buoyancy
- Magnetic confinement \Rightarrow pressure-driven modes

Constraints : pressure equilibrium ; mass and magnetic flux conservation

Interchange and Ballooning MHD modes

Plasma magnetically confined

Most actively studied in **thermonuclear fusion**, **magnetospheric physics** and **solar physics**
but ignored in **jets** and **discs**

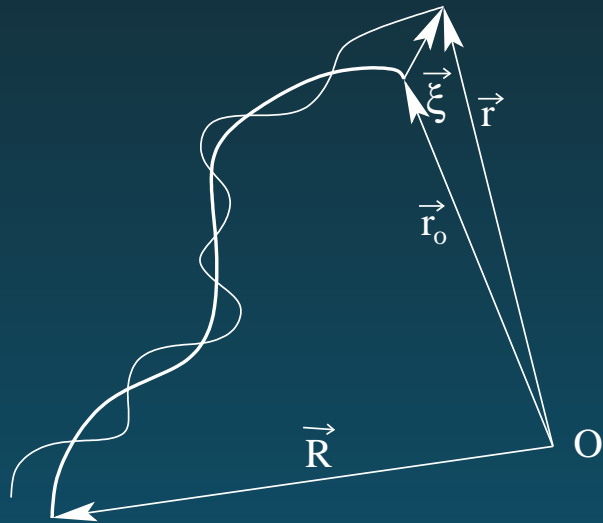


- **Interchange mode**
- Axial symmetry \Rightarrow "sausage"
- $m = 1 \Rightarrow$ helical or "kink"

- Localized mode : **Ballooning**
- **Curvature stabilizing or not**
- highly constraining thermonuclear fusion

Linear Study : Perturbations

Infinitesimal displacement of fluid particles $\vec{\xi}$



Lagrangian perturbations :

$$\delta\rho = -\nabla \cdot (\rho_0 \vec{\xi})$$

$$\delta\vec{B} = -\nabla \times (\vec{\xi} \times \vec{B}_0)$$

$$\delta P = -\vec{\xi} \cdot \nabla P_0 - \gamma P_0 \nabla \cdot \vec{\xi}$$

- Equations written in terms of ξ_{\parallel} and ξ_{\perp}
- Evolution determined by the momentum equation

General Set of Linear MHD Equations

$$\frac{\partial^2 \xi_{\parallel}}{\partial t^2} = \mathcal{F}_{\parallel}(\vec{\xi}) + \frac{1}{\rho_0} \frac{\partial}{\partial s} \left[\rho_0 C_S^2 \left(\vec{\xi} \cdot \vec{\mathcal{K}}_{\rho} + \nabla \cdot \vec{\xi} \right) \right] \\ + V_A^2 \left(\vec{\mathcal{K}}_{b_{\perp}} - \vec{\mathcal{K}}_c \right) \cdot \left(\Gamma^{-} \cdot \vec{\xi}_{\perp} + \frac{\partial \xi_l}{\partial s} \vec{e}_l + \frac{\partial \xi_A}{\partial s} \vec{e}_A \right)$$

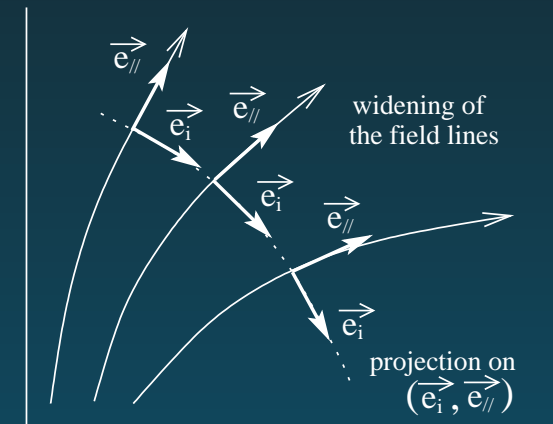
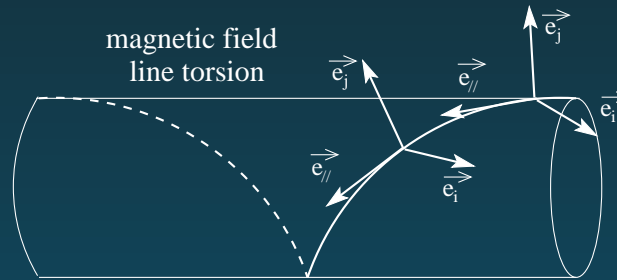
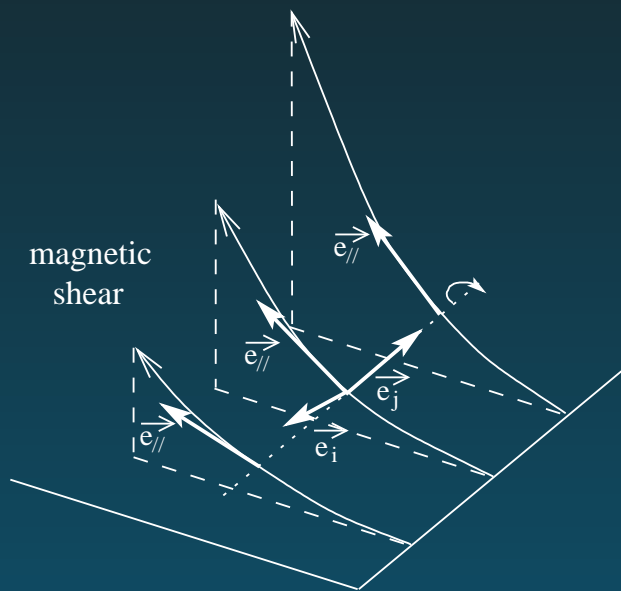
$$\frac{\partial^2 \vec{\xi}_{\perp}}{\partial t^2} = \vec{\mathcal{F}}_{\perp}(\vec{\xi}) + \frac{1}{\rho_0} \nabla_{\perp} \left\{ \rho_0 C_S^2 \left(\vec{\xi} \cdot \vec{\mathcal{K}}_{\rho} + \nabla \cdot \vec{\xi} \right) + \rho_0 V_A^2 \left[\left(\vec{\mathcal{K}}_{b_{\perp}} + \vec{\mathcal{K}}_c \right) \cdot \vec{\xi}_{\perp} + \nabla \cdot \vec{\xi}_{\perp} \right] \right\} \\ + V_A^2 \left\{ \left(\frac{\partial \delta b_l}{\partial s} \vec{e}_l + \frac{\partial \delta b_A}{\partial s} \vec{e}_A \right) + \Gamma^{+} \cdot \vec{\delta b}_{\perp} + 2 \delta b_{\parallel} \vec{\mathcal{K}}_c + \left(\vec{\delta b} \cdot \vec{\mathcal{K}}_c \right) \vec{e}_{\parallel} + \kappa_{b_{\parallel}} \vec{\delta b}_{\perp} \right\}$$

with $\delta b_{\parallel} = -\nabla \cdot \vec{\xi}_{\perp} - \left(\vec{\mathcal{K}}_{b_{\perp}} + \vec{\mathcal{K}}_c \right) \cdot \vec{\xi}_{\perp}$

and $\vec{\delta b}_{\perp} = \Gamma^{-} \cdot \vec{\xi}_{\perp} + \frac{\partial \xi_l}{\partial s} \vec{e}_l + \frac{\partial \xi_A}{\partial s} \vec{e}_A + \left(\vec{\xi} \cdot \vec{\mathcal{K}}_c \right) \vec{e}_{\parallel}$

Geometry of the Basic State Magnetic Field

Tensor Γ^\pm

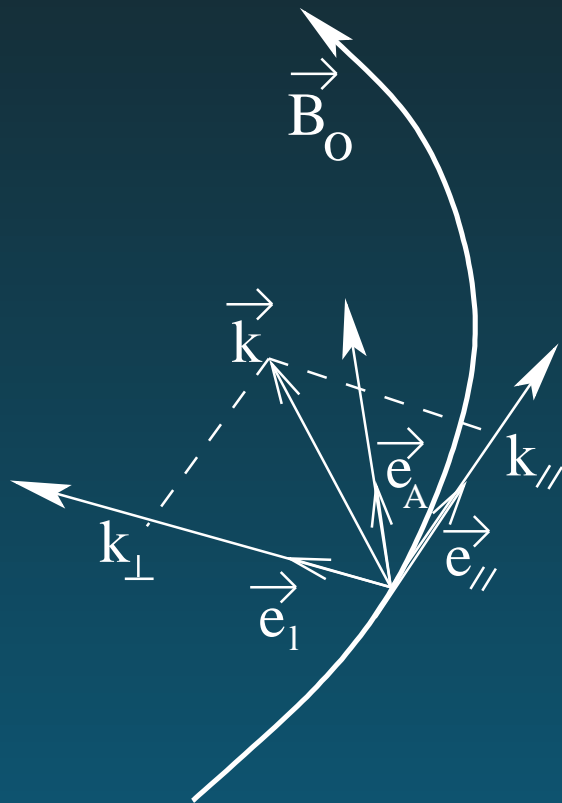


$$\Gamma_{ij}^\pm = \vec{e}_i \cdot [(\vec{e}_\parallel \cdot \nabla) \vec{e}_j \pm (\vec{e}_j \cdot \nabla) \vec{e}_\parallel]$$

$$\Gamma_{ii}^\pm = \pm \vec{e}_i \cdot [(\vec{e}_i \cdot \nabla) \vec{e}_\parallel]$$

Linearized MHD Equations

First stage : to linearize the MHD



- Hypotheses

- ★ Ideal MHD and barotropic equation of state
- ★ **Perturbations study** \Rightarrow linear equations
- ★ **Heterogeneous plasma** and **any magnetic structure**

- We get a general differential system

- ★ Two coupled equations for the displacement
- ★ Derivation following the magnetic field lines
- ★ Still valid in the presence of external forces

Further analytical study requires more simplifications

Derivation of the “Ballooning” equations

- Restrictions on the perturbation variations
 - ★ **Ordering** imposed : $k_{\perp} L_0 \gg 1$ and $k_{\perp} \gg k_{\parallel} \Rightarrow L_0 \omega \sim V_A$ and $\xi_{\parallel} \sim \xi_A \gg \xi_l$
 - ★ **Slowly quasi-transversal** propagation (Newcomb, 1961)
 - ★ **fast magnetosonic** waves, mostly compressive, **decoupled**
- Transversal momentum equation : $\delta P_m + \delta P_{th} = 0$
- **No particular mode assumed** : neither Fourier transform, nor WKB approximation
- Evolution equations include **heterogeneities** and **magnetic field geometry** :

$$\frac{\partial^2 \xi_A}{\partial t^2} - \mathcal{D}_A \cdot \xi_A = \mathcal{C}_A \cdot \xi_{\parallel} + \mathcal{F}_A(\vec{\xi})$$

$$\frac{\partial^2 \xi_{\parallel}}{\partial t^2} - \mathcal{D}_{\parallel} \cdot \xi_{\parallel} = \mathcal{C}_{\parallel} \cdot \xi_A + \mathcal{F}_{\parallel}(\vec{\xi})$$

Purely MHD Equilibrium

- **Pressure-tension** transversal equilibrium : $\beta \vec{\mathcal{K}}_\rho = \vec{\mathcal{K}}_c - \vec{\mathcal{K}}_{b_\perp}$ with $\beta = C_S^2/V_A^2$
- Differential system :

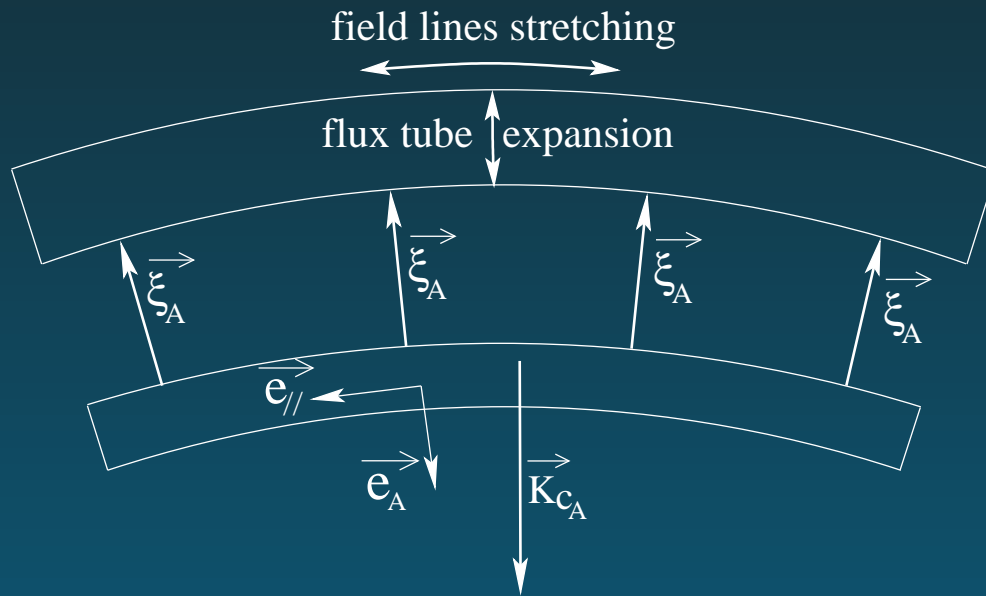
$$\frac{\partial^2 \xi_{\parallel}}{\partial t^2} - C_S^2 \frac{\partial}{\partial s} \nabla \cdot \vec{\xi} = 0$$

$$\frac{\partial^2 \xi_A}{\partial t^2} - V_A^2 \left[\frac{\partial^2}{\partial s^2} + \mathcal{K}_{b_{\parallel}} \frac{\partial}{\partial s} + 2\beta \mathcal{K}_{c_A} \mathcal{K}_{\rho_A} + \frac{\partial}{\partial s} \Gamma_{AA}^- + \mathcal{K}_{b_{\parallel}} \Gamma_{AA}^- + \left(\Gamma^+ \Gamma^- \right)_{AA} \right] \xi_A = 2 C_S^2 \mathcal{K}_{c_A} \nabla \cdot \vec{\xi}$$

- Topology of the magnetic field in $\mathcal{K}_{b_{\parallel}}$ and Γ^+ : widening, torsion and **shear**
- **Coupled** by $\nabla \cdot \vec{\xi} \equiv$ **curvature and parallel variations**

No Parallel Variation

Constant pressure \Rightarrow destabilization by the fluctuation of the transversal tension



$$\delta T_A = 2\rho_0 V_A^2 \mathcal{K}_{cA} \delta b_{||}$$

$$\delta b_{||} = \frac{\beta}{1 + \beta} (\mathcal{K}_{\rho A} - \mathcal{K}_{cA} - \mathcal{K}_{bA}) \xi_A$$

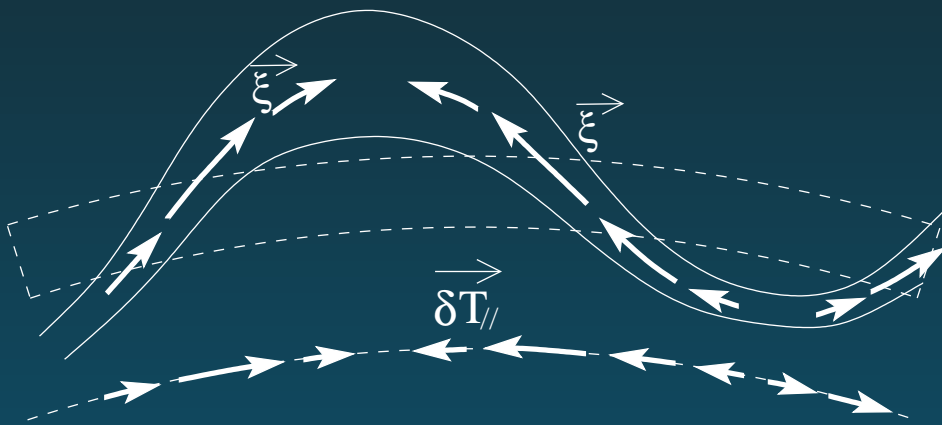
Instability criteria : $\mathcal{K}_{cA} \mathcal{K}_{\rho A} > \frac{2}{1 + \beta} \mathcal{K}_{cA}^2$

In the Presence of Parallel Variations

Crucial role of the fluctuations of the parallel tension \Rightarrow interpretation less obvious

But in the case :

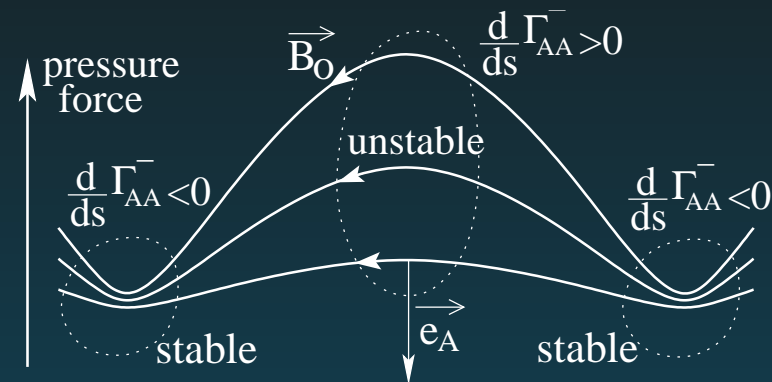
- Shape of the mode $\exp[i(k_{\parallel}s - \omega t)]$
- Limit $L_0 k_{\parallel} \ll 1$



$$\omega^2 = -2\beta \frac{\mathcal{K}_{c_A} \mathcal{K}_{\rho_A}}{k_o^2} k_{\parallel}^2 V_S^2$$

- Sufficient condition for instability : $\mathcal{K}_{c_A} \mathcal{K}_{\rho_A} > 0$
- The plasma can be **instable** in spite of a stabilizing δT_A

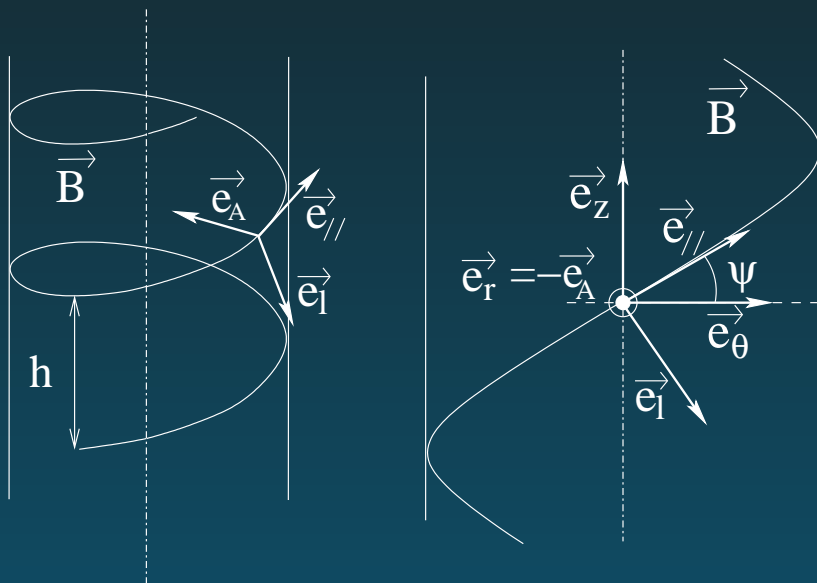
Geometric Terms



- Geometric terms \Rightarrow contribution in δT_A
- The stability depends on the **curvature** and its zero-order **variations**
- **Widening** and **torsion** \Rightarrow **curvature stabilizing** or **destabilizing**
- **Localized** modes of **Ballooning**

Cylindrical Asymptotic Rotating Jet

Cylindrical column , axial symmetry and vertically invariant



- Quasi-solid Rotation without vertical movement :
 $v_{o\theta} = r\Omega_o$ et $v_{oz} = 0$
- Cylindrical magnetic surfaces :
 $\vec{B}_o = B_o \vec{e}_{\parallel} = B_{o\theta} \vec{e}_{\theta} + B_{oz} \vec{e}_z$
- Instability maximized if $\vec{e}_A \parallel \vec{\mathcal{K}}_c$
 $\Rightarrow \vec{e}_A = -\vec{e}_r$

Exact Fourier transform on the magnetic surfaces :

$$\xi \rightarrow \xi(r) \exp[i(m\theta + k_z z - \omega t)]$$

Non-rotating Jet

- Analytically tractable 4th-order equation of dispersion : **Alfvén and slow magnetosonic modes**
- Only one root can be negative :

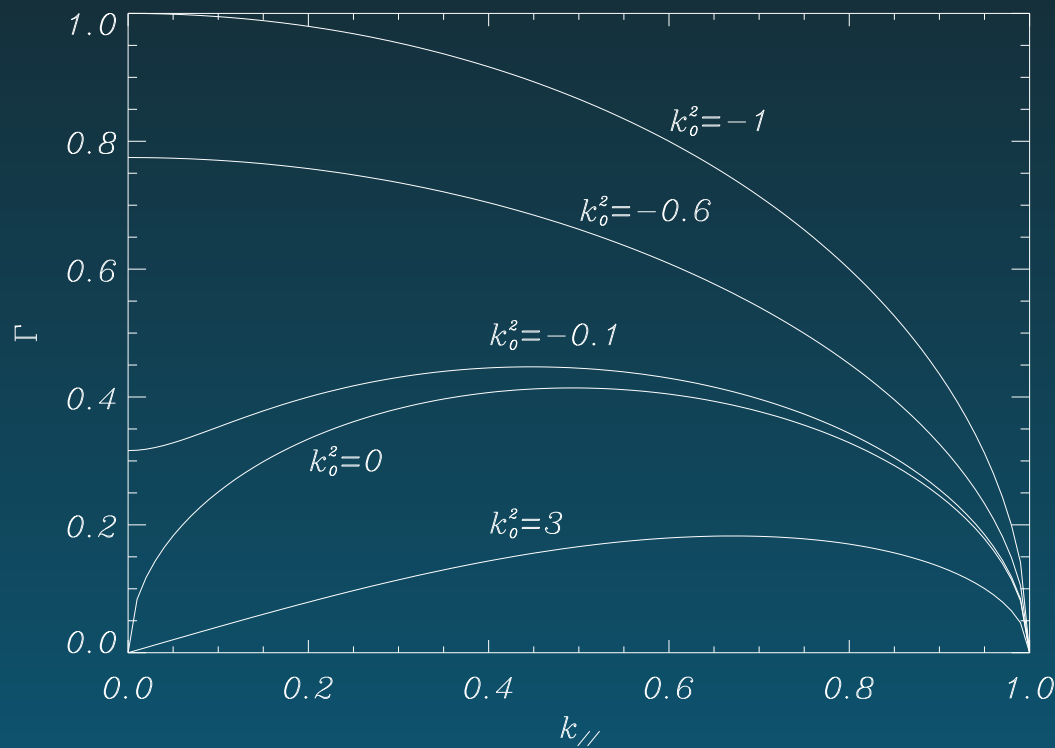
$$\omega_{-}^2 = \frac{V_A^2}{2} \left\{ \frac{1 + 2\beta}{1 + \beta} k_{\parallel}^2 + k_o^2 - \sqrt{\left(\frac{1 + 2\beta}{1 + \beta} k_{\parallel}^2 + k_o^2 \right)^2 - \frac{4\beta}{1 + \beta} k_{\parallel}^2 (k_{\parallel}^2 - k_c^2)} \right\}$$

$$k_c^2 = \underbrace{\left(\Gamma^+ \Gamma^- \right)_{AA}}_{\text{Geometry}} + \underbrace{2\beta \mathcal{K}_{cA} \mathcal{K}_{\rho A}}_{\text{Interchange}} \quad \text{et} \quad k_o^2 = -k_c^2 + \underbrace{\frac{4\beta}{1 + \beta} \mathcal{K}_{cA}^2}_{\text{Compression}}$$

$$\omega^2 < 0 \text{ when } k_c^2 > k_{\parallel}^2 > 0$$

Nature of the Solutions with $\omega^2 < 0$

Instability in the limit $k_{\parallel} \rightarrow 0$: two different behaviors



- $k_o^2 > 0 \Rightarrow$ Magnetosonic mode :

$$\omega_-^2 \simeq -\frac{k_c^2}{|k_o^2|} k_{\parallel}^2 V_S^2$$

- $k_o^2 < 0 \Rightarrow$ Alfvénic mode :

$$\omega_-^2 \simeq -|k_o|^2 V_A^2$$

$k_{\parallel \max}$ and Γ_{\max} depend on k_o^2

Coherence of this Approach

- $\vec{e}_r // \vec{e}_A \Rightarrow$ relaxation of the iconal equation

$$\xi(r) \exp(i\Phi) \quad \text{with} \quad k_{\perp} = \vec{e}_l \cdot \nabla \Phi$$

- Magnetic shear \Rightarrow fast variations toward \vec{e}_A i.e. $\partial/\partial r \propto k_{\perp}$
- But ξ_A must slowly variate \Rightarrow cylindrical studie valid in the vicinity of the resonance surfaces

$$\text{Magnetic surfaces such as for each modes } k_{\parallel} = \vec{e}_{\parallel} \cdot \nabla \Phi = 0$$

- The alfvénic mode is more relevant except the Z pinch configuration

$$\Rightarrow B_{o\theta} \gg B_{oz} \quad \text{et} \quad \frac{d \ln |\psi|}{d \ln r} \sim 1$$

- Existence of a radial global mode which maximize the instability (Goedbloed & Sakanaka, 1974)
 \Rightarrow Rotation ?

Study Including Rotation

- Modification of the equilibrium scales

$$\beta\mathcal{K}_{\rho_A} = \mathcal{K}_{c_A} - \mathcal{K}_{b_A} - \frac{\chi}{r} \quad \text{with} \quad \chi = \frac{r^2\Omega_o^2}{V_A^2}$$

- Equilibrium $\Rightarrow \chi \lesssim 1$
- Effect of the rotation :
 - ★ Coriolis force stabilizing : epicyclic oscillations
 - ★ Inertia force and δT_A of the same nature
- New coupling \Rightarrow equation of dispersion non-analytically tractable

Interiors of Jets Instable

- Jet launched from a disc \Rightarrow plasma inertia create azimuthal magnetic field

in the vicinity of the centre :

$$\frac{B_{o\theta}}{B_{oz}} = \frac{r}{v_z} (\Omega_o - \Omega_*) \Rightarrow \begin{cases} \Omega_o - \Omega_* \rightarrow 0 \\ B_{oz} > B_{o\theta} \\ B_{o\theta}/B_{oz} \propto r^\alpha, \alpha > 1 \end{cases}$$

(Pelletier & Pudritz, 1992)

- Magnetic tension increases with $r \Rightarrow$ negative magnetic shear S
- Geometrical term strong and destabilizing :

$$\omega^2 \simeq 2 \frac{V_A^2}{r^2} \frac{B_{o\theta}^2 B_{oz}^2}{B_o^4} S < 0$$

- Choice of the mode \Rightarrow criteria linear but not quadratic S (Suydam, 1958)

Interiors of Jets Instable

- Radial variations of B_{oz}
 - ★ $B_{oz} \gg B_{o\theta} \Rightarrow B_{oz}$ confine the plasma : $\frac{d}{dr}B_{oz} < 0$
 - ★ S as well as $\mathcal{K}_{c_A}\mathcal{K}_{\rho_A}$ destabilize the plasma

- Weak role of the rotation \equiv inertial and Coriolis forces
 - ★ Always instable if B_{oz} varies in r
 - ★ Otherwise it requires a fast increase of $B_{o\theta}$ ($\propto r^4$ si $\beta = 1$)

Confinement Regions

- Confinement of the jets $\Rightarrow B_{o\theta} \gg B_{oz}$ (widening and flux conservation)
- Z-pinch configuration (Kadomtsev, 1966)

★ Instability if $0 < k_{\parallel}^2 < k_c^2 \iff \frac{d \ln |B_o|}{d \ln r} > -\frac{B_o^2 \theta}{B_o^2} + \frac{1}{2} \left(m \frac{B_{o\theta}}{B_o} + r k_z \frac{B_{oz}}{B_o} \right)^2$

- ★ Alfvénic or magnetosonic ?

$$\text{Alfvénic si } \frac{d \ln |B_o|}{d \ln r} > \frac{\beta - 1}{\beta + 1} \frac{B_o^2 \theta}{B_o^2} \Rightarrow \text{easily achieved}$$

- Rotation stabilizing but must be dynamically significant !

Conclusion

- General set of equations to describe the pressure- and magnetic shear- driven instabilities
- Jet launched from accretion disc (weak rotation) \Rightarrow interior and confinement regions interchange-unstable
- Growth rate \equiv dynamical time scale $\omega^2 \propto V_A^2 / r^2$

Intrinsic analytical difficulties $\left\{ \begin{array}{l} \text{pression- and magnetic shear-driven instabilities} \\ \text{application to magnetized accretion-ejection} \end{array} \right.$

\Rightarrow Numerical simulations