Introduction to the Pressure-Driven Instabilities in Astrophysical Jets

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DAMTP, Cambridge — 5th of February 2002

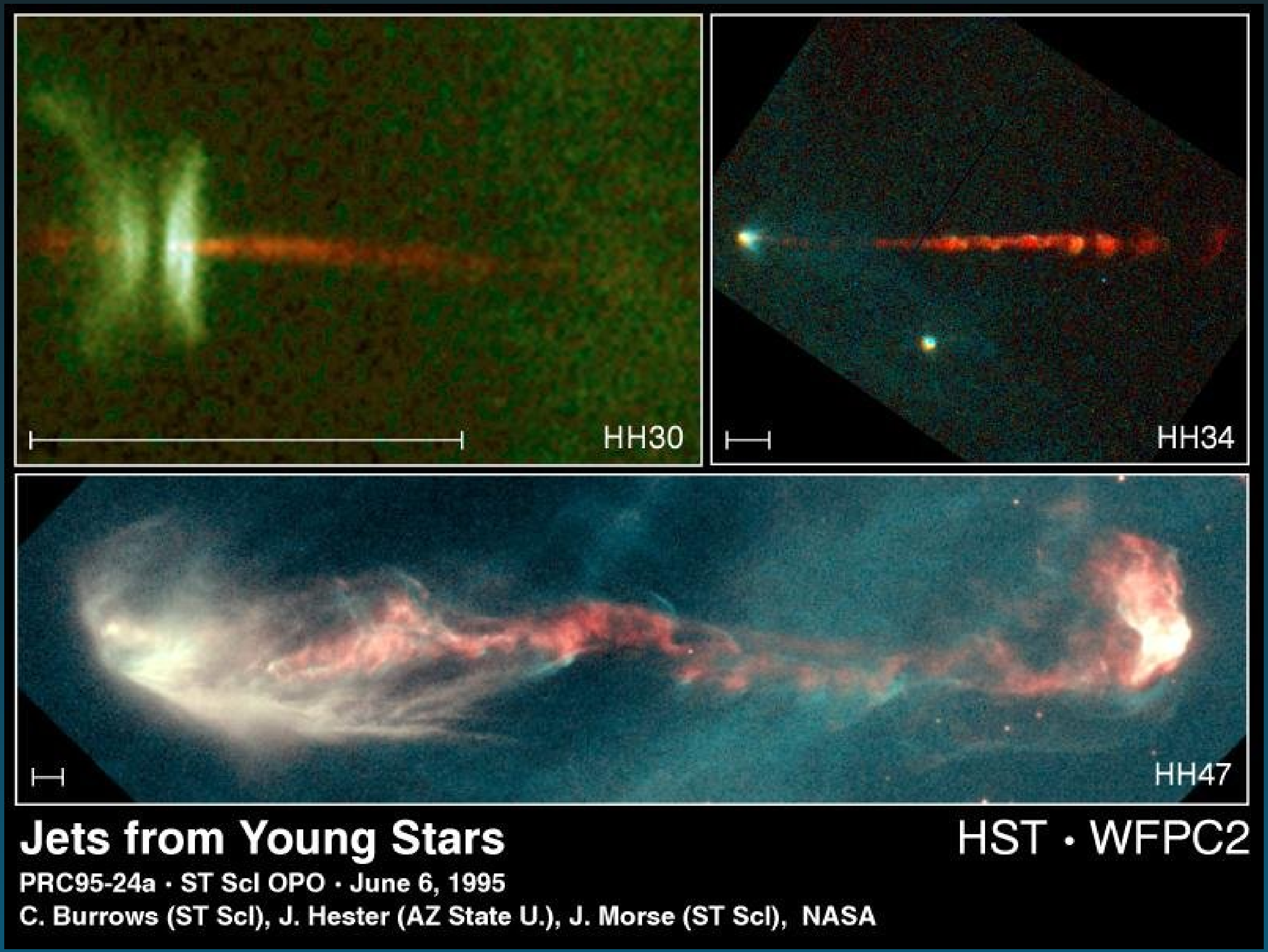
Ejection of Matter in Astrophysical Objects

Ejections of matter occur in many astrophysical environments

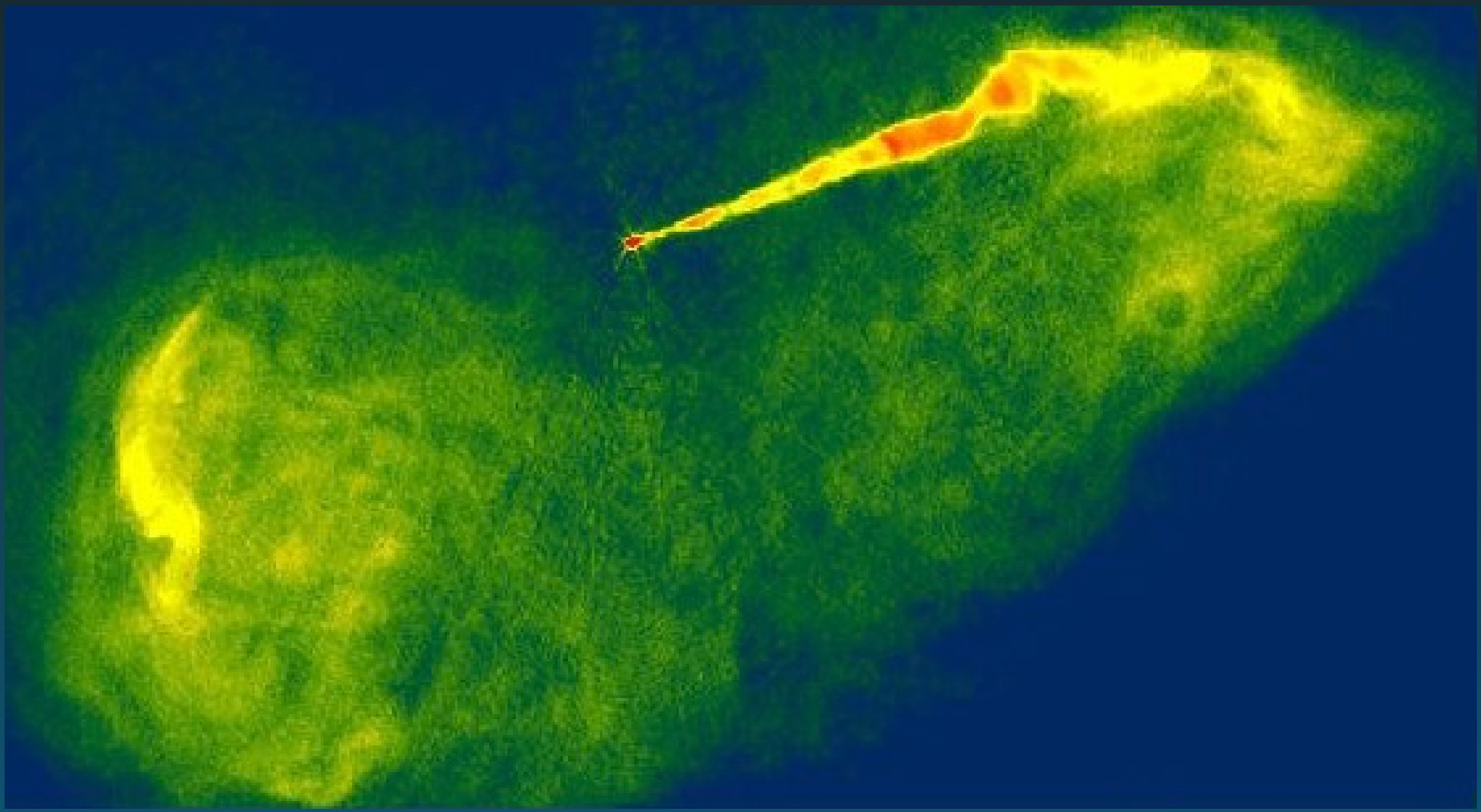
- Objects :
  - ⋆ Galactic : YSOs, stellar winds, CMEs, supernovæ, microquasars
  - ⋆ Extra-galactic : AGNs, GRBs
- Multiple :
  - ⋆ Sources of matter and ejection mechanisms
  - ⋆ Scales in space, time and energy

Stationary, highly collimated and non relativistic jets : YSOs, AGNs

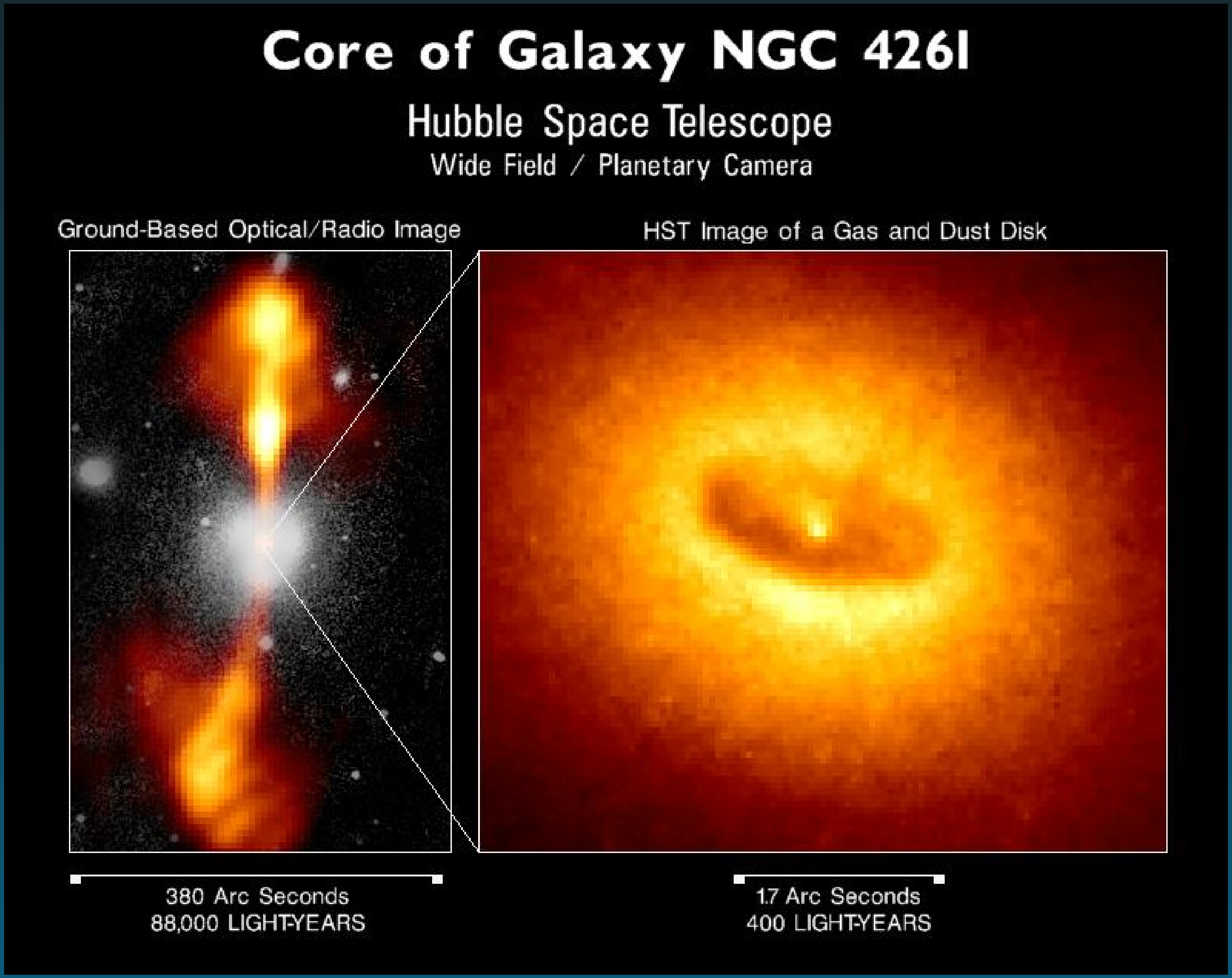
Jets in YSOs


HH30
HH34
HH47
Jets from Young Stars
HST • WFPC2
PRC95-24a • ST ScI OPO • June 6, 1995
C. Burrows (ST ScI), J. Hester (AZ State U.), J. Morse (ST ScI), NASA

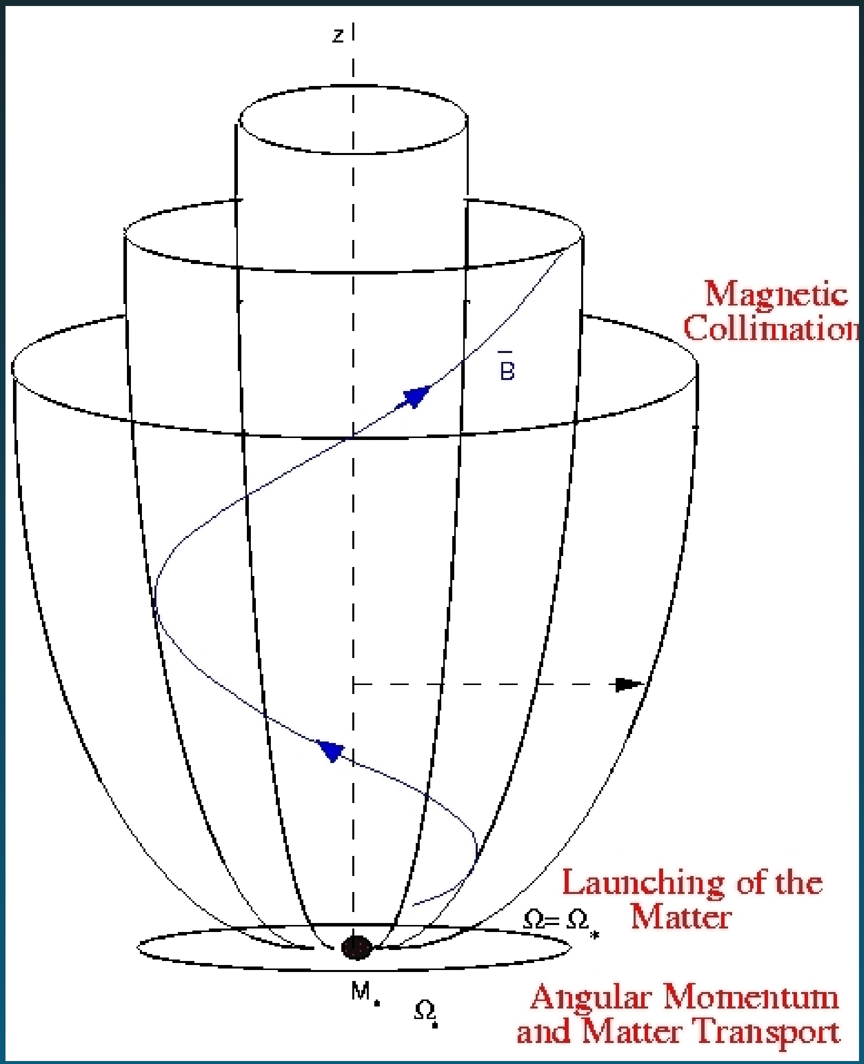
Jets in AGNs



Accretion-Ejection in AGNs



Magnetized Accretion-Ejection Structures



Motivations for this Work

- Source of energy in the interiors of jets ?
- Variations of the luminosity in jets : avalanche process ?
- Magnetized discs with $\beta = 1$: which mechanism is involved ?
- Sporadic ejection of matter : avalanche process ?

Magneto-Hydrodynamic

Plasma $\begin{cases} \text{completely ionized} \\ \text{magnetic field} \\ \text{non resistive} \end{cases}$

Continuity : $\dfrac{\partial \rho}{\partial t} = -\nabla \cdot \rho \, \vec{v}$

Induction : $\dfrac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$

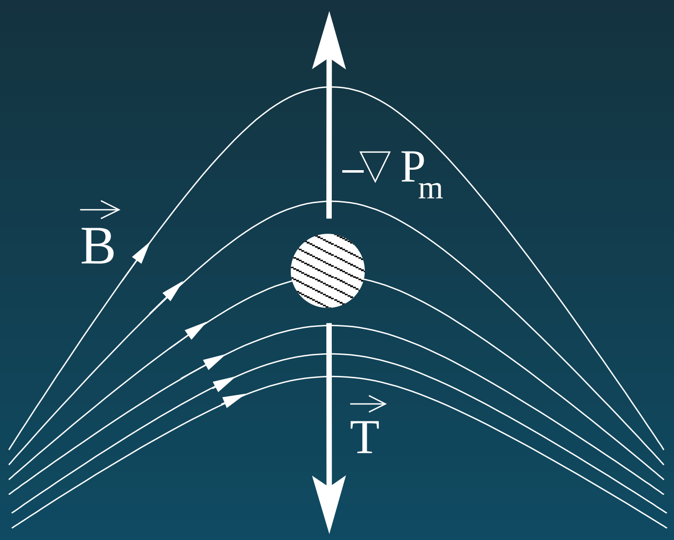
State : $P \propto \rho^{\gamma}$

Momentum : $\dfrac{\mathrm{d}\rho\vec{v}}{\mathrm{d}t} = -\nabla P + \vec{j} \times \vec{B} + \vec{\mathcal{F}}$

The magnetic field is frozen in the plasma

Characteristic scales

Laplace force : $\vec{\jmath} \times \vec{B} = -\nabla P_{\rm m} + \vec{T}$



- Pressure : $P_{\rm m} = \dfrac{B^2}{2\,\mu}$

- Tension : $\vec{T} = \dfrac{1}{\mu}\,\vec{B}\cdot\nabla\vec{B}$

Set of characteristic scales :

$\vec{\mathcal{K}}_\rho$: density — $\vec{\mathcal{K}}_b$: magnetic field — $\vec{\mathcal{K}}_c$: curvature

Energy Principle

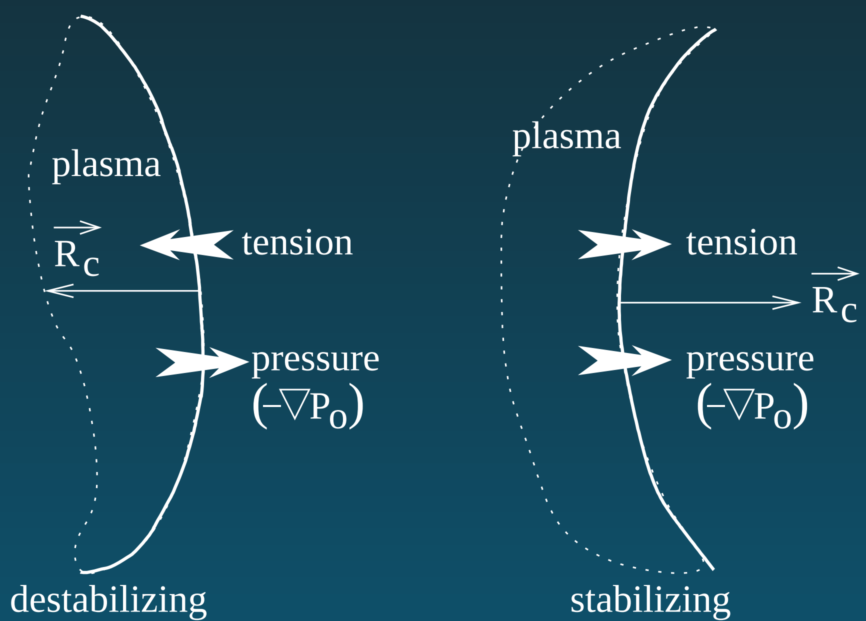
- Growth of an infinitesimal perturbation $\Rightarrow$ instability
- Conversion of potential energy in kinetic energy : $\delta W < 0$

$$\delta W_F = \frac{1}{2} \int_P \mathrm{d}^3\vec{r} \left[\underbrace{\frac{\left|\delta\vec{B}_\perp\right|^2}{\mu}}_{\text{Alfvén}} + \underbrace{\frac{B_o}{2\,\mu}\left|\nabla\cdot\vec{\xi}_\perp + 2\,\vec{\xi}_\perp\cdot\vec{\mathcal{K}}_c\right|^2}_{\text{magnetosonic}} + \underbrace{\gamma\,P_o\left|\nabla\cdot\vec{\xi}\right|^2}_{\text{sonic}} \right.$$
$$\left. \underbrace{-2\,\left(\vec{\xi}_\perp\cdot\nabla P_o\right)\,\left(\vec{\mathcal{K}}_c\cdot\vec{\xi}^*_\perp\right)}_{\text{interchange - ballooning}} - \underbrace{j_{o/\!/}\,\left(\vec{\xi}^*_\perp \times \vec{e}_{/\!/}\right)\cdot\delta\vec{B}_\perp}_{\text{kink}} \right]$$

MHD instabilities : the magnetic field increases

Non-Resitive Linear MHD Instabilities

Source $\left\{ \begin{array}{l} \text{Parallel Current} \quad (L_o k_\perp \sim 1) \\ \text{Thermal pressure} \quad (L_o k_\perp \gg 1) \end{array} \right.$ $\Longrightarrow$ Kink (works in the force-free limit)
$\Longrightarrow$ Interchange et Ballooning



Pressure Instability

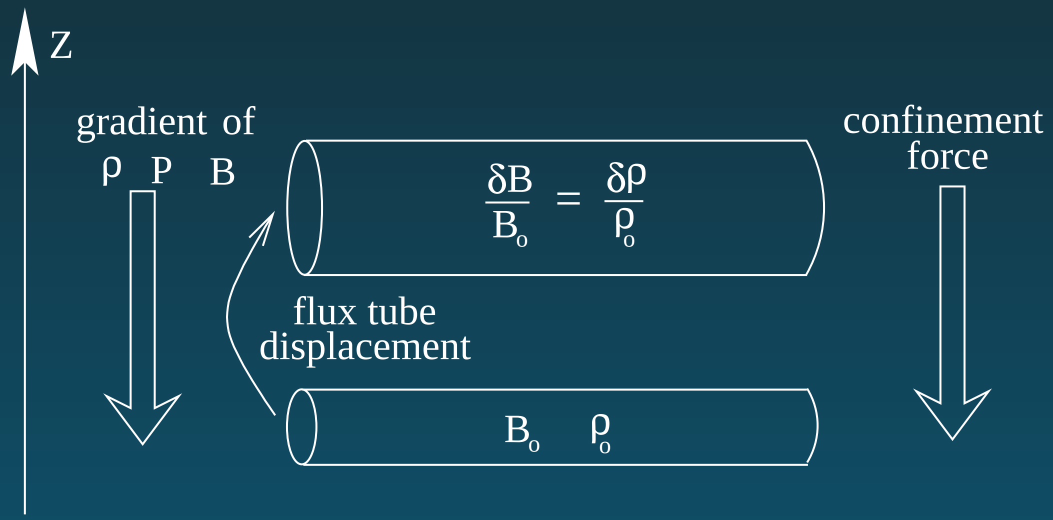
$\vec{\mathcal{K}}_c \cdot \nabla P_o < 0 \Rightarrow$ stabilizing

$\vec{\mathcal{K}}_c \cdot \nabla P_o > 0 \Rightarrow$ destabilizing

Interchange Process

Interchange of two fluid particles ⇒ decrease in potential energy ⇒ instability

Archetype : Rayleigh-Taylor instability



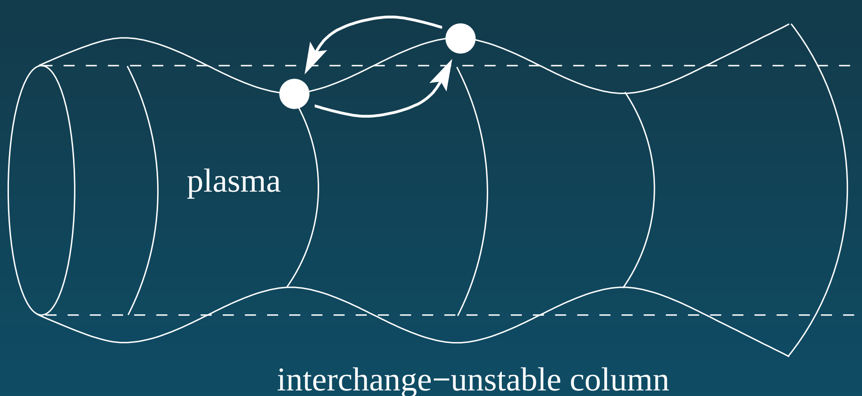
- Gravitational confinement
  ⇒Rayleigh-Taylor, convection
- In the presence of magnetic field
  ⇒Parker modes, magnetic buoyancy
- Magnetic confinement
  ⇒pressure-driven modes

Constrains : pressure equilibrium ; mass and magnetic flux conservation

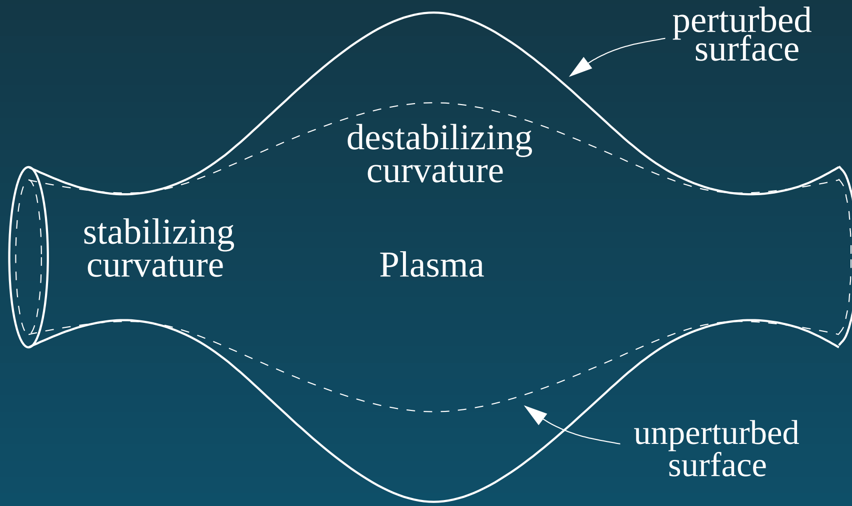
Interchange and Ballooning MHD modes

Plasma magnetically confined

Most actively studied in thermonuclear fusion, magnetospheric physics and solar physics but ignored in jets and discs



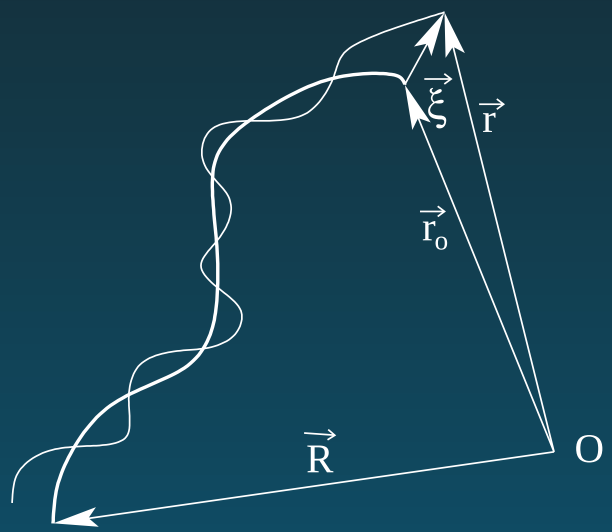
- Interchange mode
- Axial symmetry ⇒ "sausage"
- $m = 1$ ⇒ helical or "kink"



- Localized mode : Ballooning
- Curvature stabilizing or not
- highly constraining thermonuclear fusion

Linear Study : Perturbations

Infinitesimal displacement of fluid particles $\vec{\xi}$



Lagrangian perturbations :

$$\delta\rho = -\nabla\cdot\left(\rho_o\,\vec{\xi}\right)$$

$$\delta\vec{B} = -\nabla\times\left(\vec{\xi}\times\vec{B}_o\right)$$

$$\delta P = -\vec{\xi}\cdot\nabla P_o - \gamma P_o \nabla\cdot\vec{\xi}$$

- Equations written in termes of $\xi_{/\!/}$ and $\vec{\xi}_\perp$
- Evolution determined by the momentum equation

General Set of Linear MHD Equations

$$\frac{\partial^2 \xi_{/\!/}}{\partial t^2} = \mathcal{F}_{/\!/}(\vec{\xi}) + \frac{1}{\rho_o}\frac{\partial}{\partial s}\left[\rho_o\, C_S^2\, \left(\vec{\xi}\cdot\vec{\mathcal{K}}_\rho + \nabla\cdot\vec{\xi}\right)\right]$$
$$+ V_A^2\, \left(\vec{\mathcal{K}}_{b_\perp} - \vec{\mathcal{K}}_c\right)\cdot\left(\mathbf{\Gamma}^{-}\cdot\vec{\xi}_\perp + \frac{\partial \xi_l}{\partial s}\,\vec{e}_l + \frac{\partial \xi_A}{\partial s}\,\vec{e}_A\right)$$

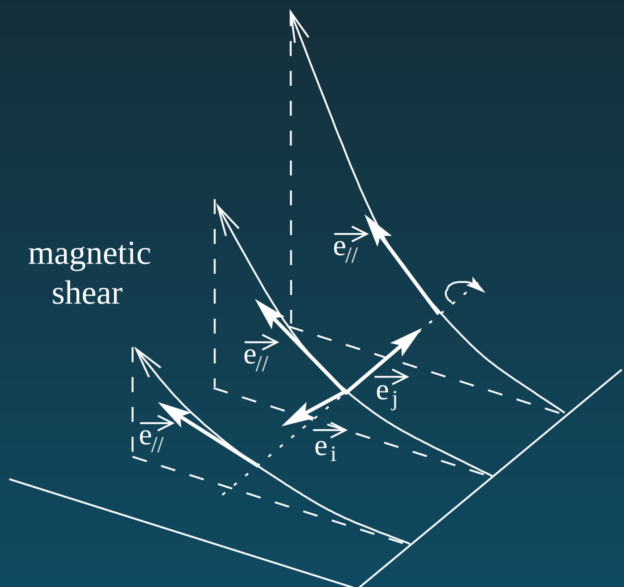
$$\frac{\partial^2 \vec{\xi}_\perp}{\partial t^2} = \vec{\mathcal{F}}_\perp(\vec{\xi}) + \frac{1}{\rho_o}\nabla_\perp\left\{\rho_o\, C_S^2\, \left(\vec{\xi}\cdot\vec{\mathcal{K}}_\rho + \nabla\cdot\vec{\xi}\right) + \rho_o\, V_A^2\left[\left(\vec{\mathcal{K}}_{b_\perp} + \vec{\mathcal{K}}_c\right)\cdot\vec{\xi}_\perp + \nabla\cdot\vec{\xi}_\perp\right]\right\}$$
$$+ V_A^2\left\{\left(\frac{\partial \delta b_l}{\partial s}\,\vec{e}_l + \frac{\partial \delta b_A}{\partial s}\,\vec{e}_A\right) + \mathbf{\Gamma}^{+}\cdot\vec{\delta b}_\perp + 2\,\delta b_{/\!/}\,\vec{\mathcal{K}}_c + \left(\vec{\delta b}\cdot\vec{\mathcal{K}}_c\right)\,\vec{e}_{/\!/} + \mathcal{K}_{b_{/\!/}}\,\vec{\delta b}_\perp\right\}$$

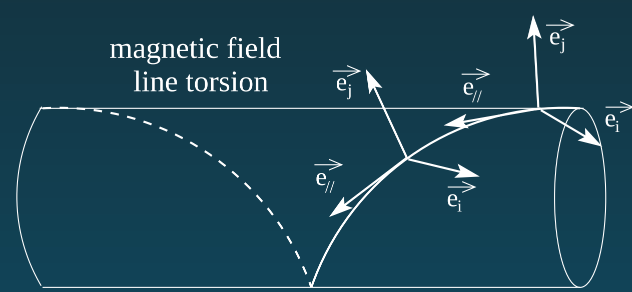
with $\delta b_{/\!/} = -\nabla\cdot\vec{\xi}_\perp - \left(\vec{\mathcal{K}}_{b_\perp} + \vec{\mathcal{K}}_c\right)\cdot\vec{\xi}_\perp$

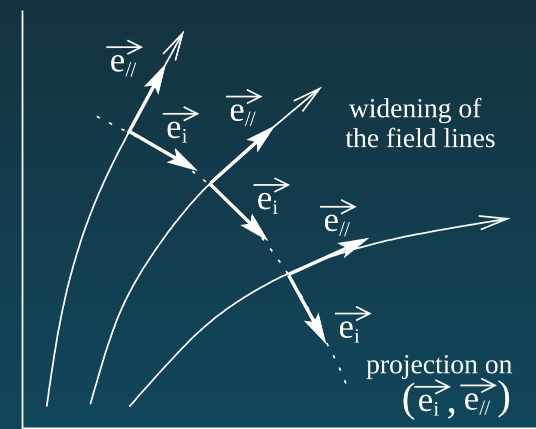
and $\vec{\delta b}_\perp = \mathbf{\Gamma}^{-}\cdot\vec{\xi}_\perp + \dfrac{\partial \xi_l}{\partial s}\,\vec{e}_l + \dfrac{\partial \xi_A}{\partial s}\,\vec{e}_A + \left(\vec{\xi}\cdot\vec{\mathcal{K}}_c\right)\,\vec{e}_{/\!/}$

Geometry of the Basic State Magnetic Field

Tensor $\mathbf{\Gamma}^{\pm}$



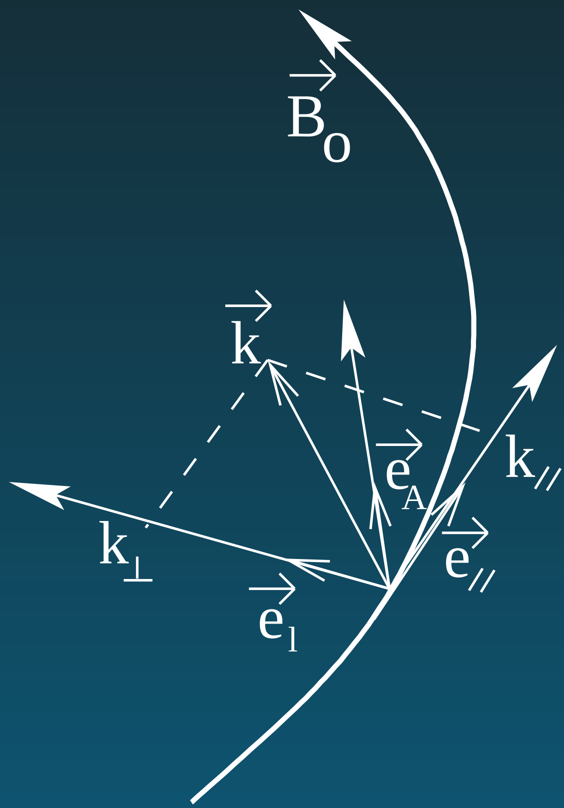




$$\mathbf{\Gamma}^{\pm}_{\mathbf{ii}} = \pm \vec{e}_i \cdot \left[\left(\vec{e}_i \cdot \nabla \right) \vec{e}_{/\!/} \right]$$

$$\mathbf{\Gamma}^{\pm}_{\mathbf{ij}} = \vec{e}_i \cdot \left[\left(\vec{e}_{/\!/} \cdot \nabla \right) \vec{e}_j \pm \left(\vec{e}_j \cdot \nabla \right) \vec{e}_{/\!/} \right]$$

Linearized MHD Equations



First stage : to linearize the MHD

- Hypotheses
  - ⋆ Ideal MHD and barotropic equation of state
  - ⋆ Perturbations study ⇒ linear equations
  - ⋆ Heterogeneous plasma and any magnetic structure
- We get a general differential system
  - ⋆ Two coupled equations for the displacement
  - ⋆ Derivation following the magnetic field lines
  - ⋆ Still valid in the presence of external forces

Further analytical study requires more simplifications

Derivation of the "Ballooning" equations

- Restrictions on the perturbation variations
  - ⋆ Ordering imposed : $k_\perp L_o \gg 1$ and $k_\perp \gg k_{/\!/} \Rightarrow L_o\omega \sim V_A$ and $\xi_{/\!/} \sim \xi_A \gg \xi_l$
  - ⋆ Slowly quasi-transversal propagation (Newcomb, 1961)
  - ⋆ fast magnetosonic waves, mostly compressive, decoupled

- Transversal momentum equation : $\delta P_m + \delta P_{th} = 0$

- No particular mode assumed : neither Fourier transform, nor WKB approximation

- Evolution equations include heterogeneities and magnetic field geometry :

$$\frac{\partial^2 \xi_A}{\partial t^2} - \mathcal{D}_A \cdot \xi_A = \mathcal{C}_A \cdot \xi_{/\!/} + \mathcal{F}_A(\vec{\xi})$$

$$\frac{\partial^2 \xi_{/\!/}}{\partial t^2} - \mathcal{D}_{/\!/} \cdot \xi_{/\!/} = \mathcal{C}_{/\!/} \cdot \xi_A + \mathcal{F}_{/\!/}(\vec{\xi})$$

Purely MHD Equilibrium

- Pressure-tension transversal equilibrium : $\beta\vec{\mathcal{K}}_\rho = \vec{\mathcal{K}}_c - \vec{\mathcal{K}}_{b_\perp}$ with $\beta = C_S^2/V_A^2$

- Differential system :

$$\frac{\partial^2 \xi_{/\!/}}{\partial t^2} - C_S^2 \frac{\partial}{\partial s} \nabla\cdot\vec{\xi} = 0$$

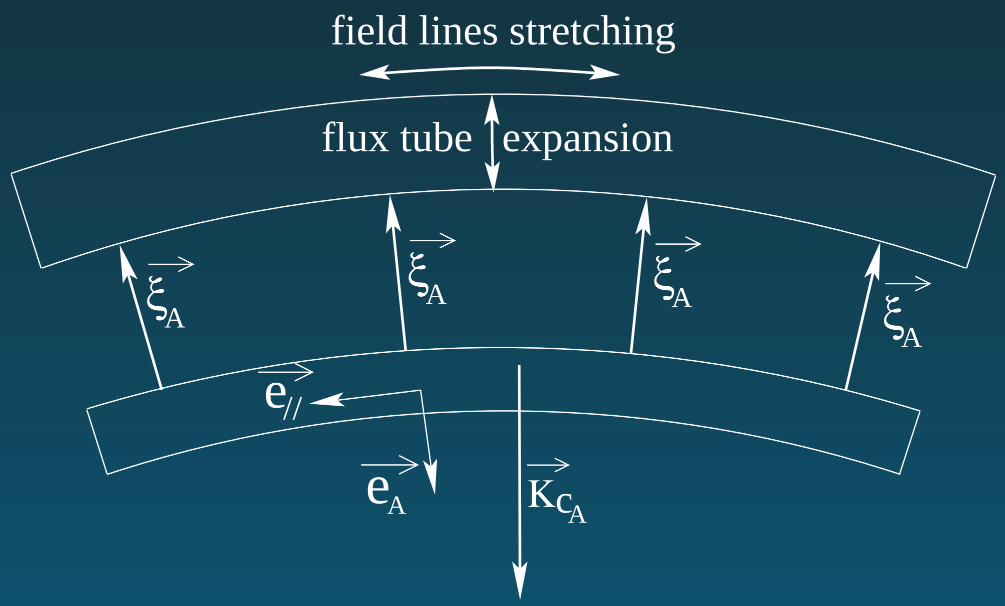
$$\frac{\partial^2 \xi_A}{\partial t^2} - V_A^2 \left[\frac{\partial^2}{\partial s^2} + \mathcal{K}_{b_{/\!/}} \frac{\partial}{\partial s} + 2\,\beta\,\mathcal{K}_{c_A}\mathcal{K}_{\rho_A} + \frac{\partial}{\partial s}\mathbf{\Gamma}^-_{\mathbf{AA}} + \mathcal{K}_{b_{/\!/}}\mathbf{\Gamma}^-_{\mathbf{AA}} + \left(\mathbf{\Gamma}^+\mathbf{\Gamma}^-\right)_{\mathbf{AA}} \right] \xi_A = 2\,C_S^2\,\mathcal{K}_{c_A}\,\nabla\cdot\vec{\xi}$$

- Topology of the magnetic field in $\mathcal{K}_{b_{/\!/}}$ and $\mathbf{\Gamma}^+$: widening, torsion and shear

- Coupled by $\nabla\cdot\vec{\xi} \equiv$ curvature and parallel variations

No Parallel Variation

Constant pressure $\Rightarrow$ destabilization by the fluctuation of the transversal tension



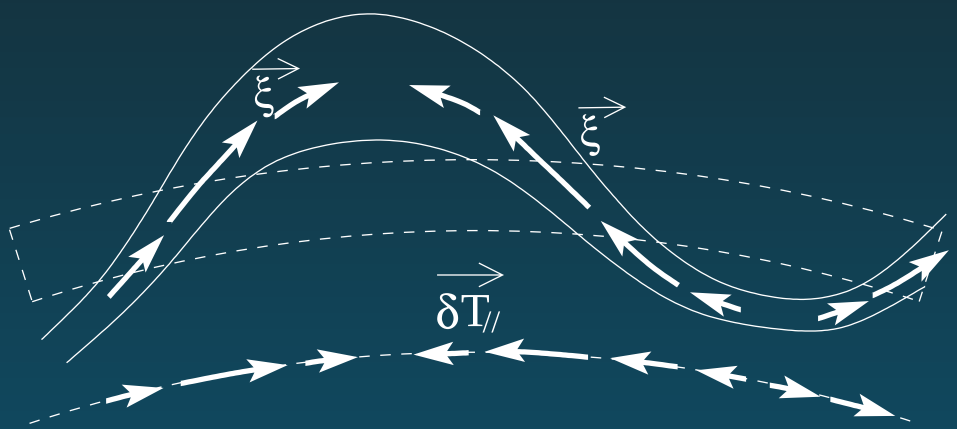
$$\delta T_A = 2\rho_o V_A^2 \mathcal{K}_{c_A} \delta b_{/\!/}$$

$$\delta b_{/\!/} = \frac{\beta}{1+\beta}\left(\mathcal{K}_{\rho_A} - \mathcal{K}_{c_A} - \mathcal{K}_{b_A}\right) \xi_A$$

Instability criteria : $\mathcal{K}_{c_A}\mathcal{K}_{\rho_A} > \dfrac{2}{1+\beta}\mathcal{K}_{c_A}^2$

In the Presence of Parallel Variations

Crutial role of the fluctuations of the parallel tension ⇒ interpretation less obvious



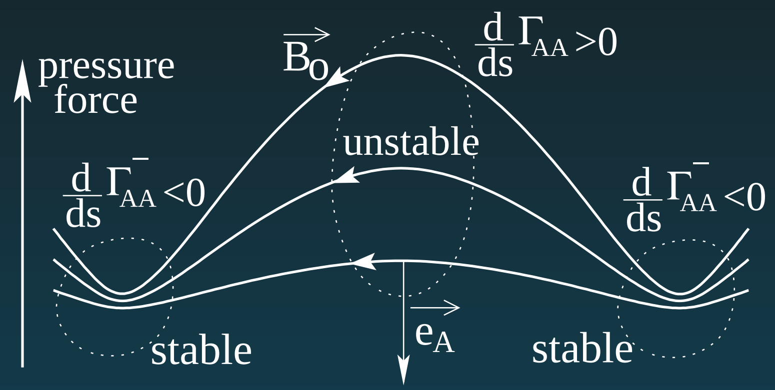
But in the case :

- Shape of the mode $\exp[i(k_{/\!/} s - \omega t)]$
- Limit $L_o k_{/\!/} \ll 1$

$$\omega^2 = -2\beta \frac{\mathcal{K}_{c_A}\mathcal{K}_{\rho_A}}{k_o^2} k_{/\!/}^2 V_S^2$$

- Sufficient condition for instability : $\mathcal{K}_{c_A}\mathcal{K}_{\rho_A} > 0$
- The plasma can be instable in spite of a stabilizing δT_A

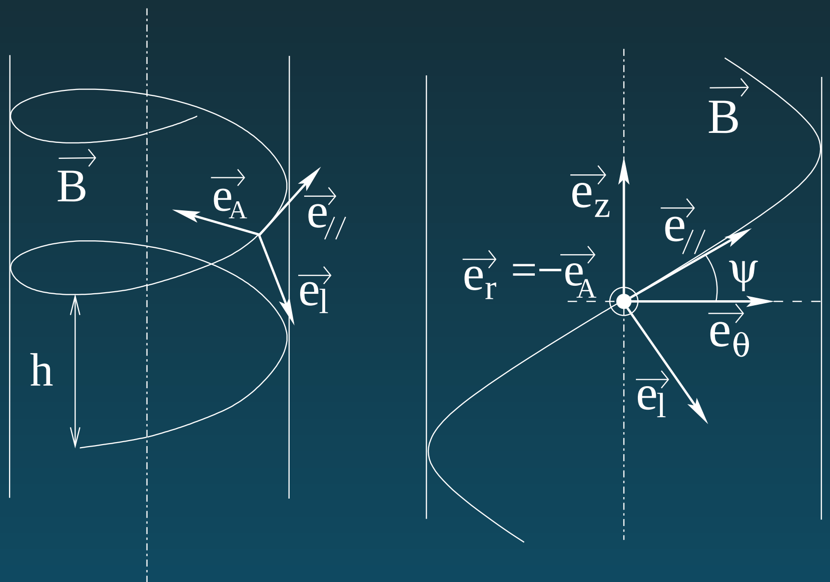
Geometric Terms



- Geometric terms $\Rightarrow$ contribution in $\delta\Gamma_A$
- The stability depends on the curvature and its zero-order variations
- Widening and torsion $\Rightarrow$ curvature stabilizing or destabilizing
- Localized modes of Ballooning

Cylindrical Asymptotic Rotating Jet

Cylindrical column , axial symmetry and vertically invariant



- Quasi-solid Rotation without vertical movement : $v_{o\theta} = r\Omega_o$ et $v_{oz} = 0$
- Cylindrical magnetic surfaces : $\vec{B}_o = B_o \vec{e}_{/\!/} = B_{o\theta}\,\vec{e}_\theta + B_{oz}\,\vec{e}_z$
- Instability maximized if $\vec{e}_A /\!/ \vec{\mathcal{K}}_c$ $\Rightarrow \vec{e}_A = -\vec{e}_r$

Exact Fourier transform on the magnetic surfaces :

$$\xi \rightarrow \xi(r)\exp[i(m\theta + k_z z - \omega t)]$$

Non-rotating Jet

- Analytically tractable 4th-order equation of dispersion : Alfvén and slow magnetosonic modes

- Only one root can be negative :

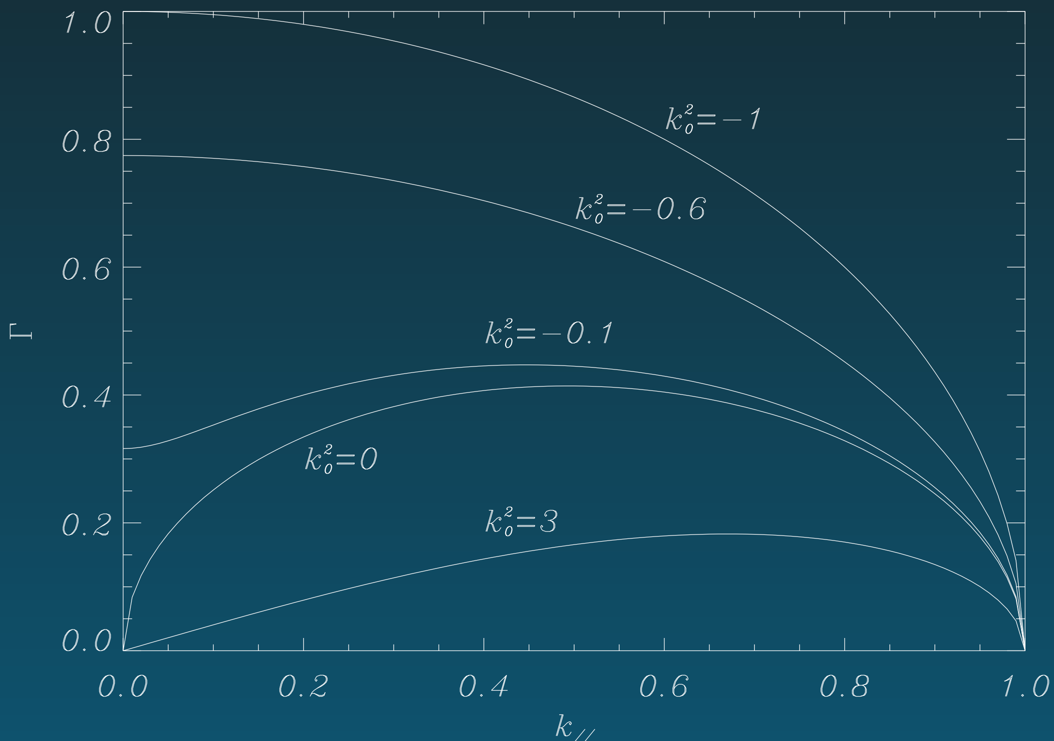
$$\omega_-^2 = \frac{V_A^2}{2}\left\{\frac{1+2\beta}{1+\beta}k_{/\!/}^2 + k_o^2 - \sqrt{\left(\frac{1+2\beta}{1+\beta}k_{/\!/}^2 + k_o^2\right)^2 - \frac{4\beta}{1+\beta}k_{/\!/}^2\left(k_{/\!/}^2 - k_c^2\right)}\right\}$$

$$k_c^2 = \underbrace{\left(\boldsymbol{\Gamma}^+\boldsymbol{\Gamma}^-\right)_{\mathbf{AA}}}_{\text{Geometry}} + \underbrace{2\,\beta\,\mathcal{K}_{c_A}\mathcal{K}_{\rho_A}}_{\text{Interchange}} \quad \text{et} \quad k_o^2 = -k_c^2 + \underbrace{\frac{4\beta}{1+\beta}\mathcal{K}_{c_A}^2}_{\text{Compression}}$$

$\omega^2 < 0$ when $k_c^2 > k_{/\!/}^2 > 0$

Nature of the Solutions with $\omega^2 < 0$

Instability in the limit $k_{/\!/} \to 0$: two different behaviors



- $k_o^2 > 0 \Rightarrow$ Magnetosonic mode :

$$\omega_-^2 \simeq -\frac{k_c^2}{|k_o^2|} k_{/\!/}^2 V_S^2$$

- $k_o^2 < 0 \Rightarrow$ Alfvénic mode :

$$\omega_-^2 \simeq -|k_o|^2 V_A^2$$

$k_{/\!/\,\mathrm{max}}$ and Γ_{max} depend on k_o^2

Coherence of this Approach

- $\vec{e}_r /\!/ \vec{e}_A \Rightarrow$ relaxation of the iconal equation

$$\xi(r)\exp(i\Phi) \quad \text{with} \quad k_\perp = \vec{e}_l \cdot \nabla\Phi$$

- Magnetic shear $\Rightarrow$ fast variations toward $\vec{e}_A$ *i.e.* $\partial/\partial r \propto k_\perp$

- But ξ_A must slowly variate $\Rightarrow$ cylindrical studie valid in the vicinity of the resonance surfaces

Magnetic surfaces such as for each modes $k_{/\!/} = \vec{e}_{/\!/} \cdot \nabla\Phi = 0$

- The alfvénic mode is more relevant except the Z pinch configuration

$$\Rightarrow B_{o\theta} \gg B_{oz} \quad \text{et} \quad \frac{\mathrm{d}\ln|\psi|}{\mathrm{d}\ln r} \sim 1$$

- Existence of a radial global mode which maximize the instability (Goedbloed & Sakanaka, 1974)
$\Rightarrow$ Rotation ?

Study Including Rotation

- Modification of the equilibrium scales

$$\beta \mathcal{K}_{\rho_A} = \mathcal{K}_{c_A} - \mathcal{K}_{b_A} - \frac{\chi}{r} \quad \text{with} \quad \chi = \frac{r^2 \Omega_o^2}{V_A^2}$$

- Equilibrium $\Rightarrow \chi \lesssim 1$

- Effect of the rotation :
  - ⋆ Coriolis force stabilizing : epicyclic oscillations
  - ⋆ Inertia force and δT_A of the same nature

- New coupling $\Rightarrow$ equation of dispersion non-analytically tractable

Interiors of Jets Instable

- Jet launched from a disc $\Rightarrow$ plasma inertia create azimumuthal magnetic field

  in the vicinity of the centre : $\dfrac{B_{o\theta}}{B_{oz}} = \dfrac{r}{v_z}(\Omega_o - \Omega_\star) \Rightarrow \begin{cases} \Omega_o - \Omega_\star \to 0 \\ B_{oz} > B_{o\theta} \\ B_{o\theta}/B_{oz} \propto r^\alpha, \alpha > 1 \end{cases}$

  (Pelletier & Pudritz, 1992)

- Magnetic tension increases with $r \Rightarrow$ negative magnetic shear S

- Geometrical term strong and destabilizing :

$$\omega^2 \simeq 2\,\frac{V_A^2}{r^2}\,\frac{B_{o\,\theta}^2\,B_{o\,z}^2}{B_o^4}\;S < 0$$

- Choice of the mode $\Rightarrow$ criteria linear but not quadratic S (Suydam, 1958)

Interiors of Jets Instable

- Radial variations of B_{oz}
  - ⋆ $B_{oz} \gg B_{o\theta} \Rightarrow B_{oz}$ confine the plasma : $\frac{\mathrm{d}}{\mathrm{d}r} B_{oz} < 0$
  - ⋆ S as well as $\mathcal{K}_{c_A} \mathcal{K}_{\rho_A}$ destabilize the plasma
- Weak role of the rotation $\equiv$ inertial and Coriolis forces
  - ⋆ Always instable if B_{oz} varies in r
  - ⋆ Otherwise it requires a fast increase of $B_{o\theta}$ $\left(\propto r^4 \text{ si } \beta = 1\right)$

Confinement Regions

- Confinement of the jets $\Rightarrow B_{o\theta} \gg B_{oz}$ (widening and flux conservation)

- Z-pinch configuration (Kadomtsev, 1966)

  ⋆ Instability if $0 < k_{/\!/}^2 < k_c^2 \Longleftrightarrow \dfrac{\mathrm{d}\ln|B_o|}{\mathrm{d}\ln r} > -\dfrac{B_{o\,\theta}^2}{B_o^2} + \dfrac{1}{2}\left(m\,\dfrac{B_{o\theta}}{B_o} + r\,k_z\,\dfrac{B_{o\,z}}{B_o}\right)^2$

  ⋆ Alfvénic or magnetosonic ?

$$\text{Alfvénic si} \quad \frac{\mathrm{d}\ln|B_o|}{\mathrm{d}\ln r} > \frac{\beta - 1}{\beta + 1}\,\frac{B_{o\,\theta}^2}{B_o^2} \quad \Rightarrow \quad \text{easily achieved}$$

- Rotation stabilizing but must be dynamically significant !

Conclusion

- General set of equations to describe the pressure- and magnetic shear- driven instabilities
- Jet launched from accretion disc (weak rotation) $\Rightarrow$ interior and confinement regions interchange-unstable
- Growth rate $\equiv$ dynamical time scale $\omega^2 \propto V_A^2/r^2$

Intrinsic analytical difficulties $\left\{ \begin{array}{l} \text{pression- and magnetic shear-driven instabilities} \\ \text{application to magnetized accretion-ejection} \end{array} \right.$

$\Rightarrow$ Numerical simulations