The role of coherent structures in the saturation of magnetorotational instabilities

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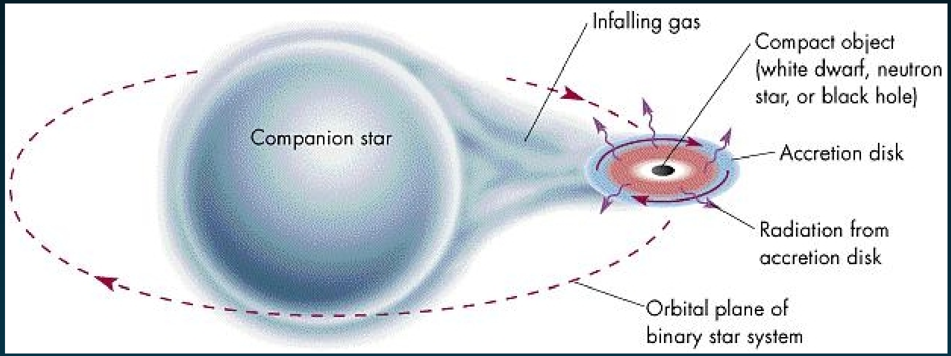
Magnetorotational instability in accretion discs

Accretion discs: systems in differential rotation under the action of a gravitational field. Orbiting gas can be accreted by the central object if its angular momentum is removed by a torque acting within or on the disc

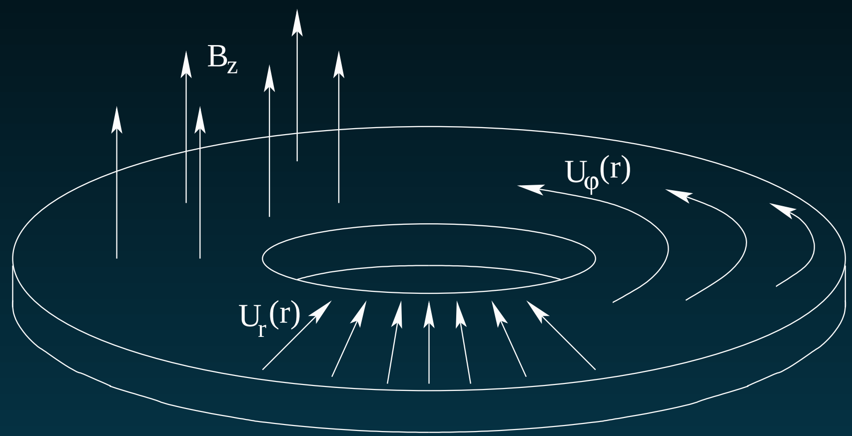


MRI: give some important insight into the nature of the mechanism leading to transport of angular momentum (e.g. Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991)

- Weakly magnetised differentially rotating flows linearly unstable if $\mathrm{d}\Omega/\mathrm{d}r < 0$
- Differential rotation source of free energy $\Rightarrow$ MRI extremely powerful $\gamma \sim r|\mathrm{d}\Omega/\mathrm{d}r|$
- Nonlinear evolution leads to 3-D turbulence and outwards transport of angular momentum

But further studies required to understand the saturation processes and to determine the level of saturation of turbulence

Global model with accretion



Evolution: incompressible non-ideal MHD

$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{U} = -\nabla\Phi - \nabla\Pi + \mathbf{B}\cdot\nabla\mathbf{B} + \nu\nabla^2\mathbf{U},$$
$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{U} + \eta\nabla^2\mathbf{B},$$
$$\nabla\cdot\mathbf{B} = \nabla\cdot\mathbf{U} = 0.$$

Basic state: magnetised Keplerian shearing flow with accretion (φ- and z-invariant)

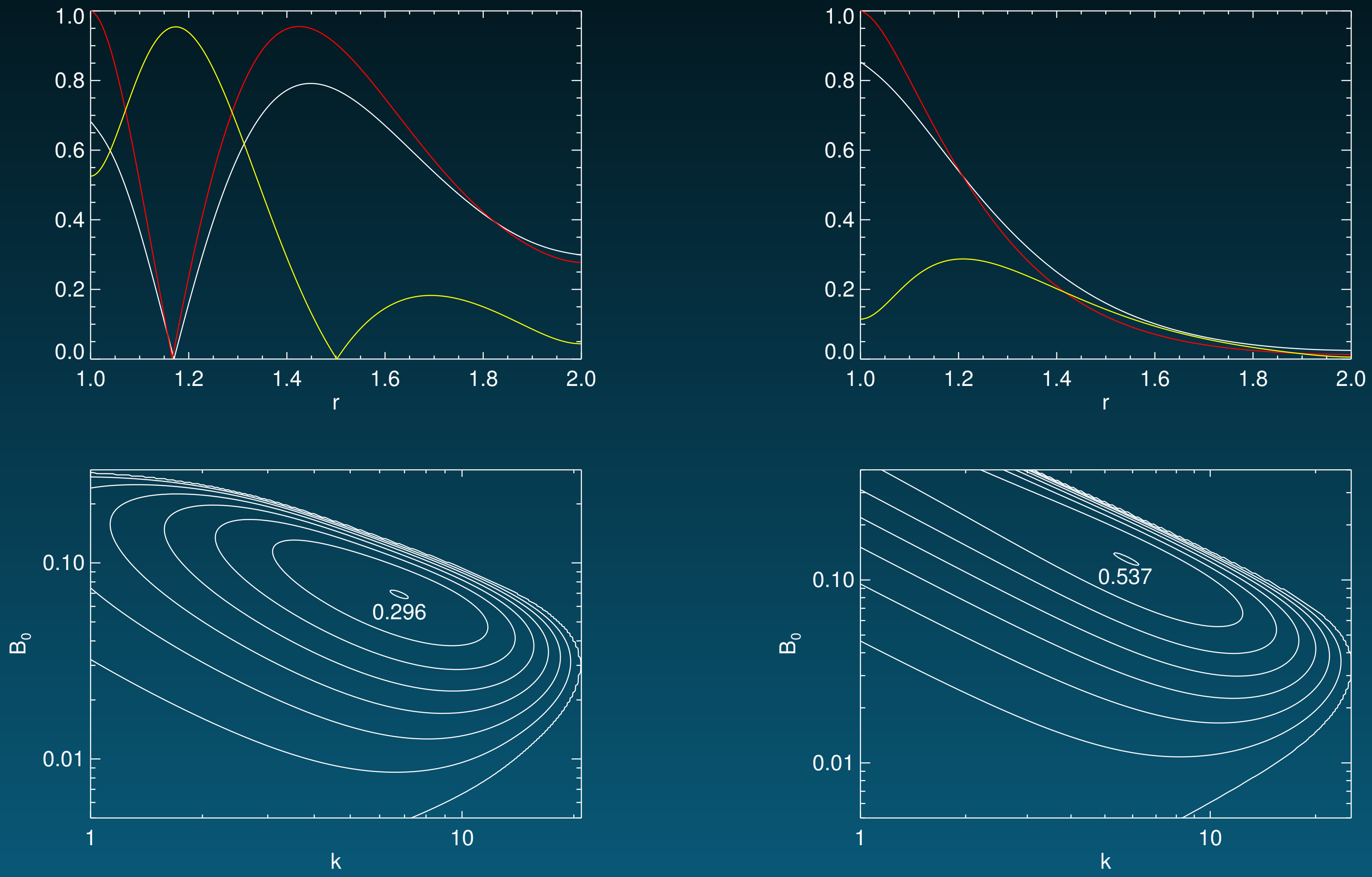
$$U_r = -3\nu/2r, \quad U_\varphi = 1/\sqrt{r}, \quad U_z = 0, \quad B_r = B_\varphi = 0, \quad B_z = B_0, \quad \Pi = \delta - 9\nu^2/8r^2$$

Radial boundary conditions: permeable (no conditions on U_r and B_r)

$$\partial_r(\sqrt{r}U_\varphi) = 0, \quad \partial_r U_z = 0, \quad \Pi = \Pi_0, \quad \partial_r(rB_\varphi) = 0, \quad \partial_r B_z = 0$$

Linear stability theory

Permeable radial boundaries permit the development of wall modes as well as body modes



Axisymmetric nonlinear evolution

Numerical investigation of the nonlinear development of axisymmetric wall modes

Features of the code developed : cylindrical geometry, pseudo-spectral, semi-implicit, adaptive time step

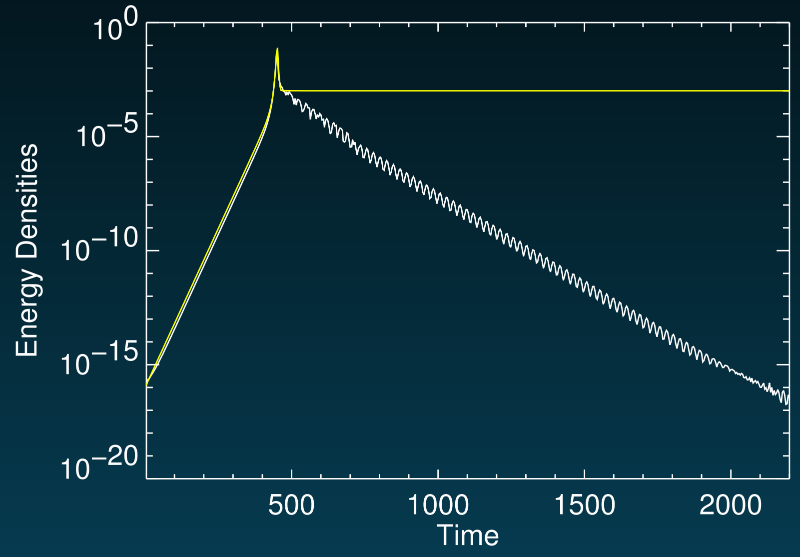
Parameters set in order to trigger only one linearly unstable wall mode:

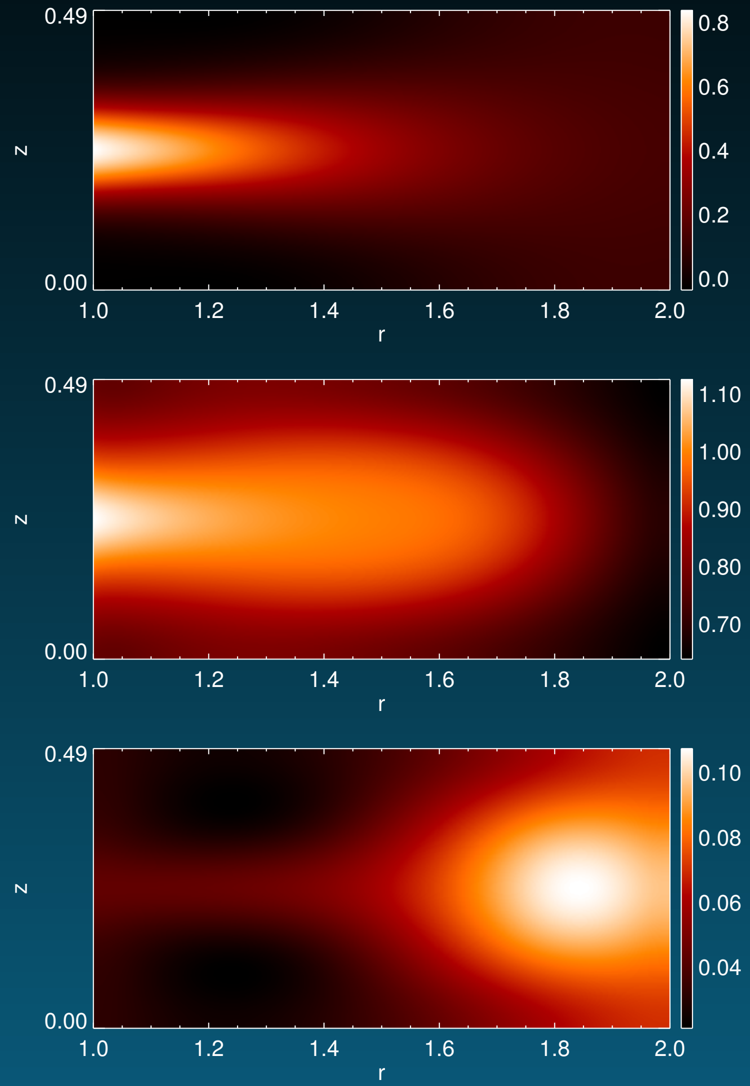
$$R_{\text{in}} = 1, \quad \Omega(R_{\text{in}}) = 1, \quad B_0 \simeq 10^{-1}\text{-}10^{-2},$$
$$H = 0.5, \quad R_{\text{out}} = \{2, 4\}, \quad \mathcal{R}_e = \mathcal{R}_m \simeq 150\text{-}500$$

We find:

- Exponential growth in agreement with the linear theory
- No turbulent motion but coherent structures: radial jet propagating outwards
- Suppression mechanism for the instability
- Cyclic evolutions

Self-consistent suppression of the instability

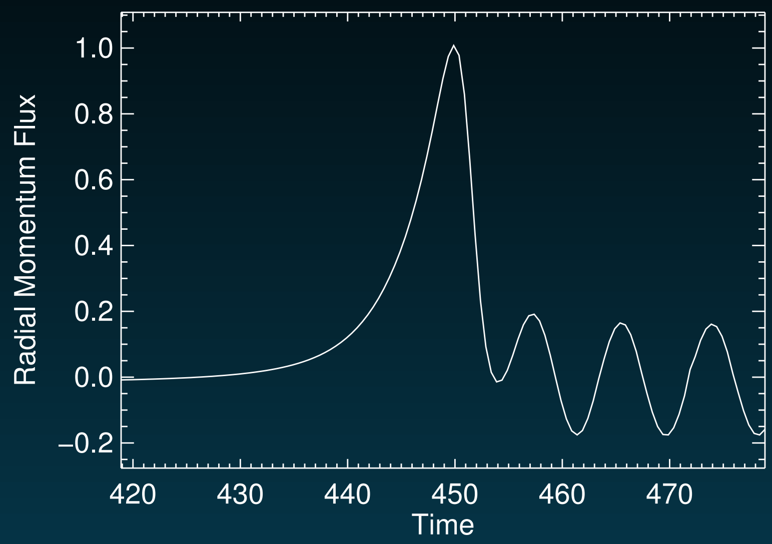


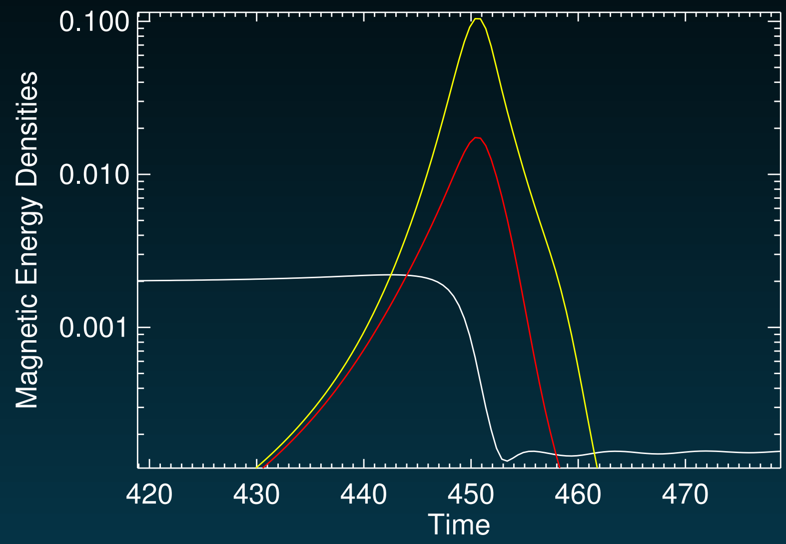


Evolution:
– exponential growth
– nonlinear readjustments
– relaxation to a new stable Keplerian equilibrium

Coherent radial jet transports magnetic flux outwards

Removing of magnetic flux





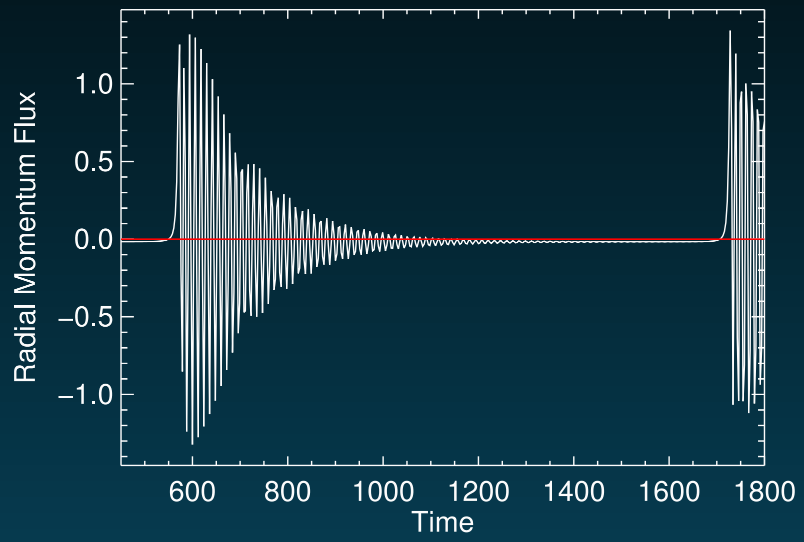
Nonlinear behaviour:

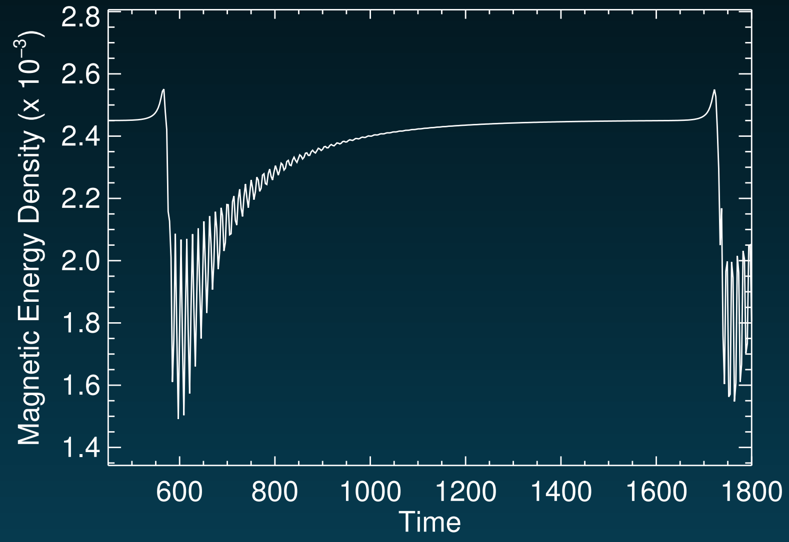
– formation of a fast radial jet: outwards flux of mass
– super-exponential growth of the magnetic energy
– advection of vertical magnetic field out of the domain
– loss of vertical magnetic field leads to instability suppression
– whole disc undergoes epicyclic oscillations (damped on a dissipative timescale)

Relaxation to a stable equilibrium: initial Keplerian flow (with accretion) with a reduced uniform magnetic field

Radial extension of the disc

Preventing removal of magnetic flux by increasing radial extent of the disc





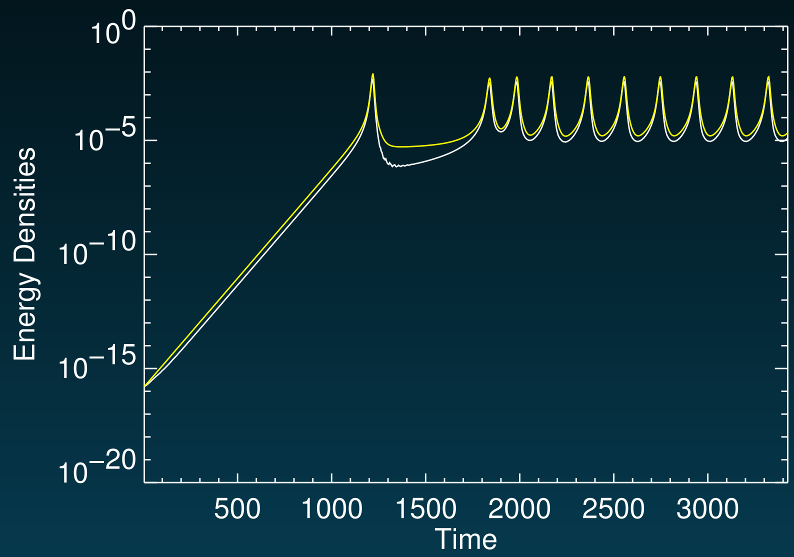
Strong radial jet: – transports B_z-flux outwards; storage in stable part of the disc
– switches-off the instability
– produces large amplitude epicyclic oscillations (viscous damping)

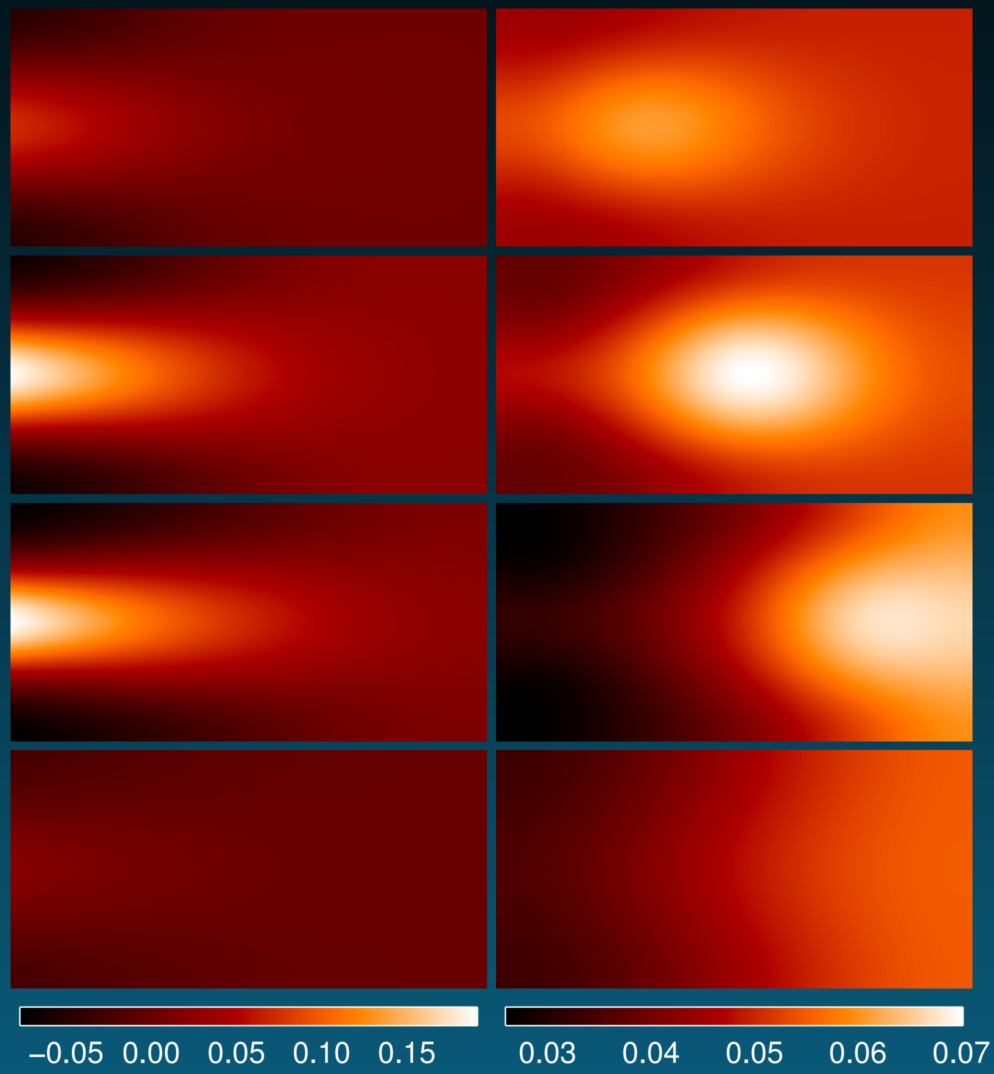
Accretion: – brings B_z-flux towards the centre
– relaxation to initial unstable state

Relaxation oscillator: cyclic behaviour & multiple timescale

Fully nonlinear periodic oscillations

Preventing removal of magnetic flux by decreasing the strength of the nonlinear jet



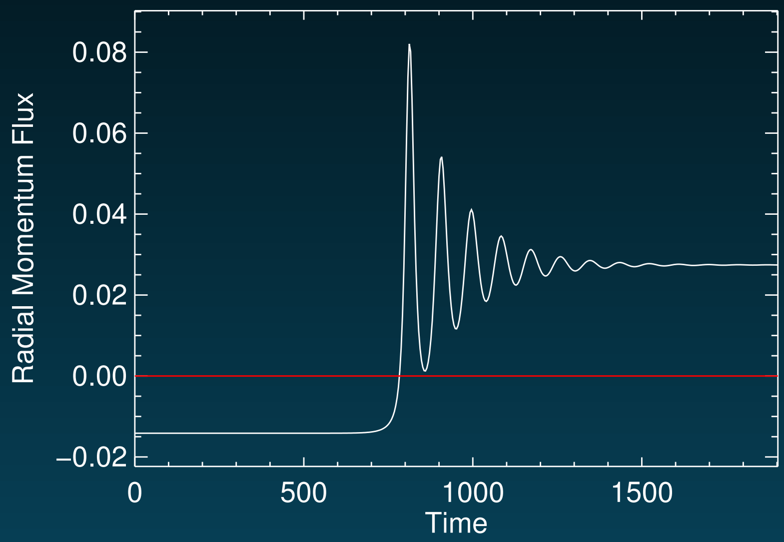


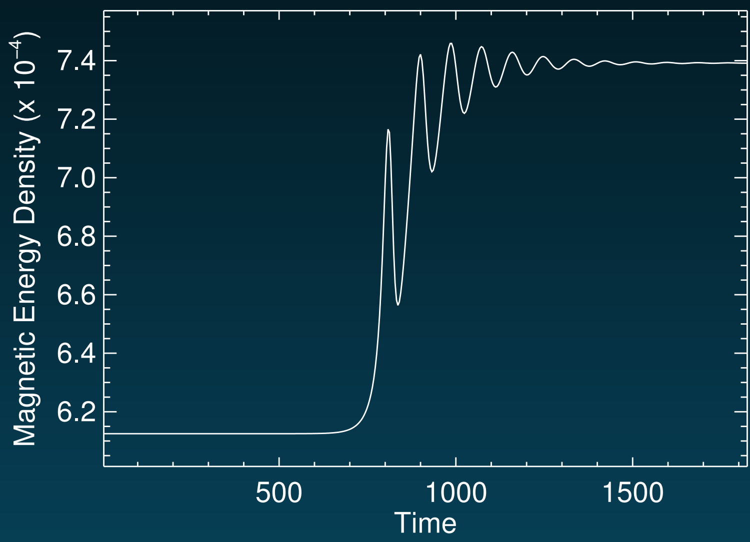
Fully nonlinear dynamics:

- interval between bursts shorter than a dissipative timescale
- no relaxation to Keplerian state
- smooth oscillations around a new (non-Keplerian) equilibrium

Non-trivial stable stationary state

Reduction of jet strength: timescale of nonlinear processes sufficiently long to permit a smooth reorganisation of the system





Evolution of the unstable wall mode:

– linear regime (MRI does not contribute to the transport)
– weak nonlinear jet evolving on a dynamical timescale
– transient phase nonlinear adjustments on a dissipative timescale
– relaxation to a non-trivial stable equilibrium: increased magnetic energy
outwards flux of radial momentum

Conclusions & Perspectives

Nonlinear evolution of MRIs leading to: – cyclic behaviours
– trivial or non-trivial equilibria

Saturation of the MRI relies on magnetic flux redistribution by nonlinear coherent structures

Model requires key features to find the solutions calculated:
- cylindrical geometry (in & out)
- explicit treatment of dissipative processes
- permeable boundary conditions
- ability to reach a steady state (no run down computation)

Correct computation of the evolution of MRIs on very different timescales (dynamical & dissipative)

Transition to less coherent — turbulent — regimes at higher Reynolds numbers:
- 2-D Kelvin-Helmholtz
- 3-D coherent structures unstable to non-axisymmetric perturbations

Stable 2-D structures identified are building blocks: 3-D chaotic flows may well exhibit some properties of 2-D stable solutions (in a statistical sens)