

The role of coherent structures in the saturation of magnetorotational instabilities

Evvy Kersalé

PPARC Postdoctoral Research Associate
Dept. of Appl. Maths — University of Leeds

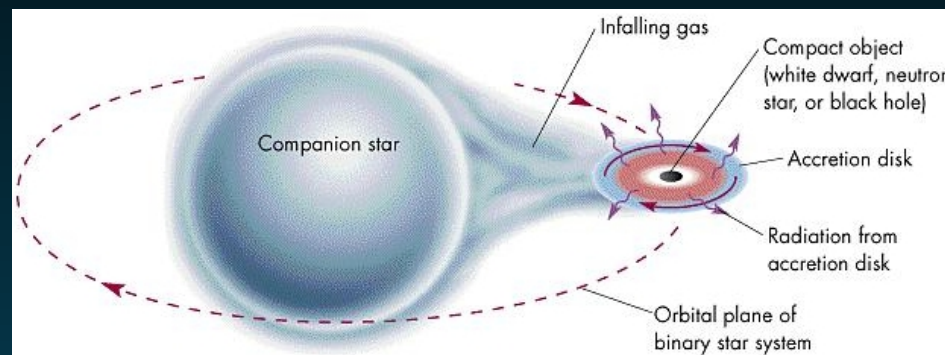
Collaboration:

D. Hughes & S. Tobias (Appl. Maths, Leeds)

N. Weiss & G. Ogilvie (DAMTP, Cambridge)

Magnetorotational instability in accretion discs

Accretion discs: systems in **differential rotation** under the action of a gravitational field. Orbiting gas can be accreted by the central object if its **angular momentum is removed** by a torque acting within or on the disc



MRI: give some important insight into the nature of the mechanism leading to transport of angular momentum (e.g. Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991)

- Weakly magnetised differentially rotating flows linearly unstable if $d\Omega/dr < 0$
- Differential rotation source of free energy \Rightarrow MRI extremely powerful $\gamma \sim r|d\Omega/dr|$
- Nonlinear evolution leads to **3-D turbulence** and **outwards transport** of angular momentum

But further studies required to understand the saturation processes and to determine the level of saturation of turbulence

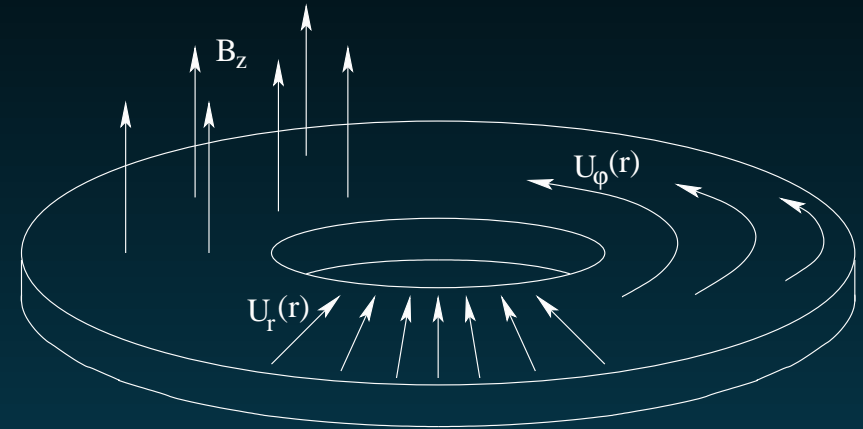
Global model with accretion

Evolution: incompressible non-ideal MHD

$$(\partial_t + \mathbf{U} \cdot \nabla) \mathbf{U} = -\nabla\Phi - \nabla\Pi + \mathbf{B} \cdot \nabla\mathbf{B} + \nu \nabla^2 \mathbf{U},$$

$$(\partial_t + \mathbf{U} \cdot \nabla) \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{U} + \eta \nabla^2 \mathbf{B},$$

$$\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{U} = 0.$$



Basic state: magnetised Keplerian shearing flow **with accretion** (φ - and z -invariant)

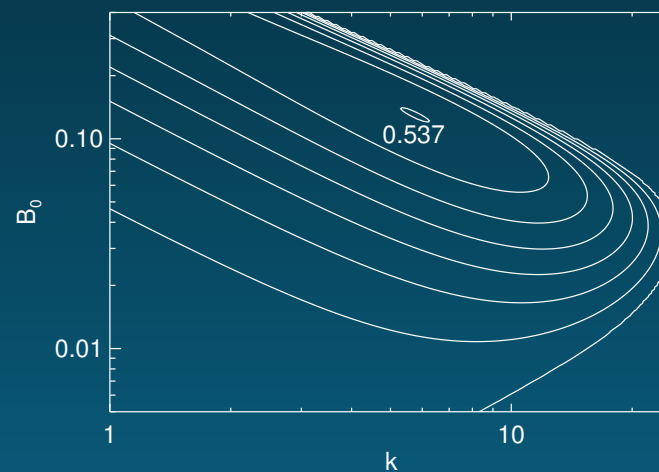
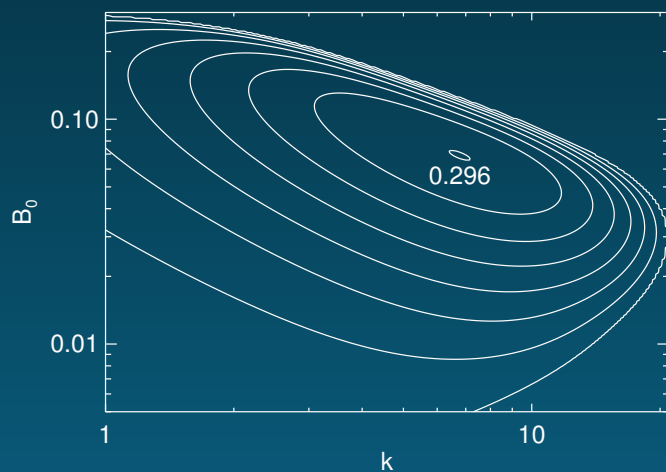
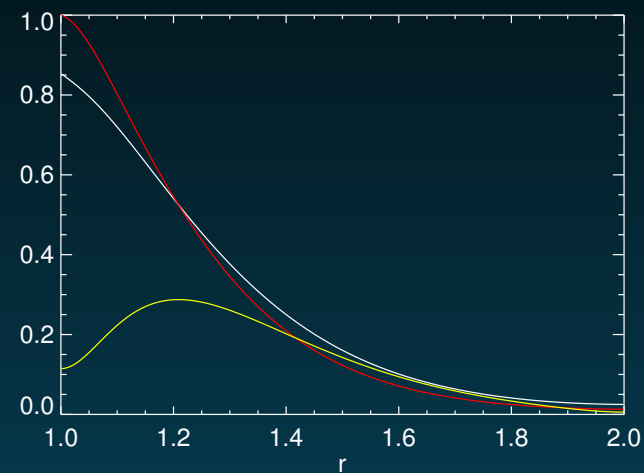
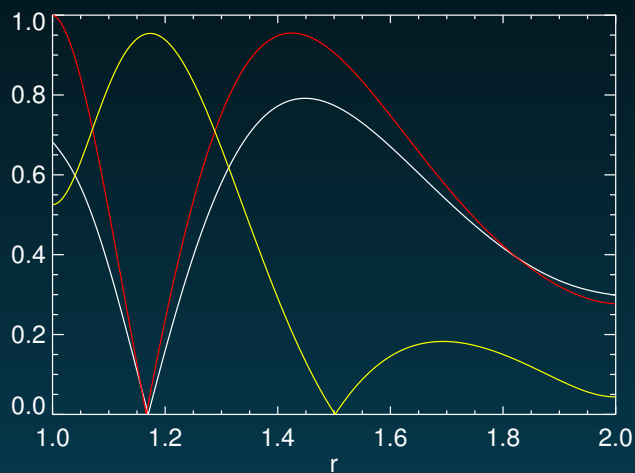
$$U_r = -3\nu/2r, \quad U_\varphi = 1/\sqrt{r}, \quad U_z = 0, \quad B_r = B_\varphi = 0, \quad B_z = B_0, \quad \Pi = \delta - 9\nu^2/8r^2$$

Radial boundary conditions: **permeable** (no conditions on U_r and B_r)

$$\partial_r(\sqrt{r}U_\varphi) = 0, \quad \partial_r U_z = 0, \quad \Pi = \Pi_0, \quad \partial_r(rB_\varphi) = 0, \quad \partial_r B_z = 0$$

Linear stability theory

Permeable radial boundaries permit the development of **wall modes** as well as **body modes**



Axisymmetric nonlinear evolution

Numerical investigation of the nonlinear development of axisymmetric wall modes

Features of the code developed : cylindrical geometry, pseudo-spectral, semi-implicit, adaptive time step

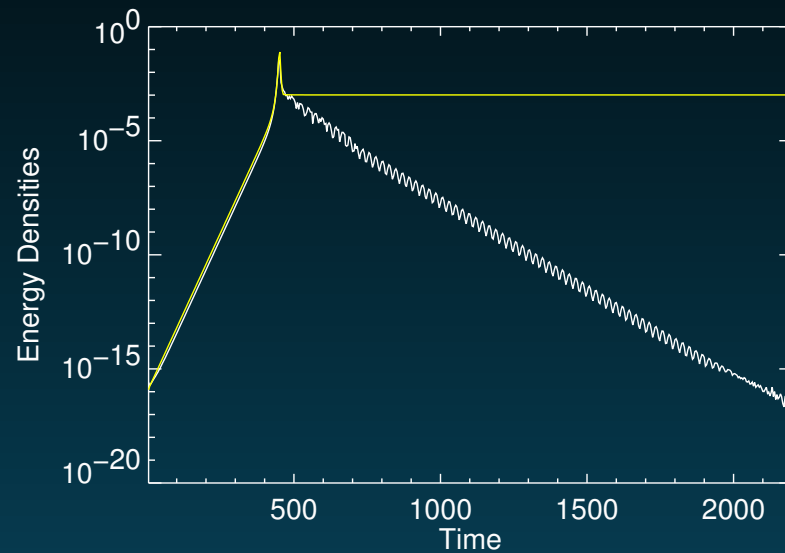
Parameters set in order to trigger only one linearly unstable wall mode:

$$R_{\text{in}} = 1, \quad \Omega(R_{\text{in}}) = 1, \quad B_0 \simeq 10^{-1}-10^{-2},$$
$$H = 0.5, \quad R_{\text{out}} = \{2, 4\}, \quad \mathcal{R}_e = \mathcal{R}_m \simeq 150-500$$

We find:

- Exponential growth in agreement with the linear theory
- No turbulent motion but coherent structures: radial jet propagating outwards
- Suppression mechanism for the instability
- Cyclic evolutions

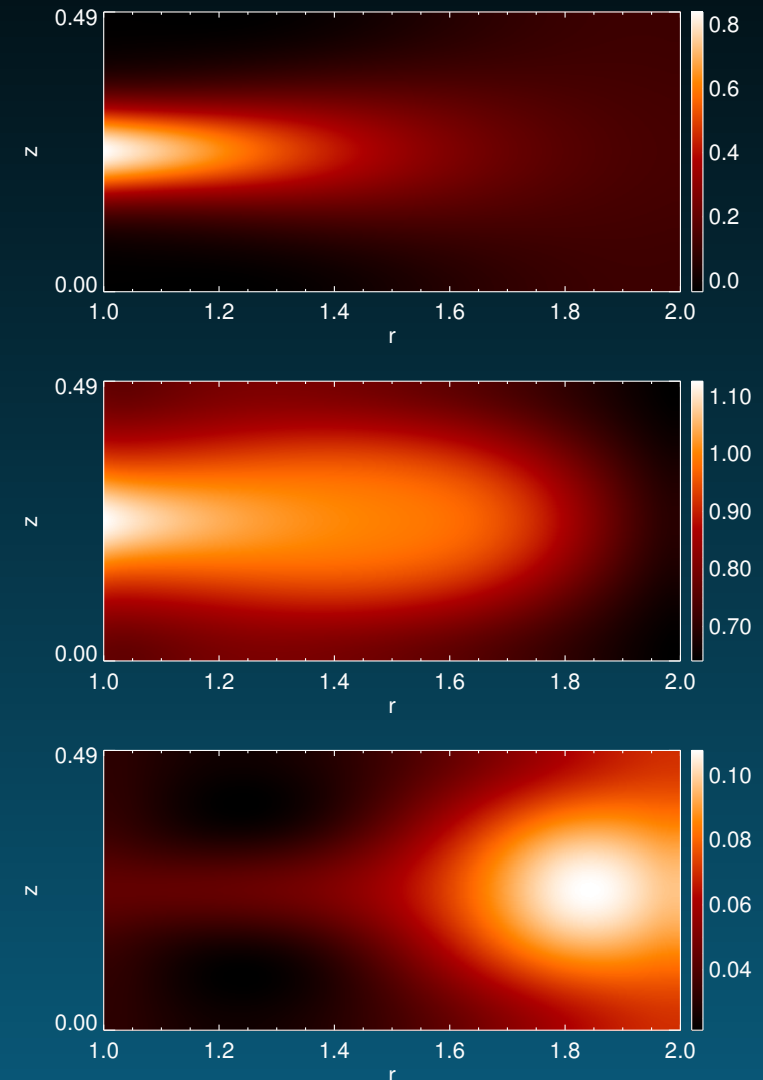
Self-consistent suppression of the instability



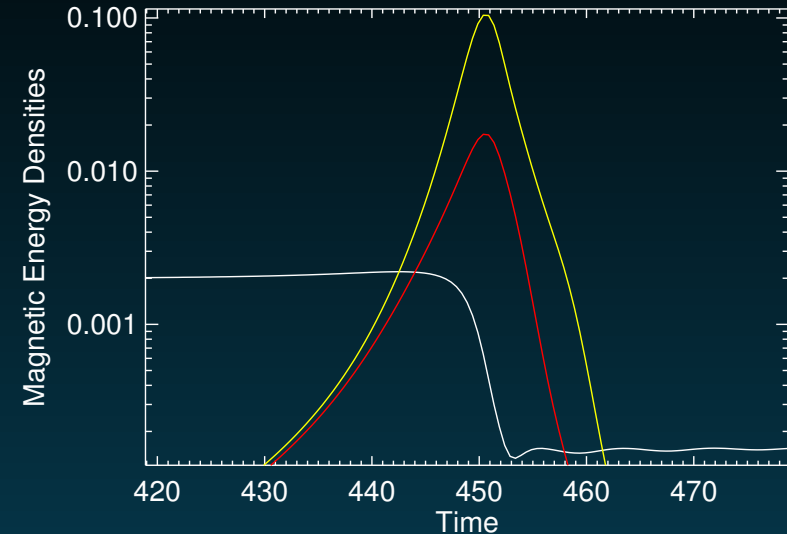
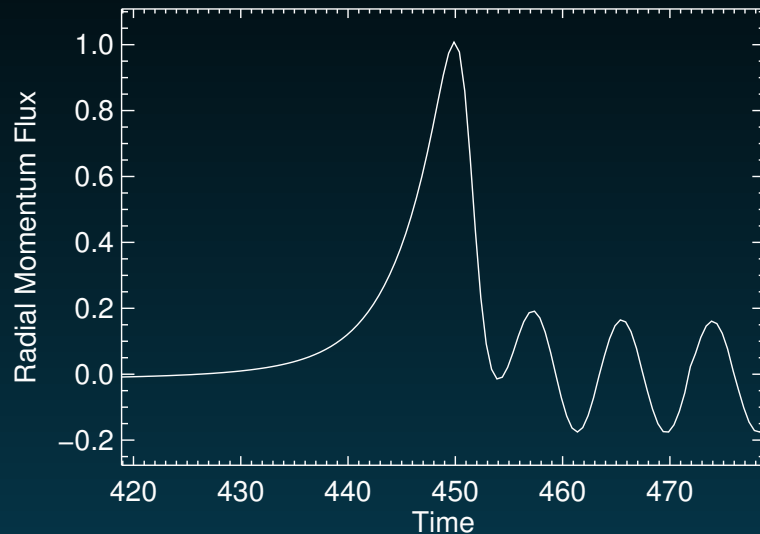
Evolution:

- exponential growth
- nonlinear readjustments
- relaxation to a new stable Keplerian equilibrium

Coherent radial jet transports magnetic flux outwards



Removing of magnetic flux



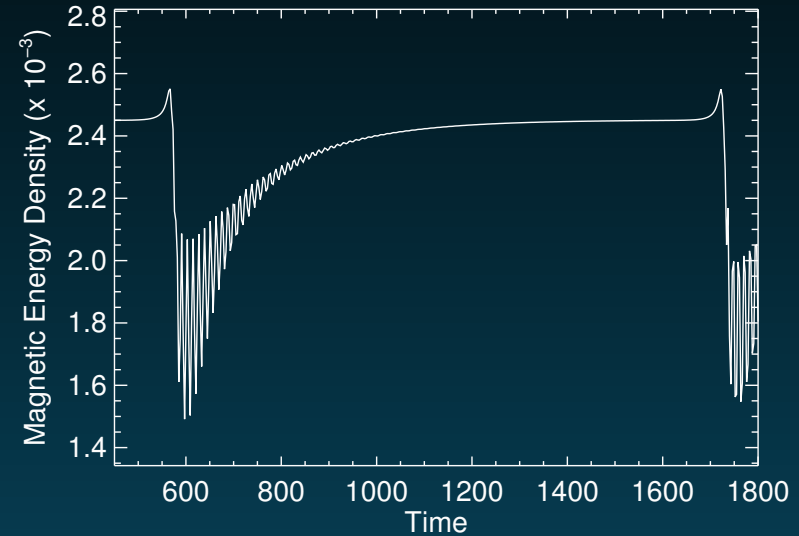
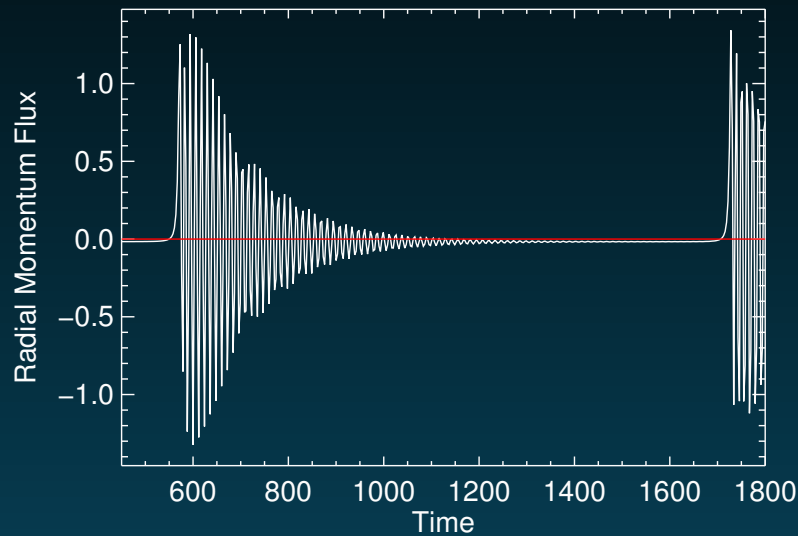
Nonlinear behaviour:

- formation of a **fast** radial jet: outwards flux of mass
- super-exponential growth of the magnetic energy
- **advection of vertical magnetic field** out of the domain
- loss of vertical magnetic field leads to **instability suppression**
- whole disc undergoes epicyclic oscillations (damped on a dissipative timescale)

**Relaxation to a stable equilibrium: initial Keplerian flow (with accretion)
with a reduced uniform magnetic field**

Radial extension of the disc

Preventing removal of magnetic flux by increasing radial extent of the disc



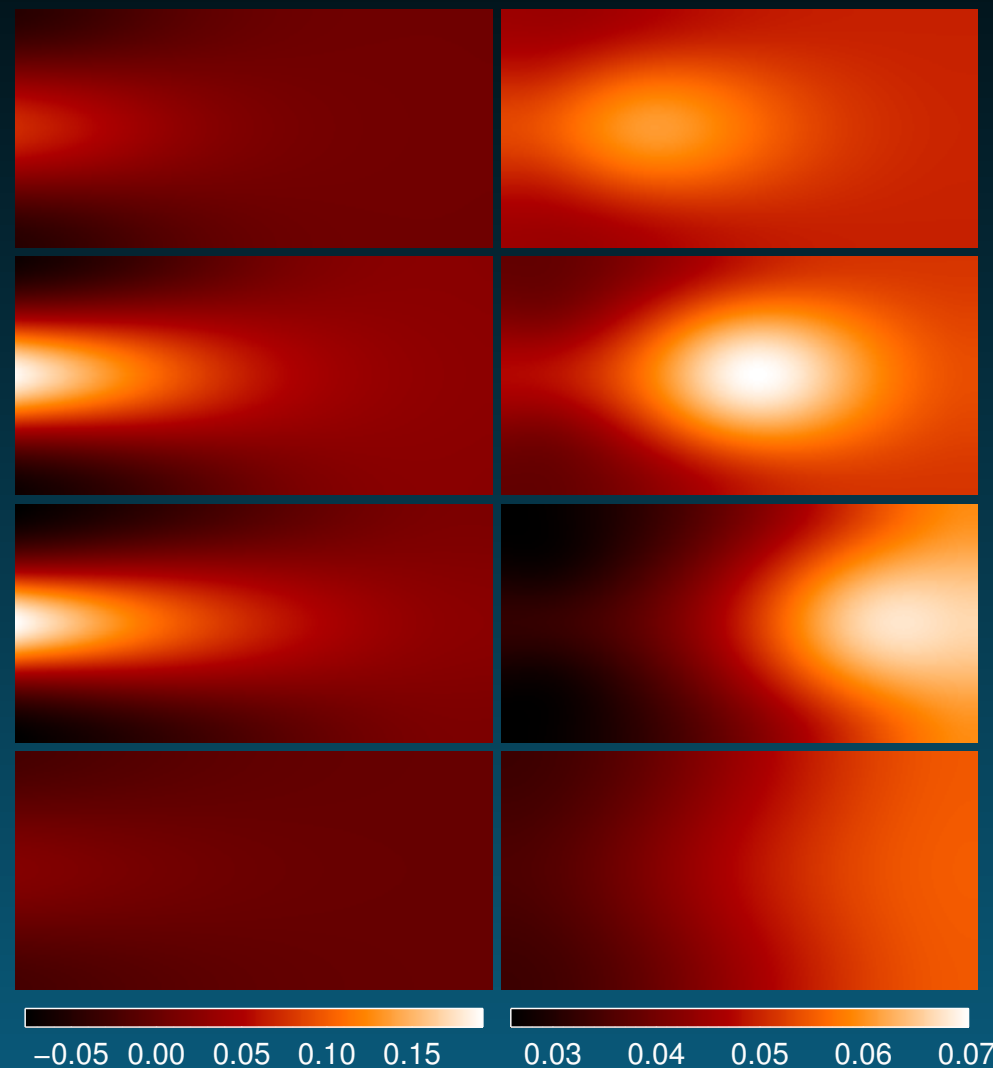
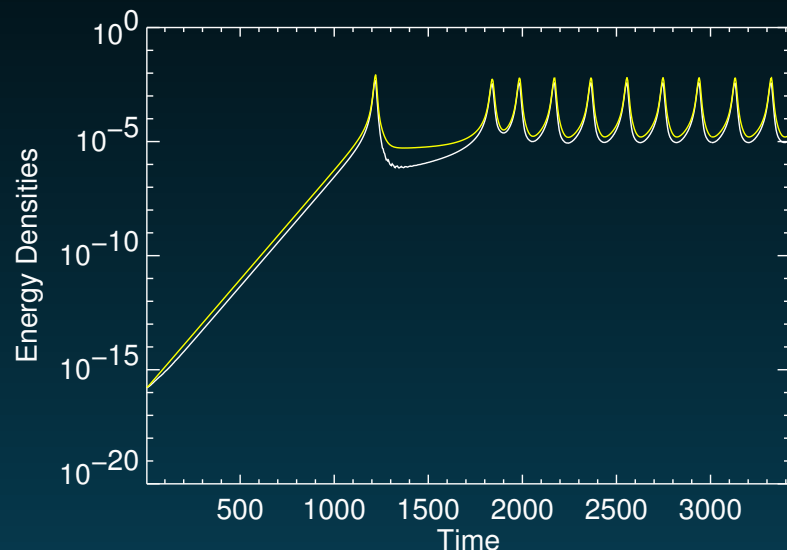
Strong radial jet: – transports B_z -flux outwards; storage in stable part of the disc
 – switches-off the instability
 – produces large amplitude epicyclic oscillations (viscous damping)

Accretion: – brings B_z -flux towards the centre
 – relaxation to initial unstable state

Relaxation oscillator: cyclic behaviour & multiple timescale

Fully nonlinear periodic oscillations

Preventing removal of magnetic flux by decreasing the strength of the nonlinear jet

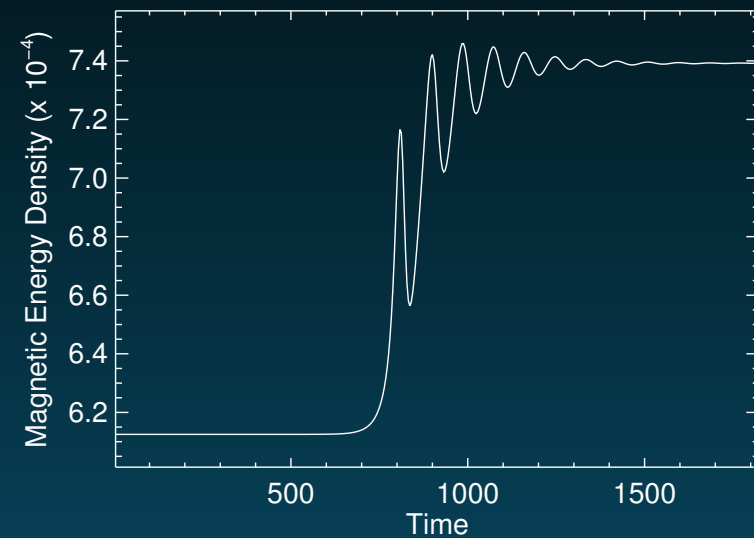
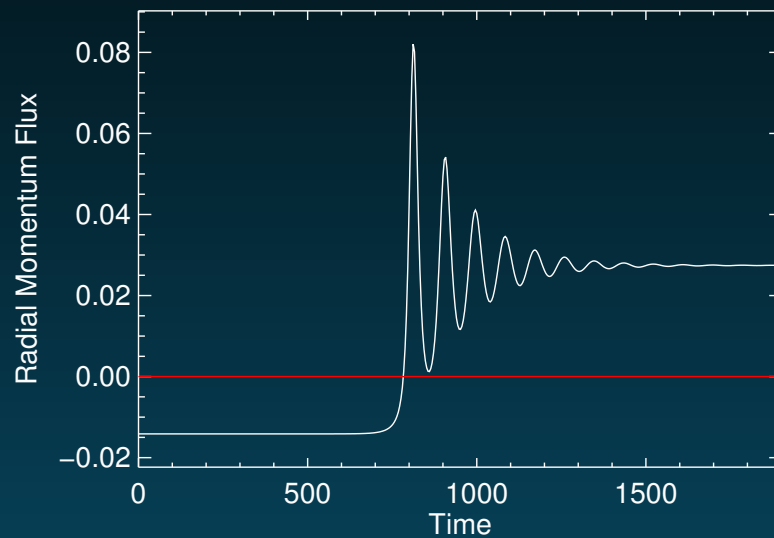


Fully nonlinear dynamics:

- interval between bursts shorter than a dissipative timescale
- no relaxation to Keplerian state
- smooth oscillations around a new (non-Keplerian) equilibrium

Non-trivial stable stationary state

Reduction of jet strength: timescale of nonlinear processes sufficiently long to permit a smooth reorganisation of the system



Evolution of the unstable wall mode:

- linear regime (MRI does not contribute to the transport)
- **weak nonlinear jet** evolving on a dynamical timescale
- transient phase nonlinear adjustments on a dissipative timescale
- **relaxation to a non-trivial stable equilibrium**: increased magnetic energy
outwards flux of radial momentum

Conclusions & Perspectives

Nonlinear evolution of MRIs leading to:

- cyclic behaviours
- trivial or non-trivial equilibria

Saturation of the MRI relies on magnetic flux redistribution by nonlinear coherent structures

Model requires **key features** to find the solutions calculated:

- cylindrical geometry (in & out)
- explicit treatment of dissipative processes
- permeable boundary conditions
- ability to reach a steady state (no run down computation)

Correct computation of the evolution of MRIs on very **different timescales** (dynamical & dissipative)

Transition to less coherent — turbulent — regimes at higher Reynolds numbers:

- 2-D Kelvin-Helmholtz
- 3-D coherent structures unstable to non-axisymmetric perturbations

Stable 2-D structures identified are building blocks: **3-D chaotic flows may well exhibit some properties of 2-D stable solutions** (in a statistical sense)