Nonlinear Magnetorotational Instability with Inflow

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Magnetorotational Instability in Accretion Discs

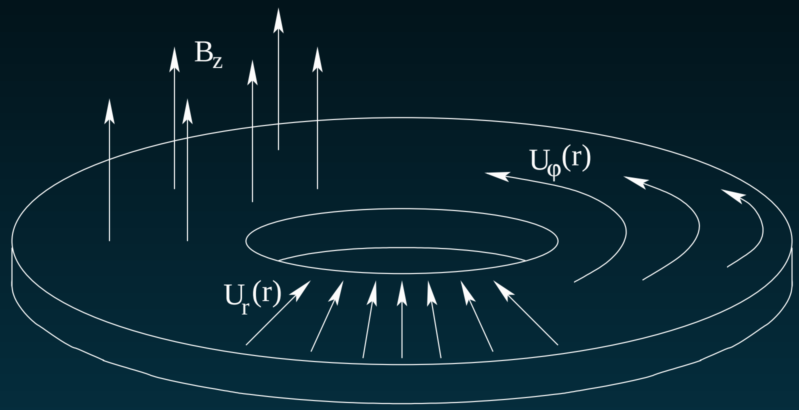
Accretion discs: systems in differential rotation under the action of a gravitational field. Orbiting gas can be accreted by the central object if its angular momentum is removed by a torque acting within or on the disc.

MRI: give some important insight into the nature of the mechanism leading to transport of angular momentum (e.g. Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991).

- Weakly magnetised differentially rotating flows linearly unstable if $\mathrm{d}\Omega/\mathrm{d}r < 0$.
- Free energy $\equiv$ differential rotation $\Rightarrow$ MRI extremely powerful $\gamma \sim r|\mathrm{d}\Omega/\mathrm{d}r|$.
- Nonlinear evolution leads to 3-D turbulence and outward transport of angular momentum.

But further studies required to understand the saturation processes and determine the level of saturation of turbulence.

Global Dissipative Study



Evolution equations: incompressible MHD

$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{U} = -\nabla\Phi - \nabla\Pi + \mathbf{B}\cdot\nabla\mathbf{B} + \nu\Delta\mathbf{U},$$
$$(\partial_t + \mathbf{U}\cdot\nabla)\,\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{U} + \eta\Delta\mathbf{B},$$
$$\nabla\cdot\mathbf{B} = \nabla\cdot\mathbf{U} = 0.$$

Basic state: magnetised Keplerian shearing flow with accretion; axisymmetric and Z-invariant

$$U_r = -3\nu/2r, \quad U_\varphi = 1/\sqrt{r}, \quad U_z = 0,$$
$$B_r = B_\varphi = 0, \quad B_z = B_0,$$
$$\Pi = \delta - 9\nu^2/8r^2$$

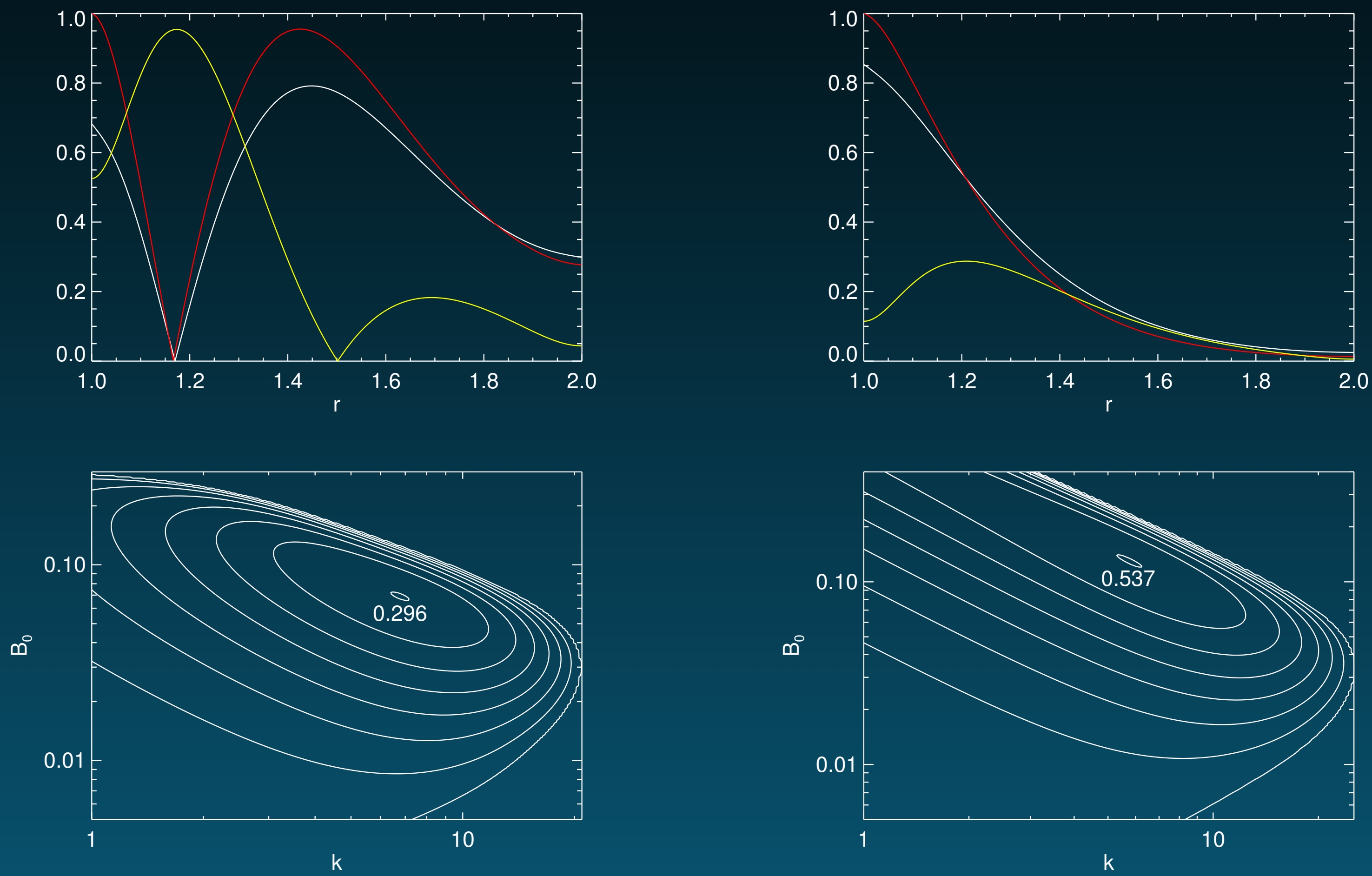
Boundary conditions: no direct condition on U_r or B_r

$$\partial_r(\sqrt{r}U_\varphi) = 0, \quad \partial_r U_z = 0,$$
$$\partial_r(rB_\varphi) = 0, \quad \partial_r B_z = 0,$$
$$\Pi = \Pi_0$$

($U_\varphi = U_{\varphi 0}$ has been found to enhance instability artificially).

Linear Stability Theory

The permeable radial boundaries permit the development of wall-modes as well as body-modes



(See Kersalé et al. 2004, ApJ)

Numerical Schemes Implemented

Spectral decomposition: $X(r, \varphi, z) = \sum_{l,m,k} \mathcal{X}_{lmk} T_l(s)\, e^{im\varphi} \{\cos, \sin\}(kz)$,
$$T_l(s) = \cos(l \cos^{-1} s) \;\&\; 2r = [(r_1 - r_2)\, s + r_1 + r_2],\; s \in [-1, +1]$$

Spatial differentiations are performed using the properties of spectral decompositions (Fourier: multiplication & Chebyshev: recurrence relations) & the nonlinear terms are computed in configuration space making use of FFTs.

The advance in time uses a semi-implicit scheme Crank-Nicholson & 4^{th} order Adams-Bashforth with adaptive time step for the linear & nonlinear terms respectively.

$$[\mathcal{I} - \Theta\, \delta t\, \mathcal{L}]\, \mathbf{U}^{n+1} + \nabla\Gamma^{n+1} = \mathcal{N_{\mathbf{u}}}^n + [\mathcal{I} + (1 - \Theta)\, \delta t\, \mathcal{L}]\, \mathbf{U}^n \text{ with } \nabla^2\Gamma^{n+1} = \nabla \cdot \boldsymbol{\mathcal{N}}_u^n$$
$$[\mathcal{I} - \Theta\, \delta t\, \mathcal{L}]\, \mathbf{B}^{n+1} + \nabla\Psi = \mathcal{N_{\mathbf{b}}}^n + [\mathcal{I} + (1 - \Theta)\, \delta t\, \mathcal{L}]\, \mathbf{B}^n \text{ with } \nabla^2\Psi = \nabla \cdot \boldsymbol{\mathcal{N}}_b^n$$

where $\Theta \in [0, 1]$, $\Gamma = \delta t\, \Pi$, $\mathcal{L} = \nu\nabla^2$ and $\boldsymbol{\mathcal{N}}$ represents the nonlinear terms

Non triviality of the differential operator $(\mathcal{I} - \Theta\, \delta t\, \mathcal{L})$ for Chebyshev polynomials in an annulus.

The azimuthal and vertical boundary conditions are automatically satisfied by the trial functions while Tau-Chebyshev methods imply some linear relations between the expansion coefficients for the radial ones. The BCs used to compute U_r and B_r come from the solenoidal constrains.

Axisymmetric Nonlinear Evolution

Numerical investigation of the nonlinear development of axisymmetric wall-modes.

The parameters are set in order to trigger only one linearly unstable wall-mode:

$$R_{\text{in}} = 1, \quad H = 0.5, \quad R_{\text{out}} = \{2, 4\}, \quad \Omega(R_{\text{in}}) = 1,$$

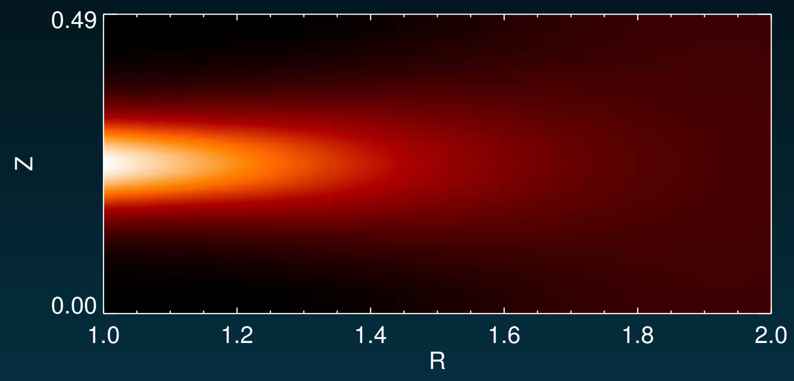
$$B_0 \simeq 10^{-1}\text{-}10^{-2}, \quad \nu = \eta \simeq 3.5 \times 10^{-3}\text{-}10^{-4}$$

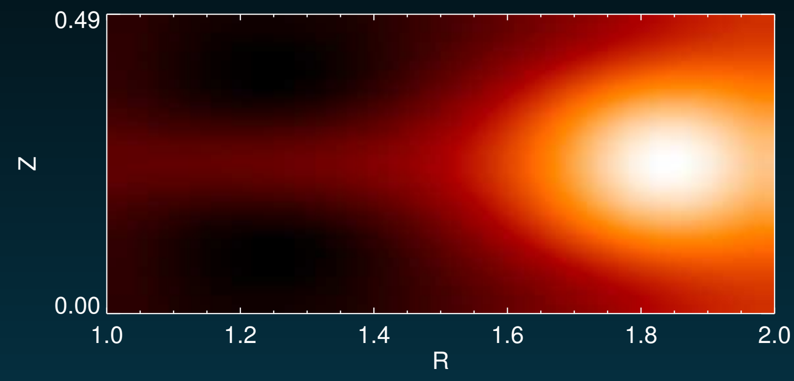
We find:

- Linear growth in agreement with the linear theory
- No turbulent motion but coherent structures: radial jet propagating outward
- Suppression mechanism for the instability
- Cyclic evolution.

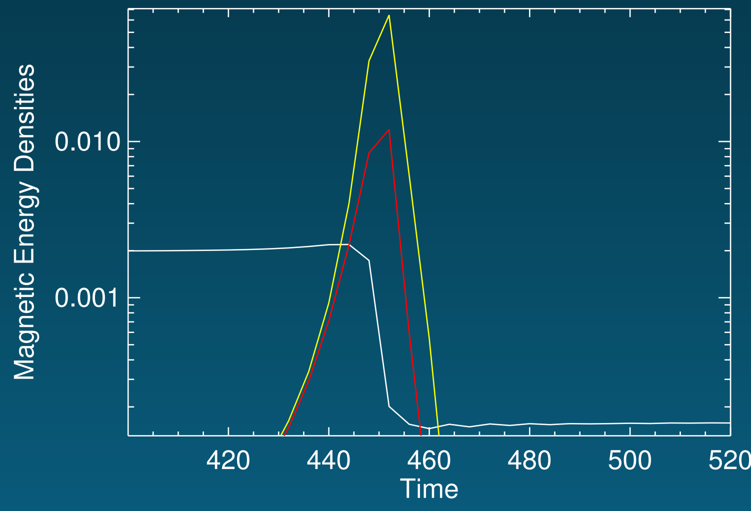
Suppression of the Instability

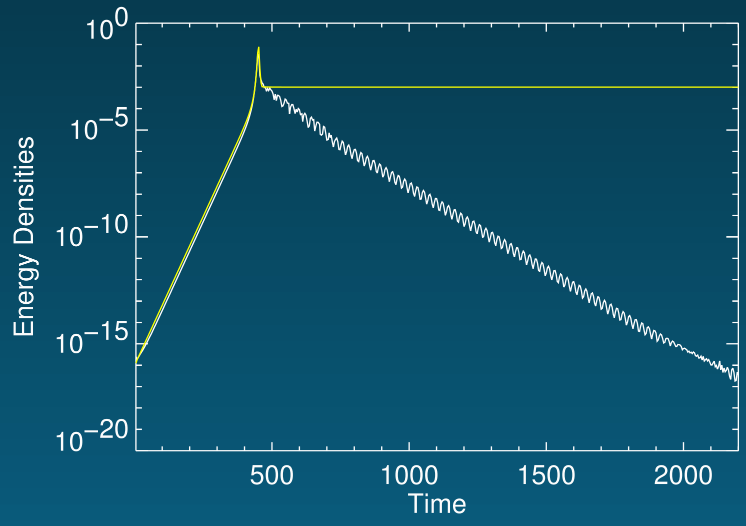
Development of coherent structures:



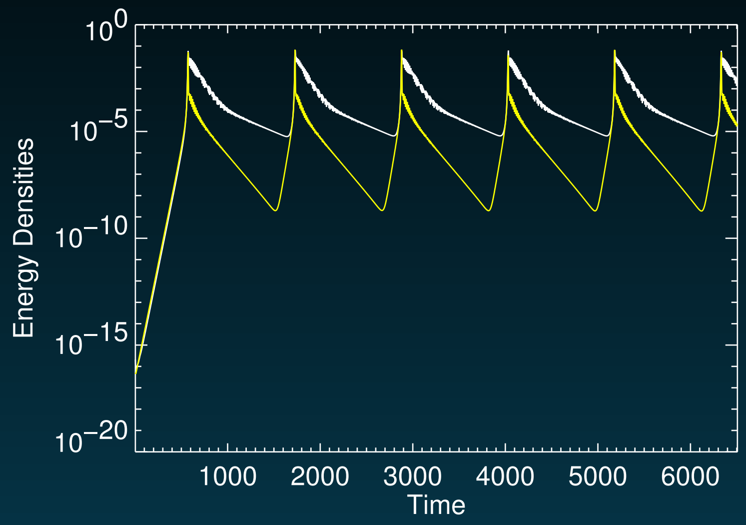


The radial jet carries B_z out of the disc. By reducing the flux of B_z the instability is suppressed and the system relaxes to its initial Keplerian state with a lower magnetic energy.



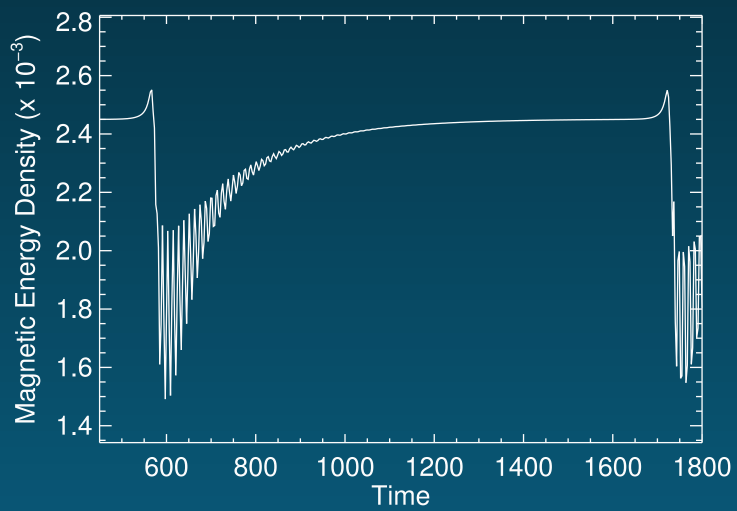


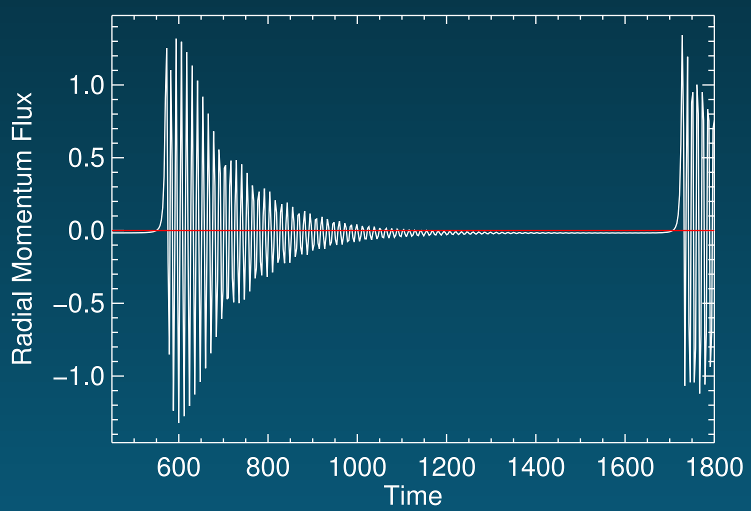
Extended Disc



Disc extended three times in radius presents a cyclic behaviour:

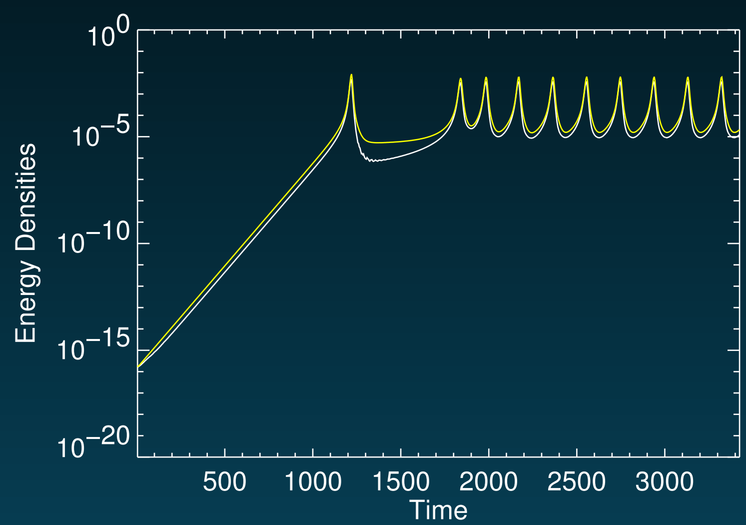
- Radial jet carries B_z-flux in the outer part of the disc and switches-off the instability.
- Violent event: leads to fast radial oscillations of the whole disc & relaxation to initial state.
- Accretion brings B_z-flux in the inner region and triggers the instability again.

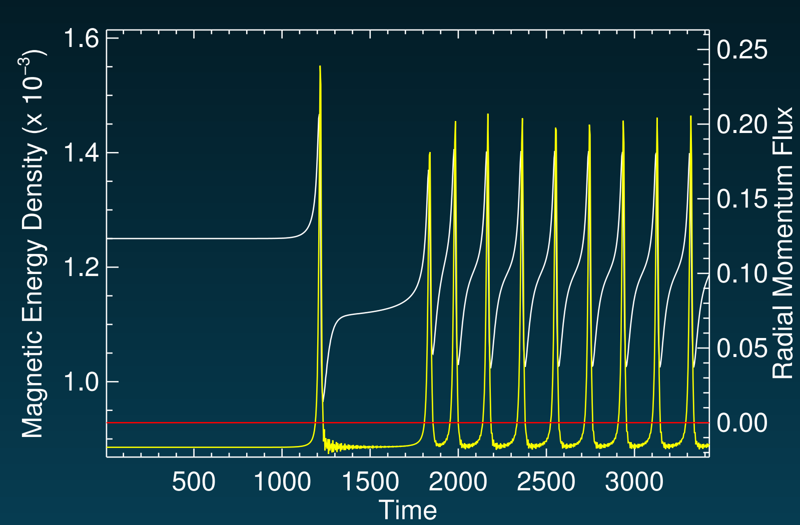




Starting with a Weaker Magnetic Field

With a reduced initial magnetic field the system presents a cyclic behaviour as in the previous case (rely on the same processes).

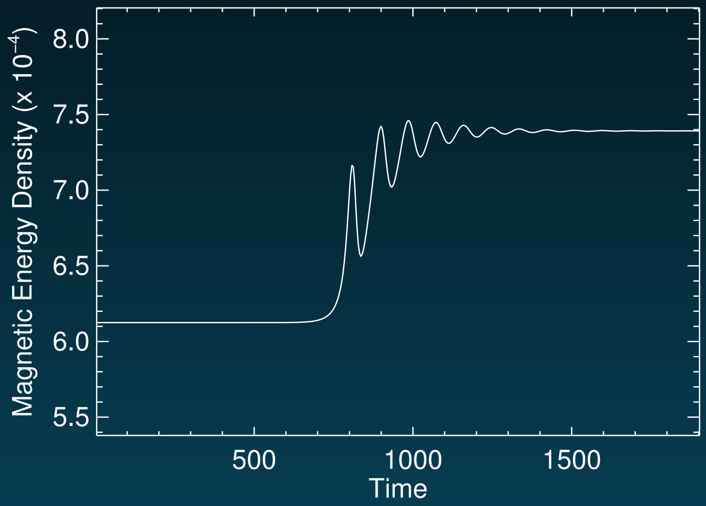


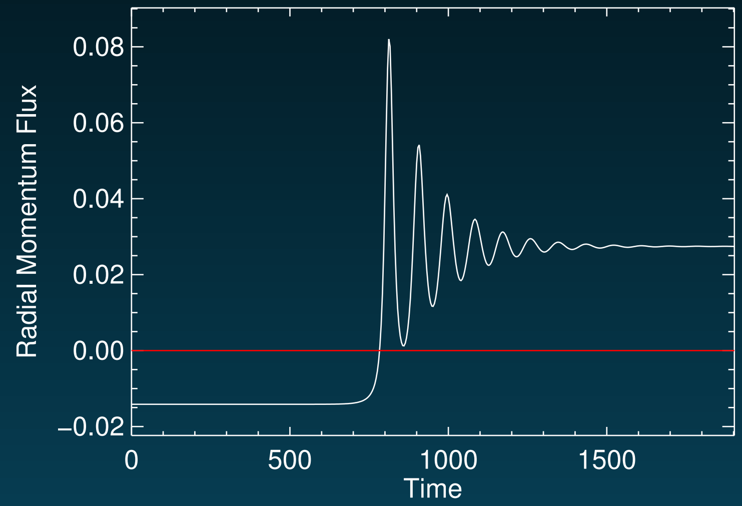


- The instability is not strong enough to expel the magnetic flux.
- A fraction of the B_z-flux initially carried out of the system (disc not extended).
- Cycles around an equilibrium different from the initial state.
- Much less violent event: no fast radial oscillation of the disc.

Evolution to a New Stable Stationary State

For some slowly growing modes with a sufficiently small initial B_z the nonlinear regime can lead to a new stable 2-D equilibrium.





- Magnetic energy increased.
- Outward radial flux of material.
- Magnetorotational instability still present.

Future Work

Numerical issues regarding:

- the fast $(\simeq \Omega^{-1})$ radial oscillations of the disc,
- the properties of differential operators for Chebyshev polynomials in an annulus.

Stability of the 2-D solutions: Kelvin-Helmholtz stability for higher Reynolds numbers.

3-D evolution of the 2-D solutions:

- transition to a turbulent regime,
- systematic parameter space survey.