

# Nonlinear Magnetorotational Instability with Inflow

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## Magnetorotational Instability in Accretion Discs

**Accretion discs:** systems in differential rotation under the action of a gravitational field. Orbiting gas can be accreted by the central object if its angular momentum is removed by a torque acting within or on the disc.

**MRI:** give some important insight into the nature of the mechanism leading to transport of angular momentum (e.g. Velikhov 1959, Chandrasekhar 1960, Balbus & Hawley 1991).

- Weakly magnetised differentially rotating flows linearly unstable if  $d\Omega/dr < 0$ .
- Free energy  $\equiv$  differential rotation  $\Rightarrow$  MRI extremely powerful  $\gamma \sim r|d\Omega/dr|$ .
- Nonlinear evolution leads to 3-D turbulence and outward transport of angular momentum.

But further studies required to understand the saturation processes and determine the level of saturation of turbulence.

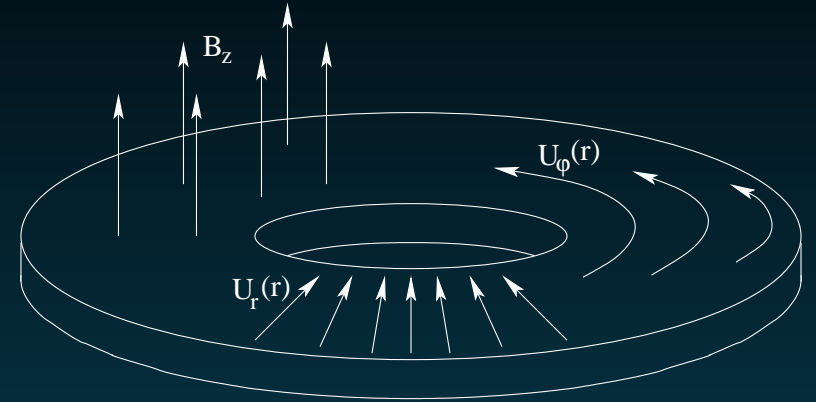
## Global Dissipative Study

Evolution equations: incompressible MHD

$$(\partial_t + \mathbf{U} \cdot \nabla) \mathbf{U} = -\nabla\Phi - \nabla\Pi + \mathbf{B} \cdot \nabla\mathbf{B} + \nu\Delta\mathbf{U},$$

$$(\partial_t + \mathbf{U} \cdot \nabla) \mathbf{B} = \mathbf{B} \cdot \nabla\mathbf{U} + \eta\Delta\mathbf{B},$$

$$\nabla \cdot \mathbf{B} = \nabla \cdot \mathbf{U} = 0.$$



Basic state: magnetised Keplerian shearing flow **with accretion**; axisymmetric and  $Z$ -invariant

$$U_r = -3\nu/2r, \quad U_\varphi = 1/\sqrt{r}, \quad U_z = 0,$$

$$B_r = B_\varphi = 0, \quad B_z = B_0,$$

$$\Pi = \delta - 9\nu^2/8r^2$$

Boundary conditions: **no direct condition on  $U_r$  or  $B_r$**

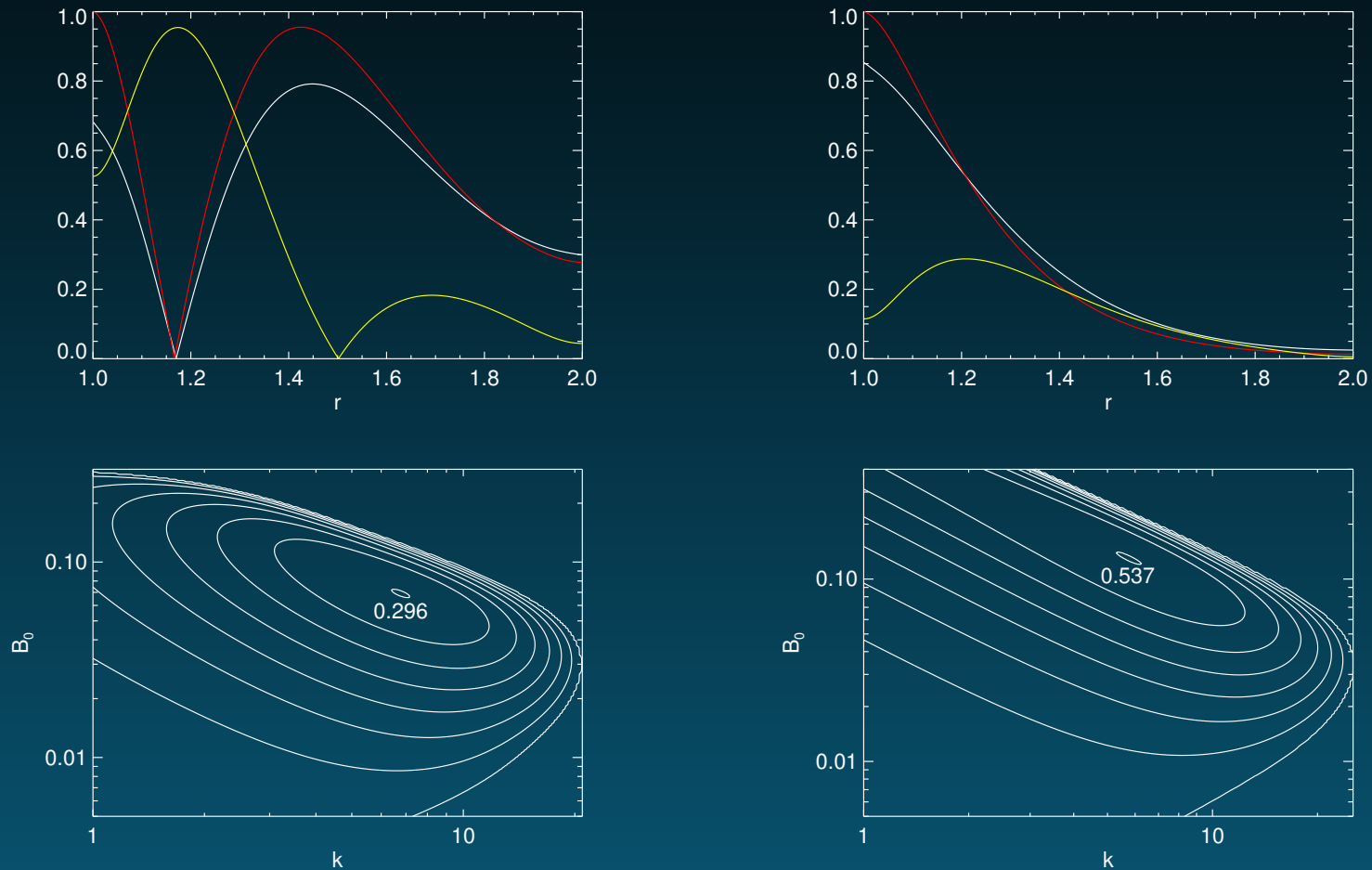
$$\partial_r(\sqrt{r}U_\varphi) = 0, \quad \partial_r U_z = 0,$$

$$\partial_r(rB_\varphi) = 0, \quad \partial_r B_z = 0,$$

$$\Pi = \Pi_0 \quad (U_\varphi = U_{\varphi_0} \text{ has been found to enhance instability artificially}).$$

## Linear Stability Theory

The permeable radial boundaries permit the development of **wall-modes** as well as **body-modes**



(See Kersalé et al. 2004, ApJ)

## Numerical Schemes Implemented

Spectral decomposition:  $X(r, \varphi, z) = \sum_{l,m,k} \mathcal{X}_{lmk} T_l(s) e^{im\varphi} \{\cos, \sin\}(kz)$ ,  
 $T_l(s) = \cos(l \cos^{-1} s)$  &  $2r = [(r_1 - r_2) s + r_1 + r_2]$ ,  $s \in [-1, +1]$

Spatial differentiations are performed using the properties of spectral decompositions (Fourier: multiplication & Chebyshev: recurrence relations) & the nonlinear terms are computed in configuration space making use of FFTs.

The advance in time uses a semi-implicit scheme Crank-Nicholson & 4<sup>th</sup> order Adams-Bashforth with adaptive time step for the linear & nonlinear terms respectively.

$$[\mathcal{I} - \Theta \delta t \mathcal{L}] \mathbf{U}^{n+1} + \nabla \Gamma^{n+1} = \mathcal{N}_u^n + [\mathcal{I} + (1 - \Theta) \delta t \mathcal{L}] \mathbf{U}^n \text{ with } \nabla^2 \Gamma^{n+1} = \nabla \cdot \mathcal{N}_u^n$$

$$[\mathcal{I} - \Theta \delta t \mathcal{L}] \mathbf{B}^{n+1} + \nabla \Psi = \mathcal{N}_b^n + [\mathcal{I} + (1 - \Theta) \delta t \mathcal{L}] \mathbf{B}^n \text{ with } \nabla^2 \Psi = \nabla \cdot \mathcal{N}_b^n$$

where  $\Theta \in [0, 1]$ ,  $\Gamma = \delta t \Pi$ ,  $\mathcal{L} = \nu \nabla^2$  and  $\mathcal{N}$  represents the nonlinear terms

**Non triviality of the differential operator  $(\mathcal{I} - \Theta \delta t \mathcal{L})$  for Chebyshev polynomials in an annulus.**

The azimuthal and vertical boundary conditions are automatically satisfied by the trial functions while Tau-Chebyshev methods imply some linear relations between the expansion coefficients for the radial ones. The BCs used to compute  $U_r$  and  $B_r$  come from the solenoidal constraints.

## Axisymmetric Nonlinear Evolution

Numerical investigation of the nonlinear development of axisymmetric wall-modes.

The parameters are set in order to trigger only one linearly unstable wall-mode:

$$R_{\text{in}} = 1, \quad H = 0.5, \quad R_{\text{out}} = \{2, 4\}, \quad \Omega(R_{\text{in}}) = 1,$$

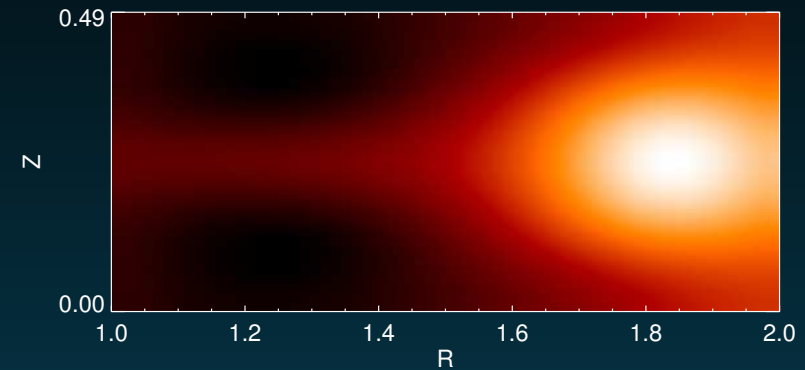
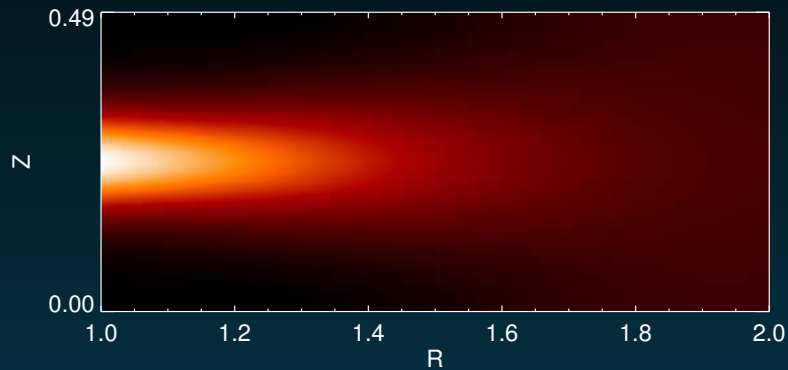
$$B_0 \simeq 10^{-1}-10^{-2}, \quad \nu = \eta \simeq 3.5 \times 10^{-3}-10^{-4}$$

### We find:

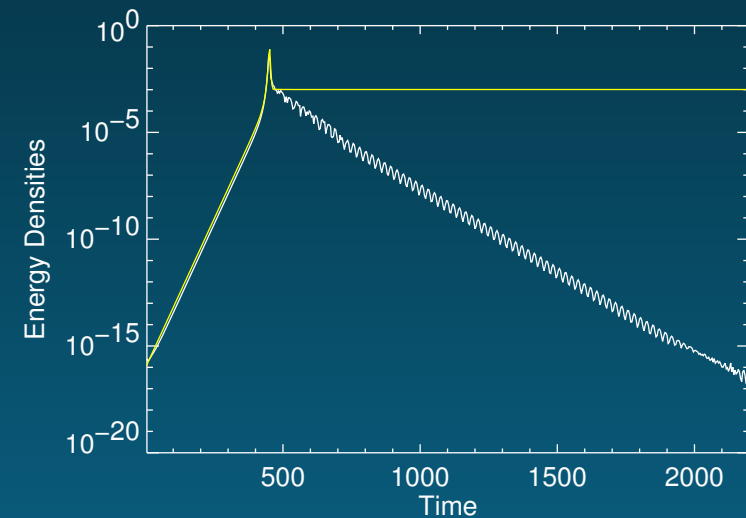
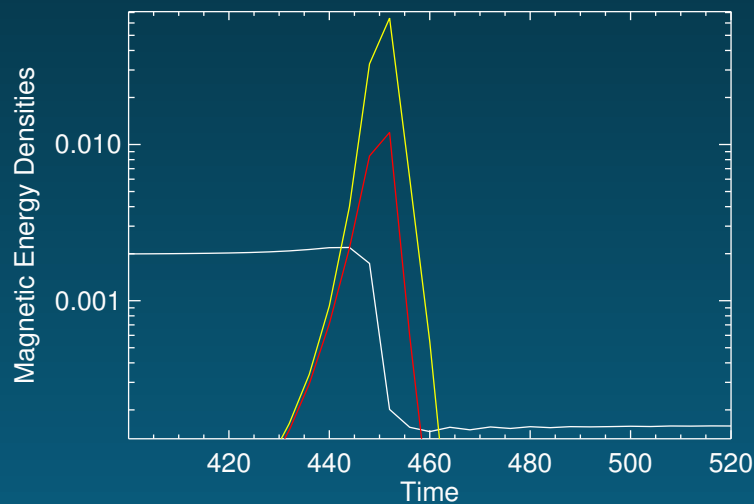
- Linear growth in agreement with the linear theory
- No turbulent motion but coherent structures: radial jet propagating outward
- Suppression mechanism for the instability
- Cyclic evolution.

## Suppression of the Instability

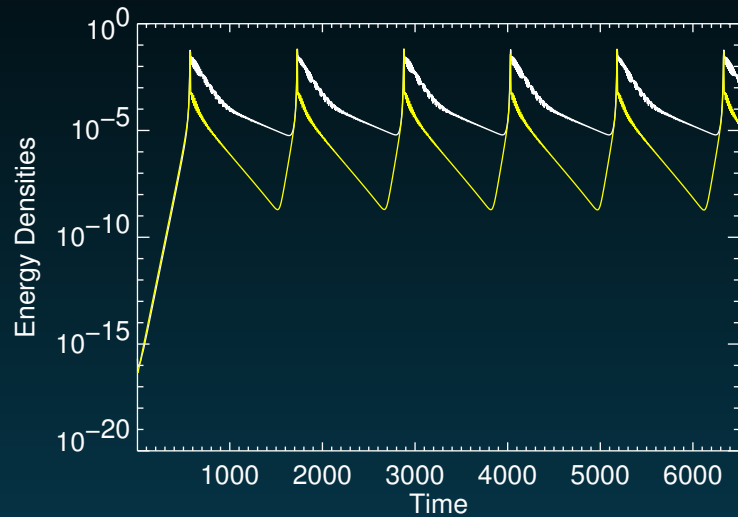
Development of coherent structures:



The radial jet carries  $B_z$  out of the disc. By reducing the flux of  $B_z$  the instability is suppressed and the system relaxes to its initial Keplerian state with a lower magnetic energy.

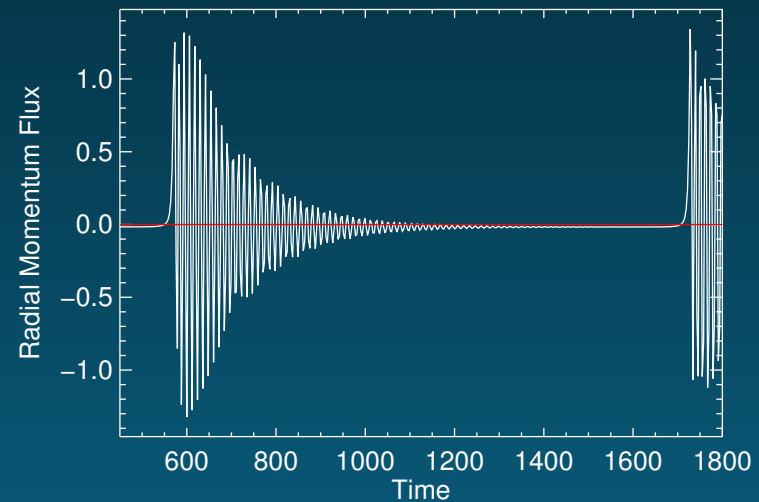
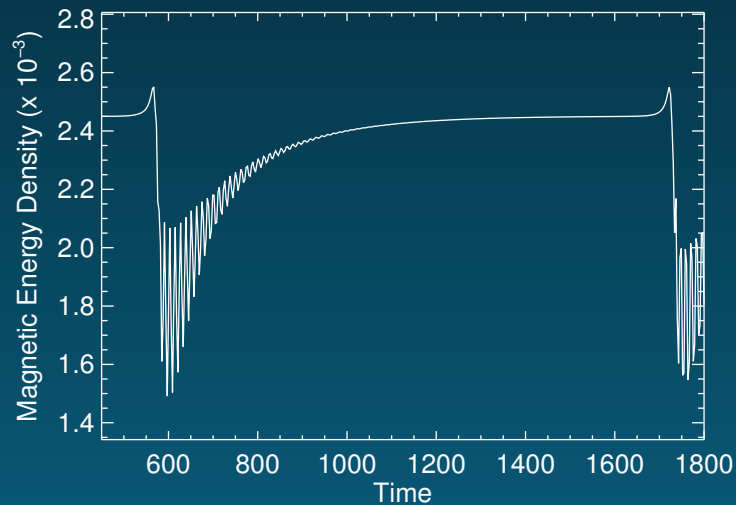


## Extended Disc



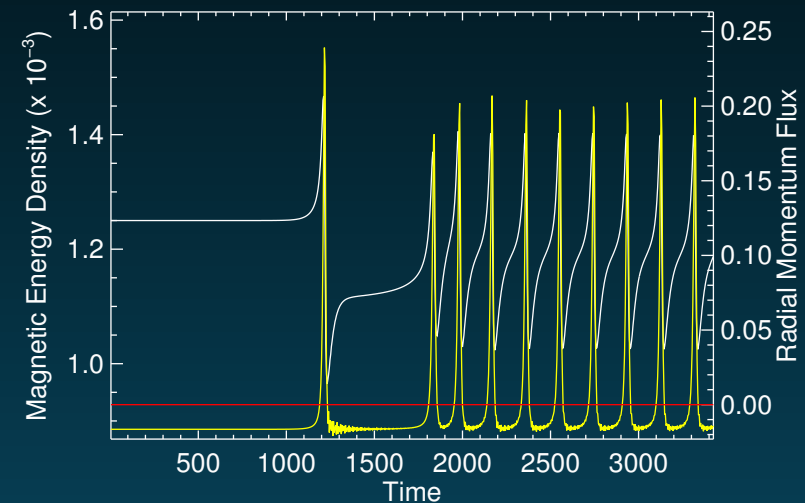
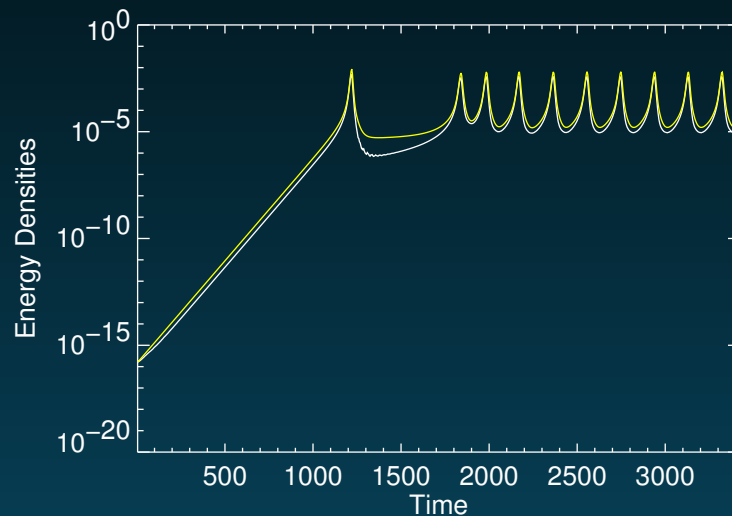
Disc extended three times in radius presents a cyclic behaviour:

- Radial jet carries  $B_z$ -flux in the outer part of the disc and switches-off the instability.
- Violent event: leads to fast radial oscillations of the whole disc & relaxation to initial state.
- Accretion brings  $B_z$ -flux in the inner region and triggers the instability again.



## Starting with a Weaker Magnetic Field

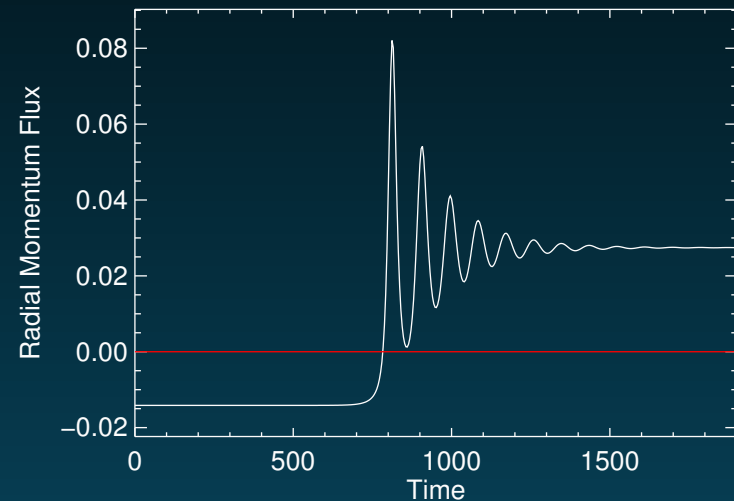
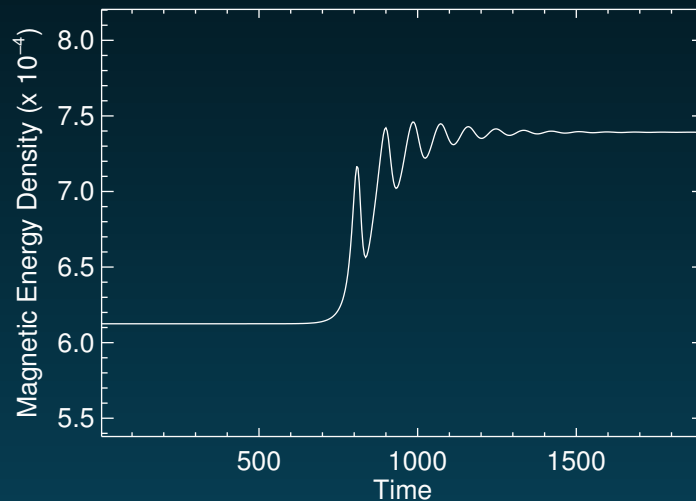
With a reduced initial magnetic field the system presents a cyclic behaviour as in the previous case (rely on the same processes).



- The instability is not strong enough to expel the magnetic flux.
- A fraction of the  $B_z$ -flux initially carried out of the system (disc not extended).
- Cycles around an equilibrium different from the initial state.
- Much less violent event: no fast radial oscillation of the disc.

## Evolution to a New Stable Stationary State

For some slowly growing modes with a sufficiently small initial  $B_z$  the nonlinear regime can lead to a new stable 2-D equilibrium.



- Magnetic energy increased.
- Outward radial flux of material.
- Magnetorotational instability still present.

## Future Work

### Numerical issues regarding:

- the fast ( $\simeq \Omega^{-1}$ ) radial oscillations of the disc,
- the properties of differential operators for Chebyshev polynomials in an annulus.

Stability of the 2-D solutions: Kelvin-Helmholtz stability for higher Reynolds numbers.

### 3-D evolution of the 2-D solutions:

- transition to a turbulent regime,
- systematic parameter space survey.