Exercise class 3

Exercise 3.1: Zero-coupon bonds

Consider a 10-year zero-coupon bond with redemption value £100. Assume an interest rate of 4% (annually compounded).

(a) What is the value of the bond? Does it depend on the redemption value? On the interest rate?

(b) What is the derivative of the answer under (a) with respect to the interest rate? Does it depend on the redemption value? On the interest rate?

(c) What is the volatility / effective duration? Does it depend on the redemption value? On the interest rate?

(d) Estimate the value of the bond if the interest rate is 5%, based on the answer to (c). Then, compute the value of the bond if the interest rate is 5% exactly. Compare the answers.

(e) What is the discounted mean term / Macaulay duration? Does it depend on the redemption value? On the interest rate?

(f) Repeat questions (a)–(e) using continuously compounded interest rates.

Exercise 3.2: Duration of an annuity

Consider a level annuity immediate lasting four years, paying £1000 at \( t = 1, 2, 3, 4 \).

(a) Compute the present value of this annuity on the basis of \( i = 0.07 \).

(b) Compute the volatility / modified duration.

(c) Use the volatility to approximate the present value of the annuity on the basis of \( i = 0.06 \). How does this compare to the exact answer?

(d) Compute the discounted mean term / Macaulay duration.
(e) The discounted mean term of an annuity with term $n$ is given by the formula

$$D = \frac{(Ia)_{\overline{n}}}{a_{\overline{n}}}$$

where

$$a_{\overline{n}} = v + v^2 + \cdots + v^n = \frac{1 - v^n}{i}$$

is the present value of an $n$-year level annuity paying 1 at the end of each year and

$$(Ia)_{\overline{n}} = v + 2v^2 + \cdots + nv^n = \frac{\overline{d}_{\overline{n}} - nv^n}{i} = \frac{1 - v^n}{vi^2} - \frac{nv^n}{i}$$

is the present value of an $n$-year increasing annuity paying 1 at the end of the first year, 2 at the end of the second year, $\ldots$, $n$ at the end of year $n$.

Check this formula for the discounted mean term. If you have time, try to prove it.

(f) We can use the second derivative to get a better approximation of how the present value changes if $i$ drops to 0.06. Thus, compute the second derivative of the present value with respect to $i$. Write down the Taylor series up to the second-order term, and use the second derivative to estimate the present value if $i = 0.06$. Compare this with the answer under (c) and the exact answer.