A great deal of effort has gone into trying to model spreading phenomena, such as the spread of behaviour, norms, and ideas. Binary-state dynamical models use two states (e.g., state 0 and 1), and each node in a network is in one of the two states [1]. This makes it possible to describe spreading phenomena by observing the spread of adoption of states: One investigates how a node in one state adopts the other state by the influence of neighbours and how the node’s change of state leads changes in other nodes’ states. Some existing binary-state models, such as the Watts threshold model [3], can exhibit cascades, in which local adoption becomes widespread throughout a network [2]. Those models tend to assume that individuals instantaneously react to their neighbours’ changes without any time delay – which is often not true in social contexts, where different agents have different response times or different degrees of laziness. In our work, we present a timer model, which includes the following mechanism: if a state-0 (inactive) node \(i\) has a timer \(\tau_i\) – which is a non-negative integer – and meets its adoption condition at time \(T\), then the node adopts state 1 at \(T + 1 + \tau_i\) and becomes an active node. The update can either be synchronous or asynchronous depending on context; we use synchronous updating in our study. Formulating our model in this way enables us to apply the timer model to any existing deterministic social-influence models where nodes change their states by influence from the other nodes [2]. Here we illustrate the incorporation of a timer using the Watts threshold model.

We demonstrate that the adoption process of a dynamical system gets delayed not only due to the size of mean of the initial distribution of the timers but also due to the heterogeneity of the initial distribution of the timer. As an example, one can observe in Fig. 1 that the timer model reaches steady state earlier in Fig. 1a, where the timers are initially homogeneously distributed, than in Fig. 1b where the timers are initially heterogeneously distributed. However, heterogeneous timers can accelerate the adoption process depending on the way timers are distributed. We seek to understand the relationship between the delay of the adoption process and the initial distribution of timers, and we derive a pair approximation that incorporates a timer by modifying a pair approximation of the Watts threshold model presented in [1], and it exhibits good agreement with the numerical results of the Watts thresholds model with a timer on random graphs. We also examine this model on real networks.

Fig. 1: Numerical results (red squares) and the analytical results (blue curve) of the Watts threshold model with a timer on Erdős-Rényi random graphs. The timers are initially distributed as a Gaussian with mean \(\mu_\tau = 3\) and standard deviation \(\sigma_\tau = 0\) (i.e., homogeneous timers) and \(\sigma_\tau = 10\) (i.e., heterogeneous timers). The number of nodes is \(N = 1000\), the mean degree is 5, the fraction of seed nodes is \(\rho_0 = 0.001\), and the thresholds are distributed as a Gaussian with mean \(\mu_\phi = 0.3\) and standard deviation \(\sigma_\phi = 0.2\). We average over 100 realizations of an Erdős-Rényi graph.