Random numbers are the digits 0, 1, . . . , 9 occurring in random order and independently, each with probability 1/10. They can be found in Tables, e.g., New Cambridge Elementary Statistical Tables, Table 27, or can be produced on a computer by pseudorandom number generators, which are deterministic methods yielding long sequences which eventually repeat themselves depending on the values of any control parameters.

The following table of random numbers was produced in R and will be used in examples later.

Table 1

| 92 | 91 | 07 | 14 | 82 | 22 | 50 | 70 | 75 | 15 | 64 | 40 | 07 | 42 | 18 | 52 |
| 69 | 12 | 71 | 34 | 77 | 73 | 66 | 86 | 46 | 91 | 84 | 72 | 27 | 98 | 82 | 16 |
| 27 | 21 | 71 | 35 | 44 | 91 | 58 | 07 | 06 | 90 | 77 | 48 | 60 | 62 | 96 | 62 |
| 23 | 14 | 62 | 65 | 96 | 13 | 72 | 62 | 53 | 68 | 56 | 03 | 18 | 73 | 17 | 52 |
| 14 | 55 | 95 | 79 | 24 | 48 | 42 | 76 | 99 | 89 | 14 | 81 | 57 | 45 | 47 | 63 |
| 83 | 73 | 64 | 50 | 31 | 43 | 63 | 02 | 56 | 91 | 95 | 57 | 17 | 72 | 14 | 40 |
| 43 | 98 | 73 | 38 | 17 | 00 | 98 | 16 | 81 | 68 | 85 | 50 | 37 | 88 | 03 | 49 |
| 36 | 95 | 63 | 81 | 07 | 30 | 81 | 17 | 72 | 44 | 31 | 54 | 41 | 07 | 43 | 39 |
| 47 | 59 | 29 | 29 | 30 | 24 | 20 | 26 | 85 | 37 | 40 | 93 | 25 | 32 | 45 | 86 |
| 71 | 50 | 22 | 16 | 34 | 29 | 80 | 58 | 06 | 99 | 89 | 09 | 66 | 21 | 84 | 94 |
| 31 | 35 | 78 | 93 | 79 | 32 | 44 | 16 | 09 | 65 | 52 | 75 | 37 | 06 | 42 | 61 |
| 42 | 43 | 03 | 85 | 44 | 27 | 29 | 04 | 91 | 62 | 64 | 41 | 84 | 80 | 12 | 98 |
| 68 | 96 | 53 | 53 | 89 | 36 | 07 | 00 | 00 | 25 | 63 | 76 | 70 | 57 | 94 | 31 |
| 84 | 93 | 55 | 62 | 53 | 60 | 55 | 98 | 45 | 85 | 56 | 33 | 05 | 66 | 38 | 69 |
| 15 | 83 | 65 | 12 | 03 | 22 | 73 | 43 | 09 | 95 | 04 | 46 | 08 | 89 | 71 | 85 |

Each single digit in the table occurs with probability 1/10. The presentation in pairs is done simply to make it easier to read off the numbers. Thus, the first pair are ‘nine’ and ‘two’, not ‘ninety-two’.

Taken singly, we have just ten random numbers each occurring with probability 1/10.

Suppose we need more than ten numbers?

Taking pairs of digits, 00, 01, . . . , 99, we have 100 possible different numbers. Each number occurs with probability \( \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \), by the multiplication principle for independent events.

Taking triples of digits, 000, 001, . . . , 999, we have 1000 possible different numbers. Each number occurs with probability \( \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1000} \), by the multiplication principle for independent events, and so on.

How did we get Table 1?

Put digits 0,1,...,9, in a vector `digits`, then use the `sample` command to sample 400 values with replacement from `digits`.

I word processed them into the paired format to make them easier to use, and more like random numbers presented in books.

```r
rand<-numeric(400)
digits<-c(0:9)
rand<-sample(digits,400,replace=TRUE,)
print(rand)
```
Random numbers are used in simulation.

Many physical problems involving random phenomena are difficult or inappropriate to solve analytically. Simulation techniques use random variables and random numbers to create conditions similar to those of real-life problems, in particular where a real-life study would be too costly or time consuming or otherwise impracticable. Simulation is usually carried out on a computer.

To perform a simulation, we repeat an experiment a large number of times to assess the probability of an event or condition occurring.

Samples of random numbers are independent only if they come from different parts of a random number table or they arise from different starting values in a random number generator.

When we use a random number table, we choose a random starting point, e.g., the top of the fourth column, and then use consecutive digits working along the rows taking digits in appropriate groups, singly, pairs or triples etc.

Sampling with replacement allows the same random number to appear more than once; without replacement - we discard repetitions of a number.

### 4.1 Simulation of random variables

The R package has facilities for generating random numbers from a wide range of discrete and continuous probability distributions, as well as random sampling with or without replacement from a discrete distribution which we can specify. We will describe some of these distributions and their uses, and use R to investigate their properties as well as use these distributions in applications.

<table>
<thead>
<tr>
<th>Probability distribution</th>
<th>command</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta</td>
<td>rbeta(n, shape1, shape2)</td>
</tr>
<tr>
<td>Binomial</td>
<td>rbinom(n, size, prob)</td>
</tr>
<tr>
<td>Cauchy</td>
<td>rcauchy(n, location = 0, scale = 1)</td>
</tr>
<tr>
<td>( \chi^2 )</td>
<td>rchisq(n, df, ncp=0)</td>
</tr>
<tr>
<td>Exponential</td>
<td>rexp(n,rate=1)</td>
</tr>
<tr>
<td>F</td>
<td>rf(n, df1, df2)</td>
</tr>
<tr>
<td>Gamma</td>
<td>rgamma(n, shape, rate = 1, scale = 1/rate)</td>
</tr>
<tr>
<td>Geometric</td>
<td>rgeom(n, prob)</td>
</tr>
<tr>
<td>Hypergeometric</td>
<td>rhyper(nn, m, n, k)</td>
</tr>
<tr>
<td>Lognormal</td>
<td>rlnorm(n, meanlog = 0, sdog = 1)</td>
</tr>
<tr>
<td>Logistic</td>
<td>rlogis(n, location = 0, scale = 1)</td>
</tr>
<tr>
<td>Multinomial</td>
<td>rmultinom(n, size, prob)</td>
</tr>
<tr>
<td>Negative binomial</td>
<td>rnbinom(n, size, prob, mu)</td>
</tr>
<tr>
<td>Normal</td>
<td>rnorm(n, mean=0, sd=1)</td>
</tr>
<tr>
<td>Poisson</td>
<td>rpois(n, lambda)</td>
</tr>
<tr>
<td>t</td>
<td>rt(n, df)</td>
</tr>
<tr>
<td>Uniform</td>
<td>runif(n, min=0, max=1)</td>
</tr>
<tr>
<td>Weibull</td>
<td>rweibull(n, shape, scale = 1)</td>
</tr>
<tr>
<td>With/without replacement</td>
<td>sample(x, size, replace = FALSE, prob = NULL)</td>
</tr>
</tbody>
</table>

Use \texttt{?command}, e.g., \texttt{?rbeta} for details about the parameter specification.
One application of random numbers is the simulation of random variables. If $X$ is a random variable and $X_1, X_2, \ldots, X_n$ are independent and have the same probability distribution as $X$, then the collection $X_1, X_2, \ldots, X_n$ is called a random sample of size $n$ for $X$.

**Example 4.1 Bernoulli trials with probability $p = 1/2$**

A Bernoulli trial is an experiment which can result in one of two possible outcomes, designated as “success” or “failure”, like tossing a coin: head or tail. There is a probability $p$ of “success” and $1 - p$ of “failure”. To simulate a random sample of 9 Bernoulli trials with probability $p = 1/2$ for a success, assign digits as follows.

Let success, $S = (0, 1, 2, 3, 4)$ and failure, $F = (5, 6, 7, 8, 9)$.

Using the first line of random numbers from Table 4.1 above,

```plaintext
Random sequence 9 2 9 1 0 7 1 4 8
 corresponds to  F S F S F S F S F
e.g., toss fair coin  T H T H T H H T
```

**Example 4.2 Bernoulli trials with probability $p = 0.135$**

To simulate a random sample of 7 Bernoulli trials with $p = 0.135$ for a success, assign triples of random digits as follows.

Let success, $S = (001, 002, \ldots, 135)$ and failure, $F = (136, 137, \ldots, 999, 000)$.

Again using the first line of random numbers from Table 4.1 above,

```plaintext
Random sequence 929 107 148 222 507 075 156
corresponds to  F S F F F S F
e.g., toss fair coin  T H T H T H H T
```

**Example 4.3 Binomial random variables based on Bernoulli trials**

A binomial random variable, $X$, is defined as the number of “successes” in a fixed number, $n$, of independent, identical Bernoulli trials. E.g., the number of heads in four tosses of a fair coin.

We can think of the binomial as a sum of Bernoulli trials.

To simulate binomial random variables where $X \sim \text{bin}(n = 3, p = \frac{1}{2})$, we can take simulations of Bernoulli trials in groups of 3. Recall Example 4.1.

```plaintext
Sequence F S F S S F S F F S S F F S F
gives $X = 1\ | \ 2\ | \ 2$
```

In R we simulate a Bernoulli trial as simply a binomial trial with $n = 1$.

Thus 10 Bernoulli trials with $p = 0.135$ are given by

```r
rbinom(10,1,0.135)
[1] 0 0 0 0 0 0 0 0 1 0
```

where ‘1’ denotes a success and ‘0’ denotes a failure.
Example 4.4 More binomial random variables
An alternative method of simulating random numbers from a binomial distribution is to use the cumulative distribution function (c.d.f.) to assign random digits to events. Consider simulating random numbers from a binomial distribution with \( n = 4 \) and \( p = 1/2 \).

\[
P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{4}{x} \left(\frac{1}{2}\right)^4, \quad x = 0, 1, 2, 3, 4.
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(X = x) )</th>
<th>( P(X \leq x) )</th>
<th>digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0625</td>
<td>0.0625</td>
<td>0001 – 0625</td>
</tr>
<tr>
<td>1</td>
<td>0.2500</td>
<td>0.3125</td>
<td>0626 – 3125</td>
</tr>
<tr>
<td>2</td>
<td>0.3750</td>
<td>0.6875</td>
<td>3126 – 6875</td>
</tr>
<tr>
<td>3</td>
<td>0.2500</td>
<td>0.9375</td>
<td>6876 – 9375</td>
</tr>
<tr>
<td>4</td>
<td>0.0625</td>
<td>1.0000</td>
<td>9376 – 9999, 0000</td>
</tr>
</tbody>
</table>

Random sequence 9291 0714 8222 5070 7515 corresponds to 3 1 3 2 3

In R, use 
\[ \text{rbinom}(10,4,0.5) \]
to generate 10 binomial random variables with \( n = 4 \) and \( p = 0.5 \).

\[ [1] \ 1 \ 1 \ 3 \ 1 \ 2 \ 2 \ 2 \ 1 \ 1 \]

Example 4.5 Geometric random variables based on Bernoulli trials
A geometric random variable, \( X \), is defined as the number of trials up to and including the first “success” in a sequence of independent, identical Bernoulli trials. To simulate geometric random variables with \( p = 1/2 \), i.e., the number of trials up to and including the first success, take the sequence of Bernoulli trials as in Example 4.1:

\[
\begin{array}{cc}
\text{Sequence} & \text{F} & \text{S} \\
gives & 2 & 2 & 1 & 2 & 1 \\
\end{array}
\]

R defines a geometric distribution in terms of the number of failures before a success, (recall \( Y \) in Example 3.6). To count trials up to and including a success, we need to add 1, thus for 10 random values from a geometric distribution with \( p = 0.5 \)

\[ \text{rgeom}(10,0.5) + 1 \]

\[ [1] \ 1 \ 1 \ 1 \ 3 \ 3 \ 2 \ 1 \ 1 \ 2 \ 5 \]
4.2 Statistical tests for random numbers

4.2.1 Test for equal frequencies

Example 4.6a Two “random” sequences!
Here are two sequences, A and B, of “random” numbers in the range 1 to 10.

Sequence A 6 6 7 5 8 6 6 6 6 7 6 8 4 6 7 5 9 2 6 8 5 10 1
Sequence B 6 7 9 9 7 3 8 5 5 8 5 10 9 3 5 9 7 3 3 10 3 8 6 7 4

The stem-and-leaf plot is fairly even for Sequence B supporting the idea of randomness, but Sequence A has far more sixes than any other value casting doubt on randomness.

4.2.2 Test for independence

What is meant by independence? One way to describe independence is to say that there is no way to predict the value of the next digit by using information about the preceding digits. We can see if there is any relationship between successive digits by plotting $X_{i+1}$ vs. $X_i$ for $i = 1, 2, \ldots$ and examining the result. If the values are unrelated, we should see a random pattern. If the resulting plot shows any kind of trend, e.g., roughly straight line or curve, then the sequence could not be regarded as random.

Example 4.6b Two “random” sequences!

The plot for sequence A clearly shows pattern while the plot for sequence B appears to be a random scatter. Note there seems to be fewer points in the plot of Sequence A - because of the runs of 6 vs 6.
Example 4.7 More “random” sequences
Here, we have two “random” sequences of 50 numbers in the range (1,1000), lcm generated by the linear congruential method, and samp generated by R’s sample command from equally likely integers 1,…,1000.

lcm
[1] 641  814  93  410  401  294  133  330  561  974  773  50  121  854
[15]  13  570  81  934  853  890  441  214  293  10  201  694  333  930
[29]  361  374  973  650  921  254  213  170  881  334  53  490  241  614
[43]  493  610  1  94  533  530  161  774

samp
[1]  749  122  157  644  426  40  158  732  876  87  214  238  484  876
[15]  35  241  141  891  571  646  368  918  286  566  659  244  792  938
[29]  215  439  218  843  572  326  295  303  334  796  982  555  715  460
[43]  564  405  39  197  416  883  928  416

Stem-and-Leaf Display: lcm
Stem-and-Leaf Display: samp
The decimal point is 2 digit(s) to the right of the | The decimal point is 2 digit(s) to the right of the |
0 | 01155899 0 | 4449
1 | 2367 1 | 2466
2 | 0114599 2 | 01224449
3 | 33367 3 | 00337
4 | 01499 4 | 1223468
5 | 3367 5 | 66777
6 | 11459 6 | 456
7 | 77 7 | 2359
8 | 15589 8 | 048889
9 | 23377 9 | 2348

The stem-and-leaf plots show a reasonably even spread of values, bearing in mind 50 is not a large sample so we would not expect it to be perfectly even. We have no evidence against randomness for either of these two sequences.
Consider the plots of lcm_{i+1} against lcm_i and samp_{i+1} against samp_i. Does either of these look suspicious?
4.3 Models of Chance

The Monte Carlo Method is an experimental approach to solving problems using simulation. We describe the steps involved in this simulation method in the context of the following example.

Example 4.8 What is the average number of girls in a three-child family?

1. **Identify the model**
   A Bernoulli trial has two outcomes, success and failure, and is therefore a suitable model for representing girl or boy for a child.

2. **Define a trial**
   Three independent, identical Bernoulli trials will represent one family.

3. **Record the statistic of interest**
   The number of successes represents the number of girls.

4. **Repeat the experiment until a sufficient number of trials is obtained**
   In a probabilistic experiment, answers differ from one run of the experiment to another because these answers are estimates of a theoretical value. The more trials we do, the closer are our results to the theoretical value.

5. **Evaluate the results**
   Find the average of the numbers recorded in Step 3.

Below are histograms of simulations of the observed numbers of girls for different numbers of families - (steps 2, 3 and 4). Notice how the shape of the distribution changes for each simulation. This reflects the randomness we expect to see in a small number of families. In the long run we expect symmetry and the histograms do appear a little more symmetric as the number of families increases. Try this yourself (see the math1815 webpage for the R program for this example).
**Example 4.8 revisited**
Alternatively, we can simulate a three-child family using the binomial distribution with \( n = 3 \) and \( p = 0.5 \) and we see similar results.

1. **Identify the model**
   A Binomial distribution with number of trials \( n = 3 \) and probability of success (= girl) \( p \) is a suitable model for representing a family with three children. The random variable is the number of girls in the family.

2. **Define a trial**
   One observation from this binomial distribution will represent the number of girls in one family.

3. 4. and 5. are the same as given above.

For the Binomial\( (n = 3, p = 0.5) \) distribution, we have \( \mu = np = 1.5 \).
In the simulation the average number of girls approaches the expected 1.5 as the number of families increases.

<table>
<thead>
<tr>
<th>Trials</th>
<th>Min.</th>
<th>1st Qu.</th>
<th>Median</th>
<th>Mean</th>
<th>3rd Qu.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0</td>
<td>1.0</td>
<td>2.0</td>
<td>1.7</td>
<td>2.0</td>
<td>3.0</td>
</tr>
<tr>
<td>50</td>
<td>0.00</td>
<td>1.00</td>
<td>2.00</td>
<td>1.52</td>
<td>2.00</td>
<td>3.00</td>
</tr>
<tr>
<td>100</td>
<td>0.00</td>
<td>1.00</td>
<td>2.00</td>
<td>1.55</td>
<td>2.00</td>
<td>3.00</td>
</tr>
<tr>
<td>500</td>
<td>0.000</td>
<td>1.000</td>
<td>2.000</td>
<td>1.492</td>
<td>2.000</td>
<td>3.000</td>
</tr>
</tbody>
</table>
Example 4.9 The Hermit’s Epidemic
Six hermits live on an otherwise deserted island. An infectious disease strikes the island. The disease has a one-day infectious period and after that the person infected is immune (cannot get the disease again). Assume the hermits are equally likely to become infected, and that one of the hermits at random contracts the disease, possibly from an infected migrating bird. He randomly visits one of the other hermits during his infectious period. If the visited hermit has not had the disease, he gets it and is infectious the following day. The visited hermit then visits another hermit. The disease is transmitted until an infectious hermit visits an immune one, whereupon the disease dies out. There is one hermit visit per day. Assuming this pattern of behaviour, how many hermits on average can be expected to get the disease?
Following the five steps of the Monte Carlo method, design a simulation experiment to solve this problem.

1. **Identify the model**
2. **Define a trial**
3. **Record the statistic of interest**
4. **Repeat the experiment until a sufficient number of trials is obtained**
5. **Evaluate the results**

```r
x<-c(1:6)
rolls<-sample(x,100,replace=TRUE)
3 2 1 6 1 5 1 1 4 4 5 1 4 6 6 5 3 1 2 2 3 5 2 2 1 1 4 6 4 6 5 3 1 3
6 1 2 1 1 3 4 6 6 4 2 1 5 3 5 2 2 3 6 2 6 2 3 3 3 6 4 4 5 5 5 4 4 5 3 6
1 1 4 6 5 6 1 4 2 4 1 1 4 6 5 6 2 6 5 2 2 1 3 2 3 6 3
```

Now complete the following table. Roll the die to see which hermit gets the disease first (3); roll the die to see which hermit is visited and gets the disease (2); continue rolling until an immune hermit is visited, 1 then 6, then 1 is immune so this trial ends, epidemic is over with 4 infected. We have used 3,2,1,6,1 from the first row of the table. Continue with the next group of numbers 5,1,1,4,4,5 for the second trial. Note that we ignore consecutive repeats of the same number.

<table>
<thead>
<tr>
<th>Trial (epidemic number)</th>
<th>Number of hermits infected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>*</td>
</tr>
<tr>
<td>2</td>
<td>*</td>
</tr>
<tr>
<td>3</td>
<td>*</td>
</tr>
<tr>
<td>4</td>
<td>*</td>
</tr>
<tr>
<td>5</td>
<td>*</td>
</tr>
<tr>
<td>6</td>
<td>*</td>
</tr>
<tr>
<td>7</td>
<td>*</td>
</tr>
<tr>
<td>8</td>
<td>*</td>
</tr>
<tr>
<td>9</td>
<td>*</td>
</tr>
<tr>
<td>10</td>
<td>*</td>
</tr>
</tbody>
</table>

|Total | 37 |
|Average | 37/10 = 3.7 |

Use the R program for this example on the math1815 webpage to look at results over many simulations.
MATH1815: Modelling in Statistics and Probability      Exercise 2

Student Assessment of this Exercise

Mark one another’s work using the solution and mark scheme to be provided, working in pairs assigned within your Statistics tutorial group. Your Statistics tutor will resolve any difficulties and will keep a record of your marks.

1. Answer true or false to the following questions.

   (i) The value found for the experimental probability of some event may never be exactly equal to the theoretical probability assigned to that same event.

   (ii) The various values of a random variable form a set of mutually exclusive events.

   (iii) The binomial parameter $p$ is the probability of a success occurring in each of $n$ trials where each trial can result in either a success or a failure.

   (iv) Pseudorandom number generators produce truly random numbers.

   (v) In a simulation of a simple questionnaire, Bernoulli trials may be used to model the answers “Yes”, “No”, “Do not know”.

   (vi) The geometric distribution may be used to model the required number of tosses of a single coin to observe a head.

   (vii) In a sequence of 200 random numbers chosen from the values 0, 1, 2, 3, we might expect to see roughly 50 occurrences of the number 3.

   (viii) In a sequence of ten random numbers chosen from the values 0 and 1, the sequence 0 1 0 1 0 1 0 1 0 1 could not possibly occur.

   (ix) Random digits may be described as independent if there is no way to predict the value of the next digit by using information about preceding digits.

   (x) In a random sequence of digits, a plot of each digit against its preceding neighbour should roughly follow a straight line.

2. Use a table of random numbers (e.g. New Cambridge Elementary Statistical Tables, Table 27) or use the R package to

   (a) simulate 36 Bernoulli trials each with probability $p = 1/3$ of a success occurring,

   (b) simulate 9 values from a binomial distribution with parameters $n = 4$ and $p = 1/3$.

   Describe the method you use in each case.

3. University staff must purchase permits to allow them to use car parking facilities. The committee responsible for fixing the price of permits wishes to gauge the strength of reaction to a proposed increase in price. Use the table of random sampling numbers below to select a random sample of 15 permit holders from the population of 1356 permit holders. Use the table beginning at the top left and working across rows.

   Explain how you allocate random numbers to the members of the population.

   2608 6238 8576 5664 5898 8321 1531 3551 3524 4017 4158 4236 5588 9981 8958 6848
   1036 2817 0180 2400 4173 8817 5847 6216 4203 6133 2098 3353 2919 6739 1238 5660
   3319 5313 1230 6924 7216 3034 4967 2501 6574 1733 8212 6436 4837 1562 0220 2609
4. In the linear congruential method of pseudorandom number generation, a sequence of non-negative integers, \( N_k, k = 1, 2, \ldots \), is obtained using the relation
\[
N_{k+1} = (lN_k + m) \mod n, \quad k = 1, 2, \ldots
\]
where \( l, m \) and \( n \) are given integers, \( n > 0, 0 < l < n, 0 \leq m < n \), and \( N_1, 0 \leq N_1 < n \), the first member of the sequence is also given. 
Taking a period \( n = 5 \), choose suitable values of \( l, m, N_1 \) and evaluate by hand the resulting sequence. By trial and error try to obtain a sequence which has maximum period and which also “looks random”, i.e. does not result in a sequence like 1, 2, 3, 4, 0, 1, 2, 3, ….

5. Two sequences, \( s_1 \) and \( s_2 \), of 50 numbers in the range 0 – 300 are presented below, and are contained in a file \texttt{ex2q4.txt} on the math1815 website. Use R to produce stem-and-leaf plots and plots of \( X_i + 1 \) against \( X_i \), \( i = 1, \ldots, 50 \), for each of the two sequences as follows. In R, type the following commands. Lines beginning with \# are comments - no need to type these.

```r
# read data into a two column matrix called sequences
sequences<-matrix(scan("http://www.maths.leeds.ac.uk/~christin/math1815/ex2q4.txt"), ncol=2, byrow=T)
print(sequences)
# assign column 1 of sequences to a vector s1, then column 2 to s2
s1<-sequences[,1]
s2<-sequences[,2]
# cat prints a title
cat("stem and leaf plot of s1:"
stem(s1, scale=1)
cat("stem and leaf plot of s2:"
stem(s2, scale=1)

# assign rows 1 to 49 of s1 to x(i) and rows 2 to 50 to x(i+1)
xi<-s1[1:49]
xiplus1<-s1[2:50]
# do plot with labelled axes and a main title
plot(xi,xiplus1,xlab="X(i)",ylab="X(i+1)",main="s1")
# print message on screen to give time for you to look at plot 1
readline("plot 1 done - press ENTER to see plot 2")
# this way of plotting s2 avoids assigning the vectors x(i) and x(i+1)
plot(s2[1:49],s2[2:50],xlab="X(i)",ylab="X(i+1)",main="s2")
```

Look at the R output on screen and say whether you judge either sequence to be a sequence of random numbers, and why. The sequences \( s_1 \) and \( s_2 \) are printed below.

\[
\begin{align*}
> \texttt{print(s1)} \\
&\texttt{[1] } 5 \ 78 \ 230 \ 38 \ 250 \ 28 \ 110 \ 98 \ 220 \ 188 \ 30 \ 138 \ 200 \ 198 \ 170 \ 68 \ 90 \\
&\texttt{[18] } 108 \ 70 \ 118 \ 210 \ 48 \ 100 \ 248 \ 0 \ 8 \ 120 \ 238 \ 150 \ 78 \ 230 \ 38 \ 250 \ 28 \\
&\texttt{[35] } 110 \ 98 \ 220 \ 188 \ 30 \ 138 \ 200 \ 198 \ 170 \ 68 \ 90 \ 108 \ 70 \ 118 \ 210 \ 48
\end{align*}
\]

\[
\begin{align*}
> \texttt{print(s2)} \\
&\texttt{[1] } 100 \ 166 \ 112 \ 121 \ 81 \ 196 \ 231 \ 50 \ 216 \ 48 \ 16 \ 91 \ 181 \ 227 \ 119 \ 41 \ 15 \\
&\texttt{[18] } 7 \ 112 \ 297 \ 222 \ 268 \ 237 \ 150 \ 60 \ 184 \ 98 \ 75 \ 184 \ 45 \ 13 \ 79 \ 225 \ 267 \\
&\texttt{[35] } 83 \ 106 \ 281 \ 140 \ 39 \ 216 \ 99 \ 123 \ 46 \ 253 \ 128 \ 28 \ 213 \ 246 \ 186 \ 94
\end{align*}
\]