

**Definition —**

**The special axioms for the first order theory  $\mathcal{PA}$  for arithmetic:**

$$(PA1) (x_1 = x_2 \rightarrow (x_1 = x_3 \rightarrow x_2 = x_3))$$

$$(PA2) (x_1 = x_2 \rightarrow (x'_1 = x'_2))$$

$$(PA3) (\bar{0} \neq x'_1)$$

$$(PA4) (x'_1 = x'_2 \rightarrow x_1 = x_2)$$

$$(PA5) (x_1 + \bar{0} = x_1)$$

$$(PA6) (x_1 + x'_2 = (x_1 + x_2)')$$

$$(PA7) (x_1 \times \bar{0} = \bar{0})$$

$$(PA8) (x_1 \times x'_2 = x_1 \times x_2 + x_1)$$

and the axiom scheme

(PA9) if  $\phi(x_i)$  is a wf of  $\mathcal{L}_{\mathcal{PA}}$ , then

$$(\phi(\bar{0}) \rightarrow ((\forall x_i)(\phi(x_i) \rightarrow \phi(x'_i)) \rightarrow (\forall x_i)\phi(x_i)))$$

is an axiom of  $\mathcal{PA}$ .