

MATH 3102/ 5102 MATHEMATICAL LOGIC 2
Problems 1

1) Show that the following formulas are theorems of PA:

$$\begin{aligned}(PA2)' & t_1 = t_2 \rightarrow t'_1 = t'_2 \\ (PA3)' & 0 \neq (t_1)'\end{aligned}$$

(where t_1, t_2 are terms of PA).

2) Show that $t = r \vdash_{\text{PA}} (r = s \rightarrow t = s)$, and hence that $\vdash_{\text{PA}} t = r \rightarrow (r = s \rightarrow t = s)$.

3)^D Show that $r = t, s = t \vdash_{\text{PA}} r = s$, and hence that $\vdash_{\text{PA}} r = t \rightarrow (s = t \rightarrow r = s)$.

4)^D Show by induction (in the metalanguage) that $\vdash_{\text{PA}} \overline{m+n} = \overline{m} + \overline{n}$ for any non-negative integers m and n .

[In questions 2-4, you can assume the Deduction Theorem, $\vdash_{\text{PA}} t = r \rightarrow r = t$, and all the theorems $(PA1)' - (PA8)'$ got by replacing variables by terms in $(PA1) - (PA8)$.]

HAND IN SOLUTIONS TO TWO QUESTIONS.