

University of Leeds, School of Mathematics
MATH1050
Workshop 5
(Dated: Monday, 1 December 2008)

1. Provide the definition of a complex number.
2. Given the following complex numbers $z_1 = (1, -1)$ and $z_2 = (1, 1)$, compute
 - (a) $z_1 + z_2$,
 - (b) $z_1 \cdot z_2$,
 - (c) z_1^2 ,
 - (d) z_2^2 ,
 - (e) $|z_1|$,
 - (f) $|z_2|$,
 - (g) \bar{z}_1 ,
 - (h) \bar{z}_2 .
3. Express the following complex numbers in polar representation, $z = r e^{i\theta}$, with $r \geq 0$ and $-\pi \leq \theta \leq \pi$.
 - (a) $(1 + \sqrt{-3})^2$,
 - (b) $\frac{1+i\sqrt{3}}{1-i\sqrt{3}}$,
 - (c) $\frac{1+i}{1-i}$,
 - (d) $(2 + 3i)(1 - 2i)$.
4. Find the following roots:
 - (a) the three cube roots of 1,
 - (b) the two square roots of i ,
 - (c) the three cube roots of $-8i$,
 - (d) the six sixth roots of 64.
5. Find the solutions of the following equations:
 - (a) Find the four solutions of the equation
$$z^4 - 2z^2 + 4 = 0.$$
 - (b) Find the six solutions of the equation
$$z^6 + 2z^3 + 2 = 0.$$
 - (c) Find all solutions of the equation
$$x^4 + 4x^2 + 16 = 0.$$
 - (d) Find all solutions of the equation
$$x^4 + 1 = 0.$$
6. Find $\sin 9\theta$ in terms of $\sin \theta$ and $\cos \theta$.
7. Find $\cos 7\theta$ in terms of $\sin \theta$ and $\cos \theta$.
8. Find $\sin 5\theta$ in terms of $\sin \theta$ and $\cos \theta$.
9. Find $\cos 3\theta$ in terms of $\sin \theta$ and $\cos \theta$.