

Examples 5

If you want your answers to be returned to you with comments,
please hand them in at Wednesday's lecture, 1PM on 03/12/08, RSLT 19

1. By using the series

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots,$$

find a series for $\cosh x$. Hence find the sum of the series

$$\frac{2}{0!} + \frac{2}{2!} + \frac{2}{4!} + \frac{2}{6!} + \cdots.$$

2. Express

$$f(x) = \frac{e^x - 1}{x} - 1 - \frac{x}{2} \quad (x \neq 0),$$

as a power series. Differentiate term by term and hence evaluate

$$\frac{2}{3!} + \frac{3}{4!} + \frac{4}{5!} + \frac{5}{6!} + \cdots.$$

3. Find the terms of the Maclaurin series for $\frac{1}{(1+x)^2}$, as far as the term in x^3 . Give an expression for the remainder $R_4(x)$.
4. Find the terms of the Taylor series with centre 9 for \sqrt{x} as far as the term in $(x-9)^2$, giving an expression for the remainder $R_3(x)$. Use these terms to obtain an approximation for the value of $\sqrt{10}$ and to estimate the error in your approximation.