1 MATH 1050: PROPERTIES OF THE DERIVATIVE

Theorem 1.1 *If f is a constant function, $f(x) = c$, then*

$$f'(a) = 0 \qquad \text{for all reals } a.$$

Theorem 1.2 *If $f(x) = x^n$ for some integer $n \geq 1$, then*

$$f'(a) = na^{n-1} \qquad \text{for all reals } a.$$

Theorem 1.3 *If f and g are differentiable at a, then $f + g$ is also differentiable at a, and*

$$(f+g)'(a) = f'(a) + g'(a).$$

Theorem 1.4 (Product Rule) *If f and g are differentiable at a, then $f \cdot g$ is also differentiable at a, and*

$$(f \cdot g)'(a) = f'(a) \cdot g(a) + f(a) \cdot g'(a).$$

Theorem 1.5 (Quotient Rule) *If f and g are differentiable at a and $g(a) \neq 0$, then f/g is also differentiable at a, and*

$$\left(\frac{f}{g}\right)'(a) = \frac{g(a) \cdot f'(a) - f(a) \cdot g'(a)}{|g(a)|^2}.$$

Theorem 1.6 (Chain Rule) *If g is differentiable at a and f is differentiable at $g(a)$, then $f \circ g$ is differentiable at a, and*

$$(f \circ g)'(a) = f'(g(a)) \cdot g'(a).$$

Theorem 1.7 *For the the exponential function $\exp(x) = e^x$ we have*

$$\exp' = \exp.$$

The inverse of $\exp$ is the function $\log$. $\log(x)$ is defined for all x in the range of $\exp$, i.e. $x > 0$. Thus $\exp(\log(a)) = a$ for all reals $a > 0$, and $\log(\exp(y)) = y$ for all reals y.
For every real $b > 0$ put

$$b^x = e^{x \log b}.$$

The derivative of $\log$ is

$$\log'(a) = \frac{1}{a}.$$

Theorem 1.8 *For the functions $\sin : \mathbb{R} \to \mathbb{R}$ and $\cos : \mathbb{R} \to \mathbb{R}$ we have*

$$\sin'(a) = \cos(a) \text{ and } \cos'(a) = -\sin(a)$$

for all reals a.

Definition 1.1 *The* **hyperbolic sine**, **hyperbolic cosine**, *and* **hyperbolic tangent**, *respectively, are defined as follows:*

$$\begin{aligned} \sinh(x) &= \frac{e^x - e^{-x}}{2}, \\ \cosh(x) &= \frac{e^x + e^{-x}}{2}, \\ \tanh(x) &= \frac{\sinh(x)}{\cosh(x)}. \end{aligned}$$