Multiple jets and zonal flow on Jupiter

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Received 24 January 2003; revised 26 May 2003; accepted 30 May 2003; published 17 July 2003.

[1] We investigate the occurrence of multiple jet zonal flows in the 2D rotating annulus model, extended to include the possibility of boundary friction. We consider Rayleigh numbers up to 10 times critical. Without boundary friction the majority of our solutions are single-jet zonal flows, but when boundary friction is present, persistent multiple jet solutions are found much more easily. Compared to the stress-free case, the number of jets increases, though the strength of the zonal flow decreases. The dependence of these features on Ekman and Rayleigh number is discussed, suggesting that at values well beyond the reach of present 3D simulations, solutions resembling the observed jovian zonal flow may exist. The boundary condition at the metallic/insulating hydrogen interface will be an important part of any explanation of the occurrence of multiple jets on the planet Jupiter. *INDEX TERMS:* 5739 Planetology: Fluid Planets: Meteorology (3346); 5724 Planetology: Fluid Planets: Interiors (8147); 5707 Planetology: Fluid Planets: Atmospheres—structure and dynamics; 6220 Planetology: Solar System Objects: Jupiter. **Citation:** Jones, C. A., J. Rotvig, and A. Abdulrahman, Multiple jets and zonal flow on Jupiter, *Geophys. Res. Lett.*, *30*(14), 1731, doi:10.1029/2003GL016980, 2003.

1. Introduction

[2] Understanding the large scale circulation of Jupiter's atmosphere has long been a major challenge for planetary science. Detailed information about the surface flows came back from the Voyager and Galileo missions, and the increase in wind speed with depth reported by the Galileo probe adds weight to the *Busse* [1976, 1994] model. Busse suggested that the zonal flow was driven by convection in the $\sim 10^4$ km deep zone between the surface and the magnetic core. Recent numerical simulations [*Christensen* 2001, 2002; *Aurnou and Olson*, 2001], have found zonal flows of the right order of magnitude arising from driving by the Reynolds stresses, and laboratory experiments in rapidly rotating convecting systems [*Manneville and Olson*, 1996] also show that zonal flows occur naturally in such systems.

[3] The main theoretical difficulty that the Busse model has encountered is the explanation of the multiple jet structure of Jupiter's zonal flow [*Yano*, 1998]. Three-dimensional simulations [*Christensen*, 2001] at moderately small Ekman number do not reproduce the complex array of prograde and retrograde zonal jets in the jovian atmosphere. The linear theory of convection in a rotating sphere is now well understood [*Jones et al.*, 2000], but linear models also fail to produce multiple jets.

[4] As we see below, it is essential to reach very low Ekman number (equivalent to high β in an annulus model), and this is hard to achieve in a fully three-dimensional model for numerical reasons, especially as models must be run for well over a diffusion time. We therefore adopt the simpler two-dimensional Boussinesq annulus, or β-plane, model for our study, details being given in *Brummel and Hart* [1993]. Although the spherical geometry and the compressibility are important aspects of the dynamics of Jupiter's atmosphere, multiple jet formation can be usefully studied in this approximation.

2. The Annulus Model

[5] The geometry of the annulus is shown in Figure 1. It corresponds to a region of the atmosphere in the northern hemisphere, the endwall at $z = 0$ being the interface with the metallic hydrogen core, and the endwall at $z = L_z$ being the planet surface. In the planet, gravity and the mean temperature gradient are radial, not perpendicular to the rotation axis as in the annulus. Rapidly rotating convection is still expected to take a columnar form inside the tangent cylinder, but the local critical Rayleigh number is significantly higher than outside the tangent cylinder. In Jupiter, we expect strongly supercritical convection at all latitudes, so the annulus model, which also has columnar convection, is a reasonable simple model.

[6] The x coordinate corresponds to the azimuthal direction and y to the direction towards the rotation axis. The slope of the endwalls is denoted by η_B at the bottom surface and η_T at the top surface. Both slopes are assumed small in the annulus theory. The boundaries at $y = 0$, $y = L_y$, and $z = L_z$ are assumed stress-free and non-slip boundary conditions are imposed at $z = 0$. Using standard Ekman layer theory [e.g. *Greenspan*, 1968] the momentum equation and heat equation then become

$$\frac{\partial\omega}{\partial t} + \frac{\partial(\psi,\omega)}{\partial(x,y)} - \beta\frac{\partial\psi}{\partial x} = -\mathrm{Ra}\frac{\partial\theta}{\partial x} - C|\beta|^{1/2}\omega + \nabla^2\omega, \qquad (1)$$

$$\mathrm{Pr}\left[\frac{\partial\theta}{\partial t} + \frac{\partial(\psi,\theta)}{\partial(x,y)}\right] = -\frac{\partial\psi}{\partial x} + \nabla^2\theta, \qquad (2)$$

$$\mathbf{u} = -\nabla\times\psi(x,y)\,\mathbf{e}_z, \quad \omega = \nabla^2\psi, \qquad (3)$$

where ω is the z-vorticity and θ is the temperature perturbation. The system has been non-dimensionalized on the length scale L_y, time scale $\tau_\nu = L_y^2/\nu$, and temperature scale $\mathrm{Pr}\Delta T$. The Rayleigh number $\mathrm{Ra} = g\alpha\Delta T L_y^3/\kappa\nu$, and $\mathrm{Pr} = \nu/\kappa$ is the Prandtl number. Here ν and κ are the

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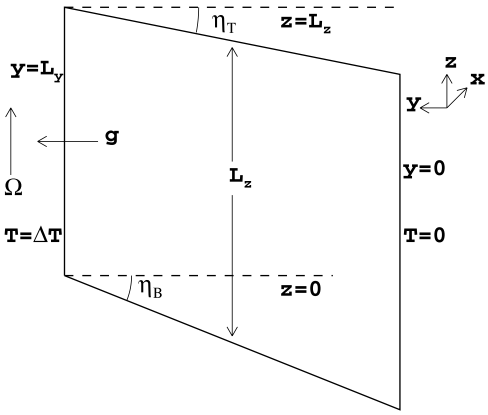

Figure 1. The annulus geometry.

kinematic viscosity and thermal diffusivity, respectively. The rotation strength is given by $\beta = 2(\eta_T - \eta_B)\Omega L_y^3/L_z\nu$. β has the opposite sign inside the tangent cylinder from the value outside, as the vertical height of the annulus now increases outward. In consequence, thermal Rossby waves propagate westward rather than eastward. Since the equations are invariant under a change of sign of β, ψ and x, solutions at $\pm\beta$ are essentially equivalent. For easy comparison with previous work we use positive β here. The term involving the constant $C = (L_y/2|\eta_T - \eta_B|L_z)^{1/2}$ arises from the Ekman layer at the bottom surface, so that we can include the effect of the Ekman layer while retaining a 2D model. A mathematically similar model has been developed by Aubert, Gillet and Cardin (preprint, 2003), in connection with laboratory experiments. They consider flow outside the tangent cylinder, and they include a similar term to represent the effect of the Ekman layer at the outer sphere. For a spherical shell, where η_B, $\eta_T \sim O(1)$ and $L_y \sim L_z$, we have $C = O(1)$, and the Ekman number $E = \nu/2\Omega L_y^2$ corresponds to $1/\beta$.

[7] The simplest model for the boundary friction term is to assume that the metallic core is a rigid rotator, bound together by the magnetic field. The evidence suggests that typical core velocities are orders of magnitude smaller than jet flow velocities [*Starchenko and Jones*, 2002], so a rigid boundary condition is a reasonable first approximation. Actually, the boundary condition will be affected by the magnetic field, and the non-Boussinesq nature of Jupiter's atmosphere may also affect the value of C. We therefore adopt the view that the value of C is a model-dependent parameter.

3. An Asymptotic Model

[8] Two different techniques have been used to explore the behaviour of the model. In the asymptotic limit of large β, [*Abdulrahman et al.*, 2000] have considered weakly non-linear convection. The streamfunction is expanded

$$\psi = \sum_{l=-1}^{1} \sum_{m=1}^{N_y} \psi_{lm} e^{ilx(k/\epsilon)} \sin m\pi y\ , \quad (4)$$

where $\epsilon = \beta^{-1/3}$. No bottom friction is included in this asymptotic model, $C = 0$. The Rayleigh number $\mathrm{Ra} = \epsilon^{-4}(r_0 + \epsilon^2 r_2)$. The wavenumber k is found by minimizing r_0. A complicated bifurcation sequence is found as r_2 is increased above critical, with chaotic time-dependence at r_2 above ≈ 50 at Pr = 1. The $m = 1$ mode dominates the zonal flow $\bar{u}_x$, (i.e. $\bar{u}_x \sim \cos \pi y$ on $0 \leq y \leq 1$) until $r_2 \approx 500$. The bar here denotes the x-average. Thereafter, persistent multiple jet solutions are found, as shown in Figure 2. A multiple jet solution is one in which the $m = 1$ part of the mean $l = 0$ component in equation (4) is not dominant, i.e. $|\psi_{0m}| > |\psi_{01}|$ for some $m \neq 1$. The asymtotic expansion is only valid for small ε, and also requires $\mathrm{Ra/Ra}_c - 1 \ll 1$.

4. The 2D Model

[9] The previous model remains valid only close to critical. However, there exists a range of supercritical Rayleigh numbers, where the effect of rotation strongly suppresses structures along the rotation axis. The 2D annulus model takes advantage of this fact. We consider this model at moderate Rayleigh number and Pr = 1. *Brummell and Hart* [1993] (referred to as BH in the following) investigated the same problem using a period $L_x = 2\pi$ along the x-axis in most of their simulations. For comparison reasons we choose the same aspect-ratio, and expand

$$\psi = \sum_{l=-(N_x-1)}^{N_x-1} \sum_{m=1}^{N_y-1} \psi_{lm} e^{ilx(2\pi/L_x)} \sin m\pi y\ , \quad (5)$$

where $L_x = 2\pi$. The code has been parallelized using the technique described in *Rotvig and Jones* [2002]. All runs in this section have a random initial state. Figure 3 shows the parameter range that was accessible to BH. Our reproduction of their results confirms that there is little evidence of multiple jet solutions in this range. In fact, only at (β, Ra, C) = (7.5×10^4, 1.0×10^7, 0.0), $\mathrm{Ra/Ra}_c = 2.5$, did we obtain a solution which has approximate zonal energy equipartion between the (l, m) = (0, 1), (0, 2) modes. The solution has been integrated 40 time units (one unit equals a viscous diffusion time τ_ν in real time). The zonal energy is 78% of the total kinetic energy.

[10] Encouraged by the asymptotic model, we have scanned the range $\beta \in [1.0 \times 10^4, 1.0 \times 10^6]$ for multiple jet solutions. In this regime, onset of convection occurs at $Ra_c \approx 0.79\beta^{1.33}$, where the marginal solution is dominated

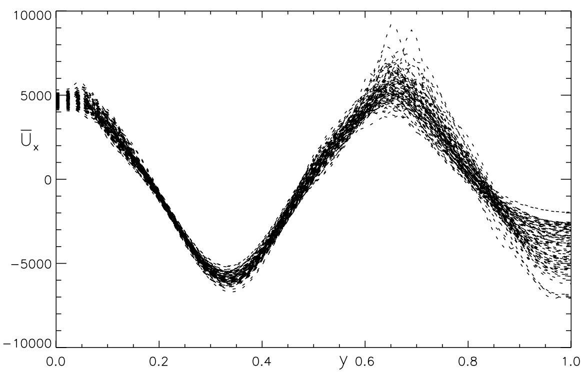

Figure 2. An $m = 3$ multiple jet solution in the asymptotic model. $r_2 = 700$, Pr = 1. $\bar{u}_x$ is plotted as a function of y at 100 times separated by 0.01 units of dimensionless time.

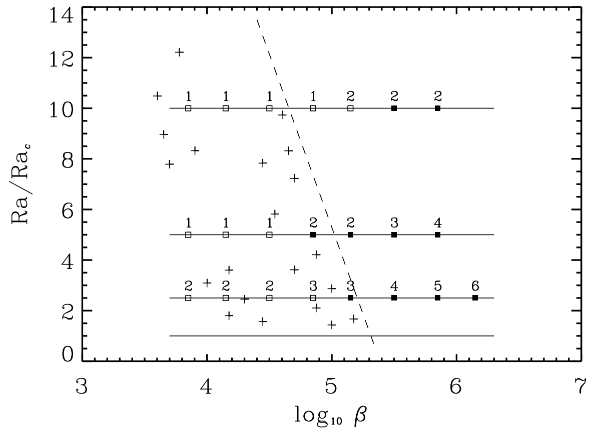

Figure 3. Parameter space section. The points considered in Table 1 in BH (except (β, Ra) = (4.0×10^3,1.0×10^6)) are depicted by +'s. The dashed line illustrates a boundary of their parameter space section. The squares display the 3 β sequences in the boundary friction case, C = 0.316. The number at each point indicates the most frequent value of the peak zonal component m_p. Solid squares denote that the two most frequent values of m_p are greater than 1 at any time after an initial transient.

by a mode with $l \approx 0.67\beta^{0.34}$. For stress-free boundary conditions we have, in general, to stay relatively close to critical to suppress a dominating (l, m) = (0, 1) mode. The solution obtained is quite sensitive to initial conditions, so that solutions with different final states can be found even for identical values of the parameters. The occurrence of multiple jet solutions for Ra/Ra$_c$ $\leq$ 3.0 is sparse. An example is obtained at (β, Ra/Ra$_c$, C) = (5.0×10^5, 2.5, 0.0). This solution has been integrated for 7.2 time units. The kinetic energy of the zonal flow, of which 99% belongs to the (l, m) = (0, 2) mode, is 88% of the total kinetic energy. Videos displaying u_x in the (x, y) plane suggest large-scale waves propagating along the x direction. However, compared to $L_x = 2\pi$, these waves are relatively localised, and we would not expect significant deviations at larger aspect-ratio. Indeed, an integration with $L_x = 4\pi$ (and double the number of longitudinal modes) reproduces the m = 2 solution.

[11] In the post-transient solution, a roughly periodic 'bursting phenomenon' occurs, [see e.g. *Christensen*, 2001; BH, 1993] in which a fairly sudden burst of convection occurs, strongly enhancing the zonal flow. The shear associated with this flow temporarily suppresses the convection, which remains weak until the zonal flow decays. Eventually, convection breaks out again, and the cycle is repeated.

[12] In the above model with stress-free boundary conditions the search for multiple jet solutions is rather bleak. E.g., very close to the above m = 2 multiple jet solution we find solutions with a dominating (l, m) = (0, 1) mode. At (β, Ra/Ra$_c$, C) = (5.0×10^5, 2.25, 0.0) the strength of the zonal flow reduces to 53%, and at Ra/Ra$_c$ = 2.00, 2.75 the solution enters a single-jet state after 0.8, 0.6 diffusion times, respectively.

[13] This situation is dramatically changed by introducing non-slip boundary conditions at the bottom surface. As noted by *Christensen* [2001], boundary friction diminishes the zonal flow. However, as shown below, boundary friction also promotes multiple jet solutions. We have carried out a systematic survey with $\beta \in [7.07 \times 10^3, 1.41 \times 10^6]$, Ra/Ra$_c$ $\in$ [2.5, 10.0], and C = 0.316. The low β region of this regime overlaps (for C = 0.0) the parameter space section investigated by BH, see Figure 3. The effect of boundary friction on marginal convection is minimal. As expected, Ra$_c$ increases slightly, the increase is 4.8% at $\beta = 7.07 \times 10^3$ and reduces to 1.7% at $\beta = 1.41 \times 10^6$. The l index of the dominating mode remains unaffected. All supercritical solutions have been integrated 6.6–10.0 time units (diffusion times) except at the numerically very expensive points in the upper right corner of Figure 3. The corner point has been integrated 0.54 units and its nearest neighbours 3.0–3.4 units. The total kinetic energy scales as $O(\beta^{0.83})$, $O(\beta^{0.93})$, and $O(\beta^{1.09})$ for Ra/Ra$_c$ equal to 2.5, 5.0, and 10.0, respectively. For all 3 β-sequences the rms temperature perturbation decreases as function of β, except at (β, Ra/Ra$_c$) = (1.41×10^5, 5.0) and (β, Ra/Ra$_c$) = (7.07×10^5,10.0), suggesting a general reduction of thermal boundary layer thicknesses at lower viscosity. For Ra/Ra$_c$ = 2.5 we find a well-defined β-scaling, $\sqrt{\left\langle|\theta|^2\right\rangle_{xy}} = O\left(\beta^{-0.28}\right)$.

[14] Table 1 gives the main results of this Letter. For Ra/Ra$_c$ = 2.5 the zonal flow gradually builds up as β is increased. For low β the peak zonal component $m_p(t)$ varies during integration. However, this situation changes at higher

Table 1. Zonal Flow Characteristics in the Boundary Friction Case, C = 0.316.

Ra/Ra$_c$	2.5		5.0		10.0	
β	m: $\langle E_k^z(m)/E_k\rangle_{t'}$ (t'/T)		m: $\langle E_k^z(m)/E_k\rangle_{t'}$ (t'/T)		m: $\langle E_k^z(m)/E_k\rangle_{t'}$ (t'/T)	
7.07×10^3	2: 3% (47%)	1: 2% (40%)	1: 51% (100%)		1: 53% (100%)	
1.41×10^4	2: 7% (58%)	1: 3% (29%)	1: 16% (74%)	2: 4% (21%)	1: 67% (100%)	
3.16×10^4	2: 10% (72%)	3: 3% (17%)	1: 10% (68%)	2: 23% (29%)	1:61% (100%)	
7.07×10^4	3: 17% (70%)	2: 7% (25%)	2: 60% (100%)		1: 25% (92%)	2: 16% (8%)
1.41×10^5	3: 33% (99%)	4: 12% (1%)	2: 54% (100%)		2: 44% (99.7%)	1: 28% (0.3%)
3.16×10^5	4: 38% (100%)		3: 65% (100%)		2: 72% (100%)	
7.07×10^5	5: 45% (100%)		4: 65% (100%)		2: 58% (100%)	
1.41×10^6	6: 48% (100%)		–		–	

At each value of Ra/Ra$_c$ and β the two most dominant wavenumbers m are shown (one m-value only if this dominates for all time), together with the time-averaged ratio of the kinetic energy of the flow associated with that component m, $E_k^z(m)$, to the total kinetic energy E_k. The averages $\langle \ldots \rangle_{t'}$ are taken only over the time-intervals t' where $m_p = m$. The ratio of the time t' when $m = m_p$ to the total integration time T is displayed in parenthesis. Only the 2 most frequent zonal components are included in the table, and in this case the sum of the numbers in parenthesis will be less than 100%. Figure 3 illustrates the most frequent value of m_p.

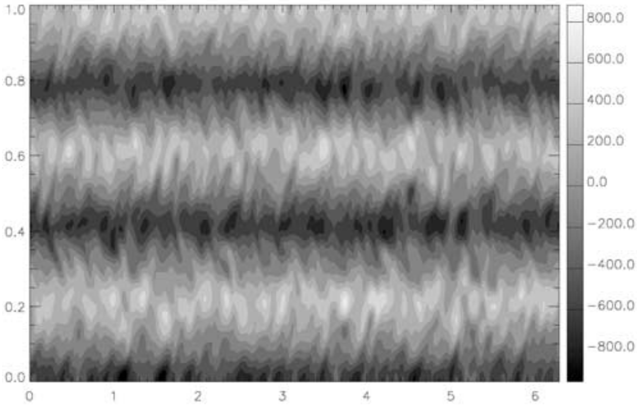

Figure 4. The velocity component u_x in the (x, y) plane at $t = 9.467$ for a multiple jet solution with boundary friction. The parameters are $(\beta, \mathrm{Ra}/\mathrm{Ra}_c, C) = (7.07 \times 10^5, 2.5, 0.316)$. Note the distinct banded structure of u_x despite the fact that the zonal energy is only 48% of the total kinetic energy.

β. Figure 4 shows a snapshot of the multiple jet solution at $\beta = 7.07 \times 10^5$ which has a single-valued $m_p = 5$. For $\beta \geq 1.41 \times 10^5$ we notice that the index of the dominating zonal mode satisfies $m_p \approx O(\beta^{0.30})$. This scaling is roughly consistent with a balance between Coriolis and inertial accelerations which gives the Rhines wavenumber $L^{-1} \sim (\beta/u_y)^{1/2} \sim O(\beta^{0.29})$ [*Rhines*, 1975]. This is obtained from the scaling $u_y \sim O(\beta^{0.42})$ found at $\mathrm{Ra}/\mathrm{Ra}_c = 2.5$. In fact, $m_p \in [4, 5]$ at $\beta = 5.0 \times 10^5$, the point in parameter space where we found the above multiple jet solution having $m_p = 2$ with stress-free boundary conditions. The dominant m in Table 1 is a generally decreasing function of only the Rayleigh parameter $\mathrm{Ra}^* = \mathrm{Ra}/\beta^2\,\mathrm{Pr}$, introduced by *Christensen* [2002], which is independent of the diffusion coefficients. This suggests that the number of jets is controlled by non-diffusive processes, although the large change in behaviour induced by bottom friction indicates diffusive processes still play a role. Thus increasing β results in m_p overshooting the number of observed jets on Jupiter. However, as seen in Table 1, increasing the Rayleigh number for fixed β reduces m_p. At $\mathrm{Ra}/\mathrm{Ra}_c = 5.0$ the zonal flow no longer increases its strength monotonically as β is increased. We observe two minima which seem to separate solutions with single-valued peak zonal components. The first minimum, which separates $m_p = 1,2$, is significant, whereas the second minimum, between $m_p = 2,3$, is only a minor dip in zonal flow strength. This pattern repeats itself at $\mathrm{Ra}/\mathrm{Ra}_c = 10.0$ but only at higher β. This β-delay will make it much more difficult to obtain numerical multiple jet solutions at strongly supercritical Rayleigh numbers.

[15] It is interesting to note that the second minima in the $\mathrm{Ra}/\mathrm{Ra}_c = 5.0, 10.0$ sequences coincide with the two local maxima in the rms temperature mentioned previously. Furthermore, Table 1 also shows that in general the zonal flow is stronger at higher Rayleigh number. Since $\mathrm{Ra}/\mathrm{Ra}_c \leq 10.0$ in our simulations we do not observe the maximum in relative strength of the zonal flow as function of Rayleigh number as observed in *Christensen* [2002]. However, if the above pattern continues to apply at the much more extreme jovian values of (β, Ra) then solutions resembling the observed jovian zonal flow may exist at these parameter values. If the magnetic field leads to lower Ekman suction near the metallic core this may also contribute to the desired solutions at jovian (β, Ra) values.

[16] Although Saturn has some evidence of multiple jet flow, it is a much less prominent feature than on Jupiter [see e.g. *Ingersoll*, 1990]. The saturnine wind speeds are greater, (despite the smaller heat flux emerging from Saturn) consistent with the view that they are less affected by the bottom boundary condition than the winds on Jupiter. The recent observations of high latitude alternating jets on Jupiter [*Porco et al.*, 2003] also suggest bottom friction may be important, but we also note that there are several alternating jets outside the tangent cylinder if the radius ratio is around 0.8. More realistic geometry and non-Boussinesq modelling are clearly desirable features for future work, but it is now clear that it is essential in any realistic model to reach very high β (or equivalently very low Ekman number) and to take proper account of the bottom boundary condition before reliable results on the zonal flow problem can be achieved.

[17] **Acknowledgments.** This work was supported by PPARC Grant No. PPA/G/S/2000/00018. The computational work was performed on the Southern UK MHD cluster at University of Exeter, supported by PPARC and the University of Exeter.

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