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Dynamos in Planets

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Significant advances in our understanding of the geodynamo have been made over the last ten years. In this review, we consider the extent to which this knowledge can be used to understand the origin of the magnetic fields in other planets. Since there is much less observational data available, this requires a 'first principles' understanding of the physics of convection driven dynamos.

1.1 Introduction

The basic structure of the interior of the Earth has been worked out by seismologists. The iron core is divided at $r_{icb} = 1220$ km, the inner core boundary (ICB), into the solid, mainly iron, inner core below and the fluid outer core above. The exact composition of the outer core is not known, but the most plausible models suggest it is a mixture of liquid iron and various impurities, probably sulphur and oxygen (Alfè *et al.* 2000). The whole core is electrically conducting. Above the core-mantle boundary (CMB), at $r_{cmb} = 3485$ km, lies the rocky mantle. The electrical conductivity of the mantle is very small, except possibly very close to the CMB itself, where iron may have leaked into the mantle. The basic structure of the other terrestrial planets, in which we include the larger satellites, is believed to be similar to that of the Earth, but the size of the iron core varies considerably, and the division between the fluid outer core and the solid inner core, if it exists, has to be computed from theoretical models. The moon's core is probably only 5% of its mass, whereas Mercury's core accounts for nearly 70% of its mass. Except in the case of the Earth, where seismological information is available, the size of a planet's core is inferred from measurements of its density distribution, which in turn is derived from its gravitational field. This can be mapped accurately by space probes.

The geomagnetic field has been recorded at magnetic observatories since the time of Gauss, and excellent data are available for the recent field from magnetic satellite observations. Historical data are also available (e.g. Jackson, Jonkers & Walker, 2000), so that the behaviour of the field over the last few hundred years can be mapped with some accuracy. Although it is not possible to reconstruct the fluid flow beneath the CMB (even if the frozen flux assumption is made, flow locally parallel to the field cannot be detected), it is nevertheless possible to estimate the typical flow velocity at 3×10^{-4} ms^{-1}, and to sketch the main features of the flow (Bloxham & Jackson, 1991). For example, the core fluid flow is primarily westward below the Atlantic, but appears to be significantly smaller below the Pacific, and there are gyres at high latitudes which may be connected with convection rolls (Longbottom *et al.* 1995). Velocities of this order give a convective turn-over time of order 10^3 years.

Paleomagnetic studies show that the geomagnetic field occasionally reverses polarity, and recently is has become clear (e.g. Gubbins, 1999) that excursions (major changes in the direction of the dipole component), and variations of the strength of the dipole (Valet & Meynardier, 1993), are far more common than full reversals. Data is available on the behaviour during a reversal (e.g. Hoffmann, 2000), which can be compared with theoretical reversal models. Reversal behaviour may be coupled to mantle convection, through an inhomogeneity in the heat flux at the CMB, which can affect core flow and hence the dynamo (e.g. Sarson *et al.*, 1997).

Our knowledge of the physical properties of core material are derived from various sources: experiments such as the diamond anvil technique (Boehler, 1993) can suggest how material at high pressure and temperature behaves. More recently, *ab initio* quantum calculations (e.g. Vocadlo *et al.*, 2000) are coming onstream. This work gives us estimates of the specific heat, the viscosity, and the thermal and electrical conductivities which are of importance in dynamo models. Braginsky & Roberts (1995) give the numerical values of many of the useful physical properties of the Earth's core, and Stevenson's comprehensive (1983) review has many of the corresponding properties in other planets. Another good source of physical data (with references) is Lodders & Fegley (1998).

This observational and experimental information is incorporated into geodynamo models. However, it is only recently that computer hardware powerful enough to solve relevant dynamo equations in reasonably realistic models has been developed. Unfortunately, it is still not possible to solve the equations numerically in the right parameter regime, but only in regimes in which the diffusivity has been artificially enhanced. As we see below, this is not necessarily a fatal objection, but it does mean that great care is needed before such simulations can be realistically applied to other planets.

Before the exploration of our solar system by space probes, our theoretical understanding of planetary interiors was insufficient to allow any worthwhile predictions of planetary magnetic fields. The rich diversity that was discovered came as a surprise. Now that far more information is available, we face the less challenging, but still very difficult, task of trying to fit the data into a coherent theoretical framework. There are many obstacles to this, some arising from lack of data about the physical conditions in planetary cores, and some from our lack of theoretical understanding of convective dynamos. The aim of this review is to explore what can be done, and where the major obstacles to progress lie.

1.2 Planetary magnetic fields

When considering the origin of their magnetic fields, it is natural to divide the planets into three groups. The terrestrial planetary dynamos are those with iron cores, which are either currently liquid, or have been liquid in the past. Such cores have an electrical conductivity σ of $\sim 4 \times 10^5$ Sm^{-1}, which corresponds to a magnetic diffusivity η of ~ 2 $\mathrm{m}^2\mathrm{s}^{-1}$. Here $\eta = 1/\sigma\mu$, and as all planetary cores have temperatures well above the Curie point, the permeability μ has its free space value. The second group are the giant planets, Jupiter and Saturn, which are sufficiently massive to have metallic hydrogen cores. This gives them a high electrical conductivity, with a corresponding magnetic diffusivity $\sim 4 \times 10^{-2}$ $\mathrm{m}^2\mathrm{s}^{-1}$. The third group consists of the outer planets Uranus and Neptune, which are not large enough to have metallic hydrogen cores, but are believed to have cores with of ionic material (Stevenson, 1983, Holme & Bloxham, 1996) with a comparatively low electrical conductivity, corresponding to a magnetic diffusivity $\sim 10^2$ $\mathrm{m}^2\mathrm{s}^{-1}$. This considerable diversity in the physical conditions means we must be very wary of simplistic 'universal' laws, such as the magnetic Bode's law, which states that the magnetic dipole moment is proportional to the planet's angular momentum.

In table 1.1, we list some of the properties of planetary magnetic fields. Magnetic fields are given in Tesla; 1 T = 10^4 gauss. The data for table 1.1 was taken from Lodders & Fegley (1998), who give the references of the source papers. The dipole component of the field is always dominant far from the planet, and hence is the easiest to measure observationally. In the case of the Earth, the dipole component is still the largest component even

Table 1.1. *Planetary magnetic fields*

	Dipole moment Tm^3	Planetary radius m	Core radius T	Max Field at CMB degrees	Dipole inclination s^{-1}	Rotation rate
Mercury	4.3×10^{12}	2.438×10^6	1.9×10^6	1.4×10^{-6}	< 10	1.24×10^{-6}
Earth	1.6×10^{16}	6.371×10^6	3.48×10^6	7.6×10^{-4}	11.5	7.3×10^{-5}
Ganymede	1.4×10^{13}	2.634×10^6	4.8×10^5	2.5×10^{-4}	~ 10	1.02×10^{-5}
Jupiter	1.5×10^{20}	6.995×10^7	5.6×10^7	1.7×10^{-3}	9.6	1.76×10^{-4}
Saturn	4.2×10^{18}	5.830×10^7	3.2×10^7	2.5×10^{-4}	0.8	1.64×10^{-4}
Uranus	3.8×10^{17}	2.536×10^7	1.8×10^7	1.3×10^{-4}	58.6	1.01×10^{-4}
Neptune	2.0×10^{17}	2.462×10^7	2.0×10^7	5.2×10^{-5}	47.0	1.08×10^{-4}

when the field is extrapolated down to the core-mantle boundary, but this is not necessarily true of other planetary magnetic fields.

The Earth and Ganymede, the largest of Jupiter's moons, are the only terrestrial planets where there is compelling evidence for a dynamo. Mercury is generally believed to have a dynamo, but its field is weak. It has a large iron core, extending to a radius of 1860 km (Spohn *et al.* 2001) compared with a total radius of 2440 km. Mercury is also a slow rotator, being locked in a 2:3 resonance with its orbital period.

The magnetic field of Ganymede was discovered by the Galileo mission. Its strength is similar to that of Io, the innermost of the Galilean satellites, but the ambient field of Jupiter at Io is of the same magnitude as the suggested internal field, so it is difficult to establish whether Io has a genuine internal field or not (Kivelson *et al.* 2001). Ganymede is further from Jupiter, where the Jovian field is much weaker, so the internal origin of Ganymede's field is not in doubt. Ganymede was not expected to have a magnetic field, as it was thought to have cooled sufficiently to stop convecting. Tidal interactions may have reheated the planet after formation (Showman *et al.* 1997).

Rocks from both Mars and the Moon show strong remanent magnetism (see e.g. Stevenson, 2001; Runcorn, 1996). Both these bodies had ancient dynamos, which have now ceased to function. Both planets seem to have had fields of roughly similar strength to the Earth's current field. Venus has no current magnetic field, and the high temperature of its surface makes it unlikely that any remanent magnetism will survive.

The giant planets Jupiter and Saturn both have strong fields; Saturn's field is unusual for its high degree of axisymmetry; in table 1.1 we note that the inclination of Saturn's dipole field is less than one degree from its rotation axis. A possible theoretical explanation has been given by Stevenson (1982). Jupiter is the only planet apart from the Earth for which we have reliable information about the secular variation (Russell *et al.* 2001), that is the rate of change of the field. This is very important information, as it enables us to estimate the typical velocity in Jupiter's core, which is $\sim 2 \times 10^{-3}$ ms^{-1}. This velocity is five orders of magnitude less than the zonal flows in Jupiter's non-magnetic atmosphere, a striking demonstration of the importance of the magnetic field to the dynamics of convection.

The outer planets Uranus and Neptune have fields with exceptionally large inclinations between the magnetic dipole axis and the rotation axis. The fields also contain quadrupole and octupole modes of comparable value near the planetary surface. We argue below that these planets are probably in a different dynamical regime from that of other planetary dynamos.

1.3 Convective driving and thermal history

We shall suppose that all the known planetary dynamos are driven by convection, with the possible exception of the possible dynamo in Io where Kerswell & Malkus (1998) proposed tidal forcing as the driving. Stevenson (1983) describes a large number of alternative mechanisms proposed to account for planetary magnetism, but none can account for the large fields actually found.

The convection can be either thermally or compositionally driven. Compositional convection can occur when there is an inner core. The light elements mixed with iron in the outer core are released when iron is deposited on the inner core, and this impurity rich buoyant material can stir the core. The amount of buoyancy can be estimated in terms of the rate of growth of the inner core and the density jump between the outer and inner core. Thermal convection comes from various sources, the cooling of the planet, radioactivity in the core, and if there is an inner core, latent heat release at the ICB.

There are major uncertainties hampering the development of thermal history models both for the Earth and for other planets. The first is the uncertainty of the initial condition, that is how much heat the planets contained immediately after formation. The second uncertainty is how much radioactive material there is in the core. The third difficulty is that mantle convection helps determine the rate of cooling (more specifically the heat flux going through the CMB), and mantle convection is a very complex subject (see e.g. Schubert *et al.* 2001). To estimate the rate of cooling it is necessary to make some assumptions about the Nusselt number-Rayleigh number relation. The rheology of the mantle is uncertain; the viscosity is very temperature dependent, and the behaviour near the Earth's surface is dominated by solid subducting slabs, so it is not even clear that a fluid dynamic model is adequate.

In planetary cores, convection is efficient and the temperature gradient is close to adiabatic and the composition is close to uniform (outside of boundary layers). The adiabatic temperature gradient T_a is given by

$$T_a^{-1} \nabla T_a = \gamma \mathbf{g}_a / u_P^2, \qquad \text{where} \qquad \gamma = \alpha u_P^2 / c_p \tag{1.1}$$

is the Grüneisen parameter, for which estimates are available (Merkel *et al.*, 2000). Gravity and the sound speed u_P are given in the preliminary reference Earth model PREM (Dziewonski & Anderson, 1981). Here α is the coefficient of thermal expansion and c_p is the specific heat at constant pressure. When solving (1.1), the temperature at the CMB must be supplied, which can, in principle, be found from mantle convection studies. The pressure is found from the hydrostatic equation $\nabla P_a = \rho_a \mathbf{g}_a$, the density ρ_a being given by PREM. The liquidus temperature at which freezing occurs increases with pressure as we go deeper into the core at a rate faster than the adiabatic temperature increases, so the inner core forms first at the centre of the planet. In principle, when the temperature and pressure are known, the freezing point and hence the location of the inner core is determined. In practice, the location of the Earth's inner core is known from seismology. This is fortunate, because the freezing point of iron is significantly depressed by the impurities in the outer core, and the exact amount of the depression is hard to estimate; for other planets, where seismology is not yet available, we have to rely on theoretical estimates to determine where the inner core lies. For the Earth, it is generally believed that the CMB is at about 4,000 °K and (1.1) then gives the ICB temperature at about 5,100 °K.

When the temperature structure is known, the next step is to use high pressure physics estimates of the thermal conductivity (45 W m^{-1} °K^{-1} is a typical value) to find the heat flux conducted down the adiabat. This is comparable to the convected flux. If the Nusselt number Nu is defined as the ratio of conducted to convected flux, then $Nu \sim 1$, very different from solar convection. Conduction down the adiabat generates entropy at a rate $\Sigma \sim 170$ MW°K^{-1}.

The energy balance is

$$\mathcal{Q}_{\rm CMB} = \mathcal{Q}_{\rm ICB} + \mathcal{Q}_L + \mathcal{Q}_S + \mathcal{Q}_G + \mathcal{Q}_R, \tag{1.2}$$

which relates the heat flux $\mathcal{Q}_{\rm CMB}$ coming out of the CMB to the small amount (0.3 TW) coming out of the inner core, $\mathcal{Q}_{\rm ICB}$. $\mathcal{Q}_S$ is the rate of core cooling, known from the time evolution of (1.1), provided T_{CMB} can be found from mantle convection studies. $\mathcal{Q}_L$ is the latent heat released at the ICB and $\mathcal{Q}_G$ is the gravitational energy liberated by the central condensation as the inner core grows. These can both be estimated in terms of the rate of growth of the inner core, which in turn depends on how the temperature structure given by (1.1) evolves with time. $\mathcal{Q}_G$ involves the density jump at the ICB. The inner core density = 12730 Kg m^{-3} and the fluid outer core density = 12160 Kg m^{-3} and the difference is the density jump across the core. However, not all this jump releases useful energy for the dynamo. There are two parts contributing (i) due to release of light material, and (ii) due to contraction on solidification. The estimates of Roberts *et al.* (2002), with the age of the inner core taken as 1.2 Gyrs, suggest that the useful part (i) gives $\mathcal{Q}_G \sim 0.5$ TW. This age for the inner core is consistent with the Labrosse *et al.* (2001) value of 1 ± 0.5 Gyr. The cooling $\mathcal{Q}_S \sim 2.3$ TW, and the latent heat $\mathcal{Q}_L \sim 4.0$ TW. If $\mathcal{Q}_R$ the radioactive term, is zero then we have $\mathcal{Q}_{CMB} \sim 7.1$ TW. However, if there is radioactivity in the core, this estimate of the CMB heat flux could be a serious underestimate. The value of 7.1 TW for CMB heat flux apparently causes no great difficulty for mantle convection models, but the same is true for larger values, too.

The flux conducted down the adiabat near the CMB is around 6 TW using the above estimates, and because the latent heat is released at the ICB, the total heat flux exceeds the conducted flux everywhere, so there is convection throughout the core on this model. However, a rather small reduction in CMB heat flux would change this. If the CMB heat flux is less than 6 TW, the top of the core is subadiabatic. It would still convect through compositional convection, and would still be close to adiabatic, but one would expect convection there to be much less vigorous; this thermally stable layer is Braginsky's 'inverted ocean', Braginsky (1993).

Energy balance does not allow us to investigate the amount of dissipation $\mathcal{Q}_D$, since the work done by buoyancy cancels the dissipation. We need to consider the entropy balance; we follow the discussion of Roberts *et al.* (2002).

$$\mathcal{Q}_D = \frac{T_D}{T_{\rm CMB}}[(\mathcal{Q}_{\rm ICB}+\mathcal{Q}_L)(1-\frac{T_{\rm CMB}}{T_{\rm ICB}})+(\mathcal{Q}_S+\mathcal{Q}_R)(1-\frac{T_{\rm CMB}}{\overline{T}})+\mathcal{Q}_G-\Sigma T_{\rm CMB}]. \tag{1.3}$$

recalling that Σ is the entropy production due to conduction down the adiabat. $\mathcal{Q}^D$ is almost entirely ohmic dissipation, viscous dissipation being orders of magnitude smaller in the core. T_D is the mean temperature at which the dissipation occurs (effectively where the dynamo operates most strongly), which clearly lies between T_{CMB} and T_{ICB}. $\overline{T}$ is the mean temperature of the outer core. Putting in numerical estimates (Roberts *et al.*, 2002) gives

$$\mathcal{Q}^D = \frac{T_D}{T_{\rm CMB}} \left(0.5{\rm TW} + \mathcal{Q}_G + 0.12\mathcal{Q}^R \right), \tag{1.4}$$

indicating that thermal and compositional convection both contribute roughly 0.5 TW, giving a total of around 1 TW to drive the dynamo. We are therefore aiming at finding a dynamo with about 1 TW of ohmic dissipation. This is consistent with the output of current dynamo models. There is still some uncertainty in the above heat flux estimates; for example Lister & Buffett (1995) estimated the conducted flux at the CMB as only 2.7 TW.

Early Earth

There is, however, a serious problem with all the above theory (Roberts *et al.* 2002). While it can explain the current geodynamo, what was happening before the inner core formed ~ 1.2 Gyr ago? According to the paleomagnetic evidence, the magnetic field dates back to at least 3.5 Gyr. It was suggested (Hale, 1987) that the field strength intensified 2.7 Gyr ago, possibly corresponding to the formation of the inner core. Before inner core formation, the latent heat and the gravitational energy sources are not available, only cooling. The power available for the geodynamo is then much reduced. Even more seriously, most, if not all, of the core would be stably stratified with the above estimate of the cooling term. It is not clear how a dynamo could be sustained under these circumstances.

If we assume the Earth was formed from material with solar abundances, there is a significant depletion of radioactive potassium, ^{40}K, in the mantle. This could either have been lost to space during the Earth's formation, which is the view favoured by most geochemists, or it could be trapped in the core. If it has been trapped in the core, then the CMB heat flux would be much greater, possibly even as much as 20 TW (Roberts *et al.* 2002), removing the difficulty with the early dynamo. Including radioactivity in the core also has the effect of altering the time at which the inner core formed, generally increasing the age of the inner core (Labrosse *et al.* 2001). Another possibility is that the primordial heat at formation was very large, and so the rate of cooling, $\mathcal{Q}_S$, is much larger than our 2.3 TW estimate, especially during the time before the inner core formed.

1.4 Physical nature of convective dynamo solutions

A number of research groups have produced three-dimensional numerical solutions of the geodynamo equations, and these have been recently reviewed by Dormy *et al.* (2001), Jones (2000) and Busse (2000). We shall therefore focus on a few particular issues here.

The geodynamo equations are usually solved in the Boussinesq approximation, which are given in e.g. Jones (2000), although Glatzmaier & Roberts have also solved the anelastic equations, which allow for variations in the properties of the Earth's core (see e.g. Glatzmaier & Roberts, 1997). To avoid complications in what is already a formidable set of equations, only one source of convection (either thermal or compositional) is usually assumed.

The dimensionless parameters that occur in the equations are the Ekman number $E = \nu/2\Omega d^2$ (d is the core radius), the Roberts number $q = \kappa/\eta$, the Rayleigh number Ra and the Prandtl number $Pr = \nu/\kappa$. Here η is the magnetic diffusivity, ν is the kinematic viscosity, and κ is the thermal diffusivity.

In the inner core, the magnetic diffusion equation is solved, and appropriate continuity conditions are applied across the ICB (see e.g. Jones *et al.* 1995). For the mechanical boundary conditions at the ICB, Glatzmaier & Roberts (1996) used no-slip, while Kuang & Bloxham (1997) used stress-free, arguing that since viscosity is artificially enhanced in the models (see below), stress-free represents the physical situation better. The different assumptions for this boundary condition appear to make a significant difference to the nature of the solutions, but the detailed reasons for this are not yet apparent.

The main problem with geodynamo solutions is that it is not possible to solve the equations in the desired parameter regime. The molecular diffusion coefficients $\kappa \sim 2 \times 10^{-5}$ m^{2}s^{-1} and $\nu \sim 10^{-6}$ m^{2}s^{-1} lead to very small values of E and q which are numerically impossible to achieve. Even if it is argued that turbulent values of these diffusion coefficients are more appropriate, and then the question of whether isotropic or anisotropic diffusion is appropriate arises, (Braginsky & Meytlis, 1990), $E \sim 10^{-9}$ which is still too small to deal with numerically.

In figure 1.1 (Sarson, 2000), we show a schematic diagram indicating which parts of the parameter space have been explored (see also Busse, 2000). It is not possible to reduce the Ekman number much below 10^{-4} in spherical codes (for plane layer codes we can do better, see below). Hyperdiffusivity (see e.g. Zhang & Jones (1997) for an explanation of what this involves), which enhances diffusion in the latitudinal and azimuthal directions, but

Fig. 1.1.
The different regimes of parameter space explored by numerical models. Dynamo action is only possible with high R_m convection. BZ and GR locate typical solutions of the 'Busse-Zhang' and 'Glatzmaier-Roberts' type.

not the radial direction, has to be used to explore the low E regime, and this introduces further uncertainties. At, for example, $E \sim 10^{-3}$, dynamos are found at mildly supercritical Rayleigh number provided $q >> 1$, the 'Busse-Zhang' regime. If $q \sim 1$ the magnetic Reynolds number is too small to give dynamo action. To correct this, we must increase the Rayleigh number, in order to increase the flow velocity. In principle, this should allow us to achieve dynamo action at lower q, but in practice raising Ra at fixed E makes the flow more chaotic and small scale, and no large scale dipole field results. We need to lower E as well as raise Ra in order to keep the flow sufficiently coherent to generate a dipole dominated field. This is the 'Glatzmaier-Roberts' regime (see also Kuang & Bloxham, 1997), but as noted above it can only be found by introducing hyperdiffusion, with its concomitant uncertainties.

There is, therefore, much less freedom to choose the parameters E and q than one would like. We still have to decide on what values of Pr and Ra to choose. Pr only affects the inertial term. For behaviour on a timescale of tens of years and greater, the inertial term is rather small, and its neglect can be formally justified by letting Pr be large. However, on molecular values at least, Pr is *small* not large. If the dynamo is in the correct low E regime, the inertial term will become less important as E is reduced and so the solutions will become *independent* of Prandtl number. Since dynamo codes are not yet run in the low E regime, it is not surprising that authors report significant Prandtl number dependence in their results. Dormy *et al.* (2001) note that since dynamo codes are not run in the correct regime, great care must be taken in interpreting the results, and in how the dimensionless variables are to be translated back into physical variables. Finally, how is the Rayleigh number to be chosen? This measures the ratio of the superadiabaticity to the diffusion coefficients, and there is no direct method of determining this. Instead, we choose the Rayleigh number so that the heat flux gives the correct value. We show below that this criterion gives us the the typical velocity of the flow, and this is similar to that of the 'westward drift' velocity.

Ohmic Dissipation

Since essentially all the magnetic energy generated by the dynamo ends up as ohmic dissipation, we can test whether our dynamo solutions have a total dissipation comparable with the available energy estimates given in the previous section.

Gubbins (1977) showed that if the field inside the core minimises the dissipation subject to the constraint that the field at the CMB is the observed field, this minimum dissipation is

$$\mathcal{Q}_{min} = \sum_{n=1}^{\infty} q_n, \qquad q_n = \frac{\eta r_{cmb}}{\mu} \frac{(2n+1)(2n+3)}{n} R_n, \tag{1.5}$$

where q_n is the dissipation from the spherical harmonics of order n, and

$$R_n = \Big(\frac{r_{earth}}{r_{cmb}}\Big)^{2n+4} (n+1) \sum_{m=0}^{n} \Big[(g_n^m)^2 + (h_n^m)^2\Big] \tag{1.6}$$

is the Mauersberger-Lowes spectrum extrapolated to the core surface (see e.g., Langel 1987), and g_n^m and h_n^m are the usual Gauss coefficients (see e.g. Backus *et al.* 1996). The Mauersberger-Lowes spectrum R_n at the CMB is well-approximated for $n \geq 3$ by

$$R_n = 1.51 \times 10^{-8} \exp(-0.1n) \text{ T}^2, \tag{1.7}$$

which leads to $\mathcal{Q}_{min} \sim 44$ MW, with the peak dissipation occurring at around $n \sim 12$. This value of n is coincidentally at about the limit of what can be observed, as higher harmonics are obscured by crustal magnetism.

Dynamo models suggest that the actual dissipation $\mathcal{Q}_D >> \mathcal{Q}_{min}$, so 44 MW is a gross underestimate. The dynamo is very inefficient, in the sense that the actual dissipation is orders of magnitude greater than the minimum necessary dissipation. The reasons are (i) most of the flux generated in dynamo models never leaves the core. The toroidal field generated is necessarily trapped in the core, but models show that only a small fraction of the poloidal field leaves the core to form the observed potential field. (ii) although the field escaping from the core is mostly dipolar in the models, the internal field has a much more complex structure than the very simple structure of the minimising field. So not only is there far more field in the core than is strictly necessary to generate the observed dipole field, its structure is also rather complex.

The upshot is that dynamo models do suggest that the dissipation is of the order of 1 TW, in agreement with the estimates of section 3. Any energy source which falls significantly short of this figure is insufficient. However, it is not yet possible to make very reliable estimates of the dissipation with the current generation of dynamo models, because the dissipation occurs mainly in the range $n \sim 10 - 40$ and this range is affected by hyperdiffusivity. An interesting theoretical question is what is the nature of the power spectrum in dynamo models. Formula (1.7) is empirical, and indeed power law spectra fit just as well. However, a recent simulation (Roberts & Glatzmaier, 2000) was fitted well by the formula

$$R_n = 1.51 \times 10^{-8} \exp(-0.055n) \text{ T}^2 \tag{1.8}$$

suggesting that an exponential law may have some as yet unknown theoretical basis. The form of this power spectrum has important implications for dynamo theory, because it connects directly with an outstanding problem at the heart of geodynamo theory, the problem of finding an adequate energy source.

1.5 Dynamical regimes in planetary cores

Rotvig & Jones (2002) and Jones & Roberts (2000) have considered plane layer models to gain some understanding of the low E dynamical regime. In this geometry we can no longer compare results with geomagnetic studies, but there are significant computational advantages in Cartesian geometry (the non-existence of useful fast Legendre transforms is the root of the problem for spectral spherical codes). We can get E small enough to get into the correct dynamical regime, where the basic balance of terms is correct. This is signalled by the magnetic field satisfying Taylor's (1963) constraint. When this is achieved, the terms in the equation of motion are in MAC balance (Braginsky, 1967), that is viscous forces and inertial acceleration are negligible, while pressure force, Lorentz force and buoyancy force are all comparable with the Coriolis acceleration.

We therefore have (Starchenko & Jones, 2002)

$$2|\mathbf{\Omega} \times \mathbf{u}| \sim |\nabla p|/\rho \sim |\mathbf{j} \times \mathbf{B}|/\rho \sim g(\alpha T_a S/c_p + \alpha_\xi \xi) \tag{1.9}$$

where S is the entropy fluctuation, ξ is the composition fluctuation, ρ is the density and α_ξ is the compositional expansion coefficient, with $\alpha_\xi \approx 0.6$ being typical for terrestrial cores.

This is a completely different balance from that in the solar convection zone, where mixing length balance occurs,

$$|\mathbf{u} \cdot \nabla \mathbf{u}| \sim U_*^2/\ell \sim g\alpha T_a S/c_p, \tag{1.10}$$

ℓ being the mixing length and U_* being a typical velocity. In non-magnetic planetary atmospheres a geostrophic balance is common,

$$2\rho\mathbf{\Omega} \times \mathbf{u} \sim -\nabla p \tag{1.11}$$

with either the thermal and viscous terms being much smaller. In laboratory convection the motion is on short length scales (tall thin rolls) so that viscous forces can be significant through the particular geometry of the motion.

Taking S_* as a typical entropy fluctuation, and ignoring compositional terms as appropriate for Jupiter,

$$2\Omega U_* \sim g\alpha T_a S_*/c_p. \tag{1.12}$$

The heat flux equation gives

$$F \sim \rho T_a U_* S_* \sim \frac{\mathcal{Q}}{4\pi r_{cmb}^2}. \tag{1.13}$$

Eliminating the entropy fluctuation S_*,

$$U_* \sim \left[\frac{g\alpha r_{cmb}\mathcal{Q}}{M\Omega c_p}\right]^{\frac{1}{2}} \tag{1.14}$$

Putting in the standard estimates for thermodynamic quantities, we obtain for Jupiter, $U_* \sim 2 \times 10^{-3}$ ms^{-1}. For the Earth, we can form the mass flux equation analogous to the heat flux equation, and we obtain (see Starchenko & Jones, 2002 for details) $U_* \sim 2 \times 10^{-4}$ ms^{-1}

These estimates are in good agreement with velocities inferred from measurements of the secular variation (Bloxham & Jackson, 1991) for the Earth, Russell *et al.* (2001) for Jupiter. This agreement provides useful evidence that MAC balance does operate in the cores of these two planets. For example, if the mixing length balance (1.10) is used in place (1.9), the typical velocity is orders of magnitude too large.

The typical magnetic field can be estimated from

$$2\rho\Omega U_* \sim |\mathbf{j} \times \mathbf{B}| \sim B_*^2/\mu r_* \tag{1.15}$$

where r_* is lengthscale of the variation of the field, $|\mathbf{B}|/|\nabla \times \mathbf{B}|$. Eliminating U_*,

$$B_* \sim \Big[\frac{g\alpha\rho_*^2\mu^2 r_*^2 r_{cmb} Q\Omega}{Mc_p}\Big]^{\frac{1}{4}} \tag{1.16}$$

How do we choose r_*? Unfortunately, this is not at all clear. Studies of flux ropes (Galloway, Proctor & Weiss, 1978) suggest $r_* \sim R_m^{-1/2}$. Numerical simulations at $R_m \sim O(10^2)$ suggest $r_* \sim d/50$, where $d = r_{cmb} - r_{icb}$. Magnetic field saturates when the stretching properties of the flow are inhibited by Lorentz force. Dynamic simulations suggest that flux ropes thicken in the fully nonlinear regime: it might therefore be that r_* eventually becomes independent of R_m at large R_m. For the Earth, it is reasonable to take $r_* \sim d/50$ as suggested by the simulations with R_m close to its terrestrial value, and we obtain $B_* \sim 0.5 \times 10^{-2}$ T, about ten times the the dipole field extrapolated to the CMB (table 1.1), a reasonable value consistent with numerical models, which indicate that about 10% of the core field escapes through the CMB.

Interestingly, if we use $r_* \sim d/50$ for Jupiter, we get a core field of $B_* \sim 2 \times 10^{-2}$ T which is reasonable if about 10% of the core field escapes to form the observed dipole. If we assume that $r_* \sim R_m^{-1/2}$, this gives a much smaller value of B_* (far smaller than the observed field), because Jupiter has a high magnetic Reynolds number.

For moderate R_m, geodynamo models typically give for the Elsasser number Λ

$$\Lambda = \frac{B_*^2}{2\mu\rho\Omega\eta} \sim 4 \Rightarrow r_* = 4d{R_m}^{-1} \sim d/50, \tag{1.17}$$

with a typically Earth-like value of $R_m \sim 200$. For Jupiter's metallic hydrogen core, we expect $R_m \sim 10^5$, and $\Lambda \sim 2 \times 10^3$. A possible problem with this is that magnetic instabilities occur when Λ is this big (e.g. Zhang, 1995). Since it is not currently possible to increase R_m in dynamo codes very significantly, we do not know whether such large values of Λ can ever be attained. A potential way out of this difficulty would be to locate Jupiter's dynamo not deep inside the conducting core, but at the interface of the metallic hydrogen core and the molecular atmosphere. Since it is likely that the electrical conductivity goes smoothly to zero with distance from the centre (Kirk & Stevenson, 1987), there must be a zone where the Elsasser number and the magnetic Reynolds number assume moderate values, and this may be a promising location for the Jovian dynamo.

Saturn may be driven by compositional as well as thermal convection, Stevenson (1982), and the uncertainty in the core energy fluxes means that typical velocities and field strengths are also uncertain. We can only compare with the field strength, and this suggests that Saturn, like Jupiter, is probably in MAC balance. A similar uncertainty holds for Ganymede; as mentioned above, the thermal history, and consequently the current core heat flux, is unknown.

Mercury's small size makes it likely that inner core nucleation started early, so that by now a large solid inner core probably exists (Stevenson *et al.* 1983). The sulphur (and other impurities) present depress the freezing point, and as the inner core grows, the relative fraction of impurity rises in the fluid left, so it is difficult to freeze the core entirely. The fluid outer core is therefore probably only a thin shell; the thickness of this shell depends on the (unkown) initial sulphur concentration, but values of ~ 100 km to ~ 500 km are plausible. The thermal stratification in the liquid iron core is likely to be stable (Stevenson *et al.* 1983) but gravitational energy $\mathcal{Q}_G \sim 6\times10^9(d/100)^2$ W, where d is the outer core thickness in km, is available from compositional convection (Stevenson, 1987). The rotation rate of Mercury is slow, and the core magnetic field is much weaker than that in other planets. These facts may well be related. If we take the shell thickness $d = r_{cmb} - r_{icb} \sim 100$ km, we obtain from MAC balance $U \sim 3 \times 10^{-3}$ ms^{-1} which implies a magnetic Reynolds number (based on length d) of over 100. To obtain the observed field strength, however, we need a rather small $r_* < 1$ km. It may be that Mercury's field has a relatively smaller dipole component than the Earth, so the core field may be more than ten times the escaping observed field. This could come about because the slower rotation and relatively stronger driving may impose less order on the flow (see figure 1.1).

The planets Uranus and Neptune present a different problem. The heat flux coming out of Uranus is about 3×10^{14} W and for Neptune 3×10^{15} W. If we assume MAC balance, we have a typical velocity of 2×10^{-4}

ms^{-1} for Uranus, and about 3 times larger for Neptune. With the large diffusivity $\eta \sim 10^2$ m^{2}s^{-1}, we have for Uranus a magnetic Reynolds number of only about 40, which is probably too small to sustain dynamo action; Neptune is marginal. MAC balance is therefore probably not possible in these planets. Holme & Bloxham (1996) also point out that the ohmic dissipation associated with the observed fields would be larger than the total heat flux if the core field is significantly larger than the observed field.

The absence of a dynamo in Venus is also of interest. Even with its slow rotation rate, Coriolis acceleration should still be important in the core, and dynamo action should result. Stevenson *et al.* (1983) suggested that slower cooling due to the high surface temperature meant that Venus' inner core had not yet formed, depriving it of much of its energy source. However, the very slow rotation must be important, because the Earth apparently maintained a dynamo before its inner core formed.

1.6 Conclusions

Although numerical convection driven geodynamo models have not yet reached the parameter regime relevant to the Earth, they are probably now not far off entering the MAC balance regime. The analysis of section 5 suggests that provided the heat flux, the rotation rate and the thermodynamic quantities are specified correctly, the typical velocity, and hence the magnetic Reynolds number, will then be predicted correctly. The magnetic field strength is harder to get right, because magnetic saturation is not so well understood, and is probably model dependent. A possible way forward is to try to improve our understanding of the magnetic energy spectrum and how it relates to ohmic dissipation.

The problem of planetary magnetic fields is closely linked to the thermal history of the planets, because this determines the amount of available thermal and gravitational energy. It also determines the point at which the core becomes stably stratified, which is most likely why Mars and the Moon have ceased to be dynamos. A serious problem is posed by recent thermal history calculations, which indicate that the solid inner core is only around 1.2 Gyr old. Current models of the Earth's thermal history do not seem to provide enough heat to drive the dynamo before this time, despite evidence for a geomagnetic field dating back 3.5 Gyr. Either more primordial heat is required, or significant radioactivity was present in the core 1.2 Gyr ago.

The uncertainities about the thermal history of the planets are even greater, and so it is not realistic to hope for a theory that can predict planetary magnetic fields solely from the physical data. Instead, we must try to piece together the thermal history of the planets using the data provided by magnetic field observations and our gradually improving understanding of dynamo theory. Our ability to build dynamo models is now getting to the point where applying the knowledge obtained from geodynamo studies to other planets, and perhaps even to extra-solar planets, could be fruitful.

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