The Project

Approximate symmetries and almost integrable equations.

Many natural systems can be modelled or described by *nonlinear* partial differential equations (PDEs), integro-differential equations, differential-difference equations or just fully discrete equations. It is very difficult, if possible at all to find a solution of nonlinear equations, except using a computer. Rather surprisingly, sizable classes of nonlinear systems are found to have an extra property, integrability, which changes the picture completely. For integrable equations there is a well developed theory which enables to find multi-parametric exact solutions, to study the general Cauchy problem, asymptotic behaviour of solutions, to find infinite hierarchies of symmetries and conservation laws. There is a deep reason why integrable equations often have fundamental and universal meaning (see, for example, F.Calogero "Why are certain nonlinear PDEs both widely applicable and integrable?", in *What is Integrability*, ed V.E.Zakharov, Springer–Verlag, Berlin Heidelberg, 1991).

During three decades of the development the theory of integrable systems proved to be very influential and seminal. Such concepts as solitons, instantons, quantum groups, exactly integrable numerical schemes, etc. take there origin in this theory. Methods from the theory of integrable systems give a new insight to classical mathematics and provide new tools to approach difficult and yet unsolved problems. Although integrable equations are important in their own right, they provide a unique possibility to investigate properties of "nearby" almost–integrable or even non-integrable systems. The concept of almost integrable systems is not well studied and presented in the literature and due to be elaborated.There is a strong feeling that almost–integrable equations are even more fundamental and important for applications than the integrable ones.

One of the basic properties of integrable equations is the existence of hidden symmetries. In the project we propose to study equations which possess hidden approximate symmetries or approximate conservation laws. Nothing is known about the behaviour of the solutions of almost integrable equations. Symmetry reductions lead to classes of exact solutions of differential equations, it is very probable that the corresponding reductions due to approximate symmetries will give important classes of approximate solutions. The subject is very new and promising, it could be a good start towards the next degree.