Completely Integrable Hamiltonian Systems*

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William Rowan Hamilton[1] (1806-65) was a brilliant mathematician and 'natural scientist', who was made professor of Mathematics **before** receiving his first degree, and became Astronomer Royal of Ireland at the age of 21. In 1834 he published his theory of dynamics.

Hamiltonian dynamics is a generalisation of Lagrangian dynamics. It is not only the most mathematically sophisticated and beautiful form of mechanics, but also the stepping stone between classical and quantum mechanics. Hamiltonian dynamics is concerned with non-dissipative (conservative) systems, which occur in most fundamental branches of physics. One of the great advantages over Lagrangian dynamics (already having superseded Newtonian dynamics) is in transformation theory. Hamilton's equations have a much larger set of symmetries than Lagrange's equations. This property can be exploited in order to solve a wider class of physically important systems, which is the essence of canonical transformation theory and the Hamilton-Jacobi method.

In recent years, Hamiltonian dynamics has received a tremendous boost from modern nonlinear dynamics, such as *Hamiltonian chaos* and (at the other end of the spectrum) **integrable systems theory**. This project concentrates on the latter aspect. It is a very large subject, so there is plenty of scope for the student moulding the project to suit his/her personal interests and abilities. After learning the basics, the student could concentrate on one or more of the following topics

1. Hamilton-Jacobi theory and the separation of variables;
2. Lax matrices and the modern theory of integrable systems;
3. Bäcklund transformations as canonical maps;
4. Canonical transformations and integrable maps.

There are many books on this subject, with a large range of difficulty. I list a few of these below.

References

[1] V.I. Arnol'd. *Mathematical Methods of Classical Mechanics*. Springer-Verlag, Berlin, 1978.

[2] A.P. Fordy. Integrable symplectic maps. In Peter A. Clarkson and Frank W. Nijhoff, editors, *Symmetries and Integrability of Difference Equations*, pages 43–55. LMS Lecture Notes 255, CUP, 1999.

[3] J.V. José and E.J. Saletan. *Classical Dynamics. A Contemporary Approach.* CUP, Cambridge, 1998.

[4] L.D. Landau and E.M. Lifshitz. *Course of Theoretical Physics Volume 1: Mechanics*. Pergamon, Oxford, 1976.

*Assignment in Applied Mathematics

[1] see http://www-groups.dcs.st-and.ac.uk/ history/Biographies/Hamilton.html