Relaxation Shock Structures

Suject Area: Hyperbolic Systems

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Many physical systems, such as compressible flow, shallow water waves, magnetohydrodynamics, traffic flow, creep waves in groundwater flow etc, are described by non-linear hyperbolic systems of equations. Such equations have the property that their solutions tend to become discontinuous even if the initial data is smooth. Although it is possible to determine the behaviour of these discontinuities by an appeal to conservation laws and causality, in reality the solutions are never truly discontinuous, but are always smoothed out by some physical process that was not included in the original equations.

The actual mechanism that prevents the solutions from being discontinuous depends upon the particular physical system, e.g. in fluid dynamics it is generally viscosity, whereas in traffic flow it is the finite size of cars. Despite this diversity, these mechanisms can be classified mathematically into two types: those, such as viscosity, that depend upon gradients and those that involve a relaxation process represented by a source term. The important difference between these is that, whereas a process that depends upon gradients always leads to a smooth solution, a relaxation process is not always able to prevent the solution from becoming discontinuous.

The purpose of this project is to look at relaxation processes in various physical systems and the theory that determines the conditions for the existence, or otherwise, of continuous solutions. Although much of this can be found in textbooks, there are a number of applications which require an extension of the standard theory. The work is analytical rather than numerical, but familiarity with Maple or Mathematica would be an advantage.