Nonaxisymmetric instabilities of convection around magnetic flux tubes

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ABSTRACT

On the surface of the Sun, magnetic flux elements collect in regions of converging flow, grow in field strength and become pores, which have been observed to exhibit nonaxisymmetric structure over a range of scales. Around a fully developed sunspot, as well as the fine scale of the penumbra, the moat sometimes shows clearly observable spoke-like structure at low azimuthal wavenumbers. We investigate the formation of azimuthal structure by computing the linear stability properties of fully nonlinear axisymmetric magnetoconvection, which takes the form of a central flux tube surrounded by a convecting field-free region. We find steady and oscillatory instabilities with a preferred azimuthal wavenumber. The unstable modes are concentrated in the convecting region close to the outer edge of the flux tube. The instability is driven by convection, and is not a magnetic fluting instability.

Subject headings: convection — MHD — Sun: magnetic fields — sunspot

1. Introduction

Pores and sunspots are prominent magnetic features on the solar surface. Many of these objects are roughly circular and appear to be axisymmetric near their centres. However, the outer edges of these structures often have pronounced nonaxisymmetric features, the most obvious being the penumbrae of sunspots. A wide range of azimuthal structures have been observed: around pores there are nonaxisymmetric downflows, hair-like radial striations and needle-like features, as well as proto-penumbral structures, while sunspots have prominent penumbrae as well as moats with spoke-like lanes of converging flow.

As pores form from inflowing magnetic flux elements on the Sun's surface, they have patches of strong downflows around the flux concentrations (Hirzberger 2003; Rimmele 2004; Stangl & Hirzberger 2005). Around the edge of pores hair-like striations have been observed with an azimuthal wavelength smaller than the surrounding granular convection (Scharmer et al. 2002; Berger et al. 2004). These striations are believed to be magnetoconvective downflow lanes. Needle-like structures have been observed surrounding pores with an internal flow towards the pore and a downflow at the end near the flux concentration (Sankarasubramanian & Rimmele 2003). A pore growing in size due to accumulated flux may evolve a rudimentary penumbral structure. This proto-penumbra is transitory in nature and may oscillate between penumbral-like filaments and elongated granules (Dorotovič et al. 2002), decay (Sobotka et al. 1999) or evolve into a fully developed sunspot penumbra (Keppens & Martínez Pillet 1996). The formation of a fully formed penumbra around sunspots is usually abrupt, with a sudden change of the magnetic field direction from vertical to inclined (Rucklidge et al. 1995; Yang et al. 2003). The sunspot penumbra is highly structured with a small azimuthal wavelength (Scharmer et al. 2002; Sobotka 2003; Thomas & Weiss 2004). The moat flow surrounding sunspots sometimes exhibits azimuthal structure, with spoke-like lanes of converging flow which have a higher average concentration of outwardly moving magnetic features (Shine & Title 2001; Hagenaar & Shine 2005).

Two types of instabilities have been proposed as mechanisms for the formation of azimuthal structure in an axisymmetric system: magnetic interchange or fluting instabilities (Parker 1975) driven by buoyancy and the Lorentz force, and convective instabilities (Spruit 1981) driven only by buoyancy. The first is a hydromagnetic instability of curved magnetic fields in a stratified atmosphere: the instability is driven by tension along the curved field lines, and opposed by a stable density stratification. The second is an instability of an axisymmetric flow configuration that prefers to develop cell-like structure; this instability does not necessarily require compressible effects or a magnetic field.

Here we present an investigation into the stability of nonlinear axisymmetric compressible magnetoconvection, focusing on instabilities that generate azimuthal structures around magnetic flux tubes. We find that a convective mechanism can occur in a steady or oscillatory fashion, producing nonaxisymmetric flows that resemble those found in the convecting regions around pores and, on a supergranular scale, in the moat around sunspots.

2. Model

The numerical grid is a three-dimensional (3D) cylindrical wedge (r, ϕ, z) with depth one unit and radius Γ. The azimuthal number $M_\phi = 2\pi/\phi_{max}$ quantifies its width, with the constraint $M_\phi \geq 4$ built into the model. On this grid the nonlinear equations for fully compressible, 3D magnetohydrodynamic (MHD) convection (Hurlburt et al. 2002) are solved, together with

$$\frac{\partial \psi}{\partial t} = -c_h^2 \nabla \cdot \mathbf{B} - \frac{c_h^2}{c_p^2}\psi, \tag{1}$$

where the variable ψ was introduced to enforce $\nabla \cdot \mathbf{B} = 0$ (Dedner et al. 2002) and couples with the MHD equations through an extra term in the induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} - \nabla \psi. \tag{2}$$

The constants c_h and c_p are set to optimize the divergence clearance. All the other symbols have their usual meaning.

Both the top and bottom boundary conditions on the domain set the magnetic field to be vertical and treat the temperature as fixed. The outside wall is slippery, perfectly conducting and thermally insulating, while regularity conditions apply to the central axis (Botha et al. 2006). The numerical domain is periodic in the azimuthal direction. In the numerical simulations we use a fourth-order Bulirsch–Stoer type time integration, with sixth-order compact finite differencing in the (r, z) plane (Hurlburt & Rucklidge 2000) and a spectral treatment azimuthally. The level of dealiasing is increased towards the central axis to maintain grid uniformity.

In order to explore whether any observed instabilities are hydrodynamic or magnetic in origin, we also solved the PDEs with the evolving magnetic field being replaced by its azimuthal average at each time step. In this way, the Lorentz force in the Navier-Stokes equation remains axisymmetric, so that the basic state does not change and magnetic effects are removed from the nonaxisymmetric mode. Given that the basic state is a central magnetic flux tube surrounded by field-free convection cells (Section 3), this nonaxisymmetric non-magnetic solution is effectively confined to an annulus, with the inner boundary the shape of the edge of the flux tube.

3. Numerical results

We obtained results with two sets of parameter values, one close to the onset of convection, and the second with more vigorous convection. The parameters in common are: $Q = 100$, $\sigma = 1.0$, $\zeta_0 = 0.2$, $\theta = 10$, $\gamma = 5/3$, and $\Gamma = 3$. (See Botha et al. (2006) for the meaning of the symbols.) The first parameter set also has Rayleigh number $R = 10^4$ and polytropic index $m = 1$, and the second has $R = 10^5$ and $m = 1.495$.

The initial state is a time independent axisymmetric solution (Hurlburt & Rucklidge 2000). It has a well defined flux tube at the centre of the cylinder and a clear separation between magnetic field and field-free convection. Figure 1 presents this state for the first parameter set: the central flux tube has radius $r = 1.42$, surrounded by two convection cells. The inner cell is orientated so that it forms a collar flow around the tube (Botha et al. 2006). The initial state for the second parameter set is similar, with a narrower flux tube (its edge at $r = 1.1$) and higher convective velocities. The initial state is perturbed with a single wave in the azimuthal direction. By changing the width of the wedge (ϕ_{max}), the stability of modes with different azimuthal wavenumbers (M_ϕ) can be investigated. The nonaxisymmetric component of the vertical velocity, integrated over the domain, is used to determine the linear growth rate for each mode.

Figure 2 shows the linear growth rates obtained with the full PDEs and with the PDEs with azimuthally averaged magnetic field. In both cases the largest growth rate is with the lowest value of M_ϕ, with decreasing growth rates as M_ϕ increases. Eventually the wedge becomes thin enough for the nonaxisymmetric modes to be damped. All the modes are steady growing or decaying modes. The eigenfunctions of the growing mode at $M_\phi = 4$ is given in Figure 3 for both the full and averaged calculations. The eigenfunctions are located primarily outside the magnetic flux tube, in the convecting region. These modes are convective modes, with the nonaxisymmetric part of the magnetic field suppressing both the growth rate (Figure 2) and the extent to which the mode penetrates the magnetic flux tube (Figure 3). This result is reminiscent of the nonaxisymmetric instability of annular convection rolls in a Boussinesq fluid, which has the largest growth rates for low values of M_ϕ (Jones & Moore 1979).

For the second set of parameter values, there were two types of linear instability: for $4 \leq M_\phi \leq 6$ we obtain steady growth, and for $M_\phi \geq 8$ oscillatory growth (Figure 4). The period of the oscillating modes depends on M_ϕ: the frequency decreases for smaller M_ϕ, and may go to zero at the transition from oscillatory to steady instability. The eigenfunctions of the steady modes look similar to those obtained for parameters $R = 10^4$ and $m = 1$ (Figure 3). An example of an oscillatory eigenfunction ($M_\phi = 12$) is presented in Figure 5, with approximately half a period between the two contour plots. Again the eigenfunction is situated in the convecting part of the domain, with maxima just outside to the magnetic flux tube as well as between the two convection cells. The zero contour line moves as the simulation evolves, which indicates that there are more than one mode present in the solution. When compared with the solution obtained with the azimuthal averages, it is clear that the nonaxisymmetric oscillations do not rely on the presence of a magnetic field. As such, they correspond to the oscillatory instability found in Boussinesq (Busse 1972) and compressible fluids (Rucklidge et al. 2000). Figure 4 shows that the absence of the magnetic field increases the growth rates of the modes; the oscillation period is also larger.

4. Discussion

We solved the nonlinear PDEs for compressible 3D magnetoconvection in a cylindrical wedge, investigating the linear stability of a nonlinear axisymmetric solution that has a well defined magnetic flux tube at the center of the cylinder, surrounded by two convection cells. The nonaxisymmetric instability can be either steady or oscillatory, depending on the parameter regime chosen, with the wavelength of optimal growth shorter for oscillating growing modes. The eigenfunctions of the instability show that it is located just outside the magnetic flux tube.

We investigated the role of the magnetic field in the instability by taking the azimuthal average of the magnetic field after each time-step. In this way the nonaxisymmetric perturbation becomes hydrodynamic, while at the same time the shape of the magnetic flux tube is preserved. This procedure shows that the instability is primarily an instability of the convection (rather than a fluting instability), and that the magnetic field is not responsible for the oscillations, although the frequency of the oscillations is enhanced by the presence of the magnetic field. In all cases (steady and oscillatory) the magnetic field inhibits the growth of the nonaxisymmetric instabilities. In contrast, a fluting instability would require more highly curved magnetic fields and different magnetic boundary conditions.

Our results are not directly relevant to the formation of penumbral structure at the edge of magnetic pores, as described by Tildesley & Weiss (2004). However, the convective nature and the location of the nonaxisymmetric instability indicate that our results are relevant to the azimuthal structure observed in the convection around pores (Sankarasubramanian & Rimmele 2003) and in the moat (Hagenaar & Shine 2005). In both cases there is a preferred azimuthal wavenumber, comparable to the scales of granulation and supergranulation respectively. Taken with the fact that oscillations have been observed around pores, it seems likely that the azimuthal structure around pores may be modeled by the higher Rayleigh number (R) calculations, while the steady moat structures would fit the lower R results. The inflowing collar surrounding the magnetic flux tube in our model can naturally be compared to the flow around magnetic pores on the Sun's surface. The flow in the moat cell is radially outward, but we would expect a similar convective instability to be found in this case.

A previous related study in Cartesian geometry investigated the linear stability of nonlinear two-dimensional Boussinesq magnetoconvection (Tildesley 2003). The initial configuration was a magnetic structure concentrated on one side of the domain, with an effective outflow at the top (opposite to our configuration), and stability to 3D perturbations was computed. All growth rates obtained were real and the wavenumbers of maximum growth rate were similar to those we observe for oscillatory modes. These wavelengths of optimal growth seemed to be robust over a wide range of parameters. In contrast to the results presented here, it was found by Tildesley (2003) that the presence of a magnetic field enhanced the growth rates. Its presence also pushed the instability closer to the boundary between the magnetic flux bundle and the field-free convection area, similar to our results. This work was extended into the nonlinear regime by Tildesley & Weiss (2004), who found that the 3D modes saturated at low amplitude. However, both these studies were carried out in relatively narrow domains (aspect ratio 1), and so may have been influenced by strong magnetic field concentrations at the outer boundary.

Preliminary nonlinear studies in cylindrical geometry (Hurlburt et al. 2000; Hurlburt & Alexander 2002) suggest that there is a critical magnetic field strength below which the structure stays axisymmetric and pore-like, while above which azimuthal structures form around the central flux tube. However, these calculations were started with a zero velocity, uniform magnetic field initial condition, rather than the fully nonlinear axisymmetric state that we use here. In addition, their calculations were carried out with much stronger magnetic fields, and their results may have been influenced by the strong magnetic field concentration at the outer boundary, as the calculations were performed in a relatively small domain (radius $\Gamma = 1$).

This linear study is the first stage in a programme of numerical simulations in which we hope to explore the development of different types of azimuthal structures around magnetic flux tubes. It would be interesting to extend our study to include different magnetic field strengths, as well as to allow magnetic boundary conditions that allow inclined fields at the top of the domain – this will be the subject of a future paper. The ultimate aim is to model the fine structure observed in and around sunspots.

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REFERENCES

Berger, T. E., Rouppe van der Voort, L. H. M., Löfdahl, M. G., Carlsson, M., Fossum, A., Hansteen, V. H., Marthinussen, E., Title, A., & Scharmer, G. 2004, A&A, 428, 613

Botha, G. J. J., Rucklidge, A. M., & Hurlburt, N. E. 2006, MNRAS, 369, 1611

Busse, F. H. 1972, Journal of Fluid Mechanics, 52, 97

Dedner, A., Kemm, F., Kröner, D., Munz, C.-D., Schnitzer, T., & Wesenberg, M. 2002, Journal of Computational Physics, 175, 645

Dorotovič, I., Sobotka, M., Brandt, P. N., & Simon, G. W., 2002, A&A, 387, 665

Hagenaar, H. J., & Shine, R. A. 2005, ApJ, 635, 659

Hirzberger, J. 2003, A&A, 405, 331

Hurlburt, N. E., & Alexander, D. 2002, in Cospar Colloquia Ser. 14, (Oxford: Pergamon), p. 19

Hurlburt, N. E., Alexander, D., & Rucklidge, A. M. 2002, ApJ, 577, 993

Hurlburt, N. E., Matthews, P. C., & Rucklidge, A. M. 2000, Solar Physics, 192, 109

Hurlburt, N. E., & Rucklidge, A. M. 2000, MNRAS, 314, 793

Jones, C. A., & Moore, D. R. 1979, Geophys. Astrophys. Fluid Dynamics, 11, 245

Keppens, R., & Martínez Pillet, V. 1996, A&A, 316, 229

Leka, K. D., & Skumanich, A. 1998, ApJ, 507, 454

Parker, E. N. 1975, Solar Physics, 40, 291

Rimmele, T. R. 2004, ApJ, 604, 906

Rucklidge, A. M., Schmidt, H. U., & Weiss, N. O. 1995, MNRAS, 273, 491

Rucklidge, A. M., Weiss, N. O., Brownjohn, D. P., Matthews, P. C., & Proctor, M. R. E. 2000, Journal of Fluid Mechanics, 419, 283

Sankarasubramanian, K., & Rimmele, T. 2003, ApJ, 598, 689

Scharmer, G. B., Gudiksen, B. V., Kiselman, D., Löfdahl, M. G., & Rouppe van der Voort, L. H. M. 2002, Nature, 420, 151

Shine, R. A., & Title, A. M. 2001, in Encyclopedia of Astronomy and Astrophysics, ed. Murdin, P., (Bristol: IOP Publishing), vol. 4, p. 3209, doi: 10.1888/0333750888/2038

Sobotka, M. 2003, Astron. Nachr., 324, 369

Sobotka, M., Vázquez, M., Bonet, J. A., Hanslmeier, A., & Hirzberger, J. 1999, ApJ, 511, 436

Spruit, H. C. 1981, Space Science Reviews, 28, 435

Stangl, S., & Hirzberger, J. 2005, A&A, 432, 319

Thomas, J. H., & Weiss, N. O. 2004, Annu. Rev. Astron. Astrophys., 42, 517

Tildesley, M. J. 2003, MNRAS, 338, 497

Tildesley, M. J., & Weiss, N. O. 2004, MNRAS, 350, 657

Yang, G., Xu, Y., Wang, H., & Denker, C. 2003, ApJ, 597, 1190

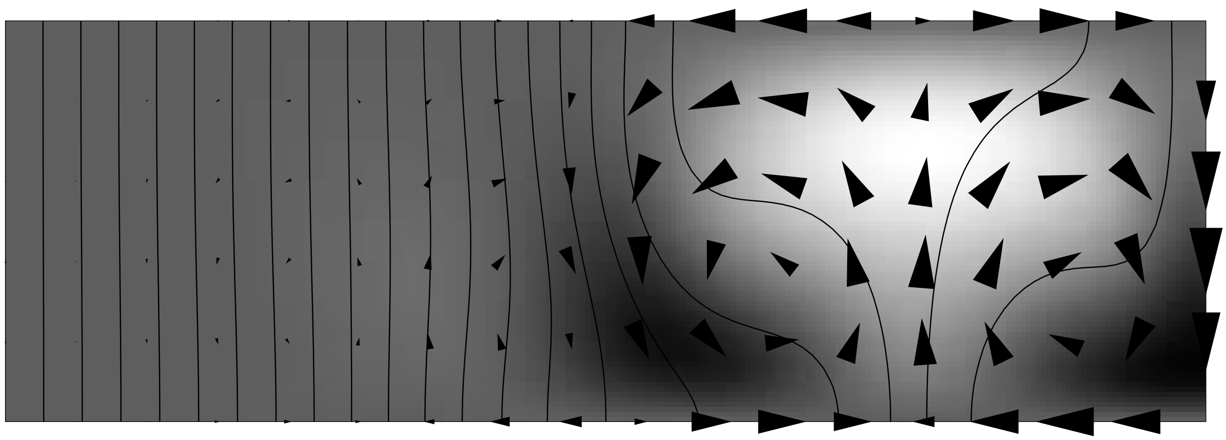

Fig. 1.— The time independent basic state for $R = 10^4$ and $m = 1$. The lines are magnetic field, arrows are the velocity field and the grey scale the temperature fluctuations relative to the unperturbed state.

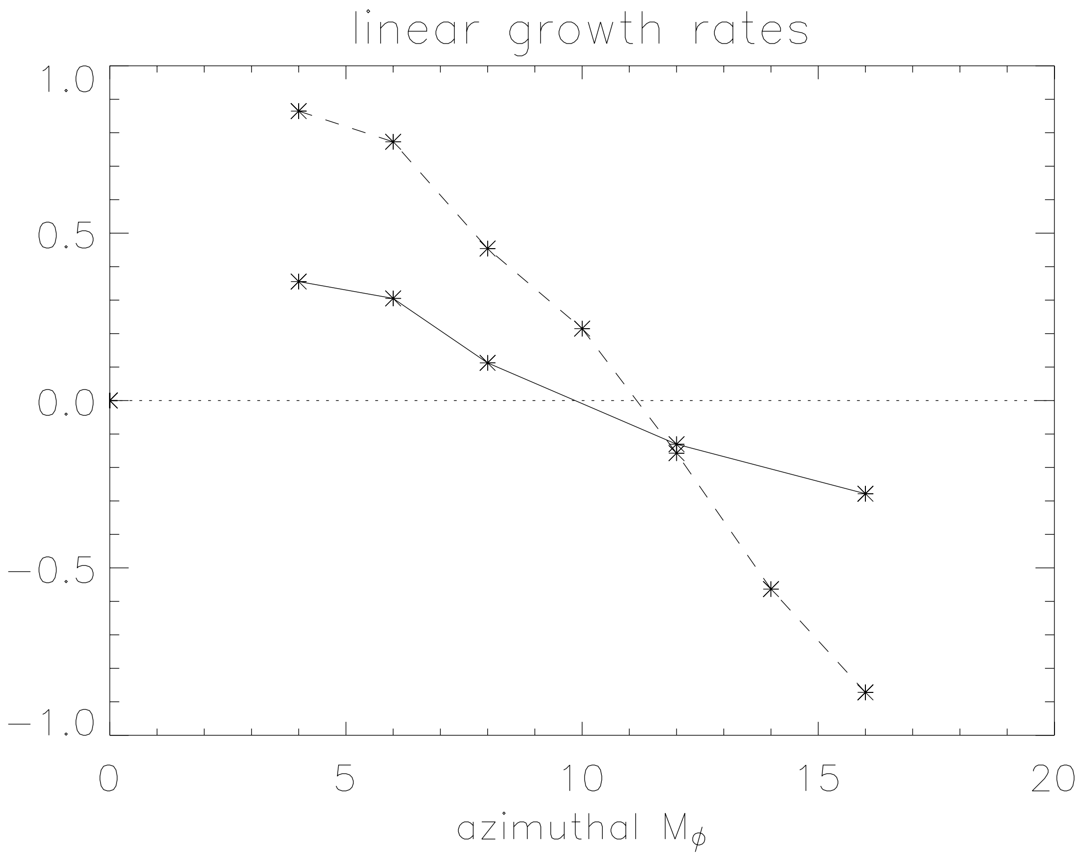

Fig. 2.— Growth rates for the steady modes for different azimuthal wavenumbers (M_ϕ), obtained from the full PDEs (solid line) and the PDEs with azimuthally averaged magnetic field (broken line), for $R = 10^4$ and $m = 1$.

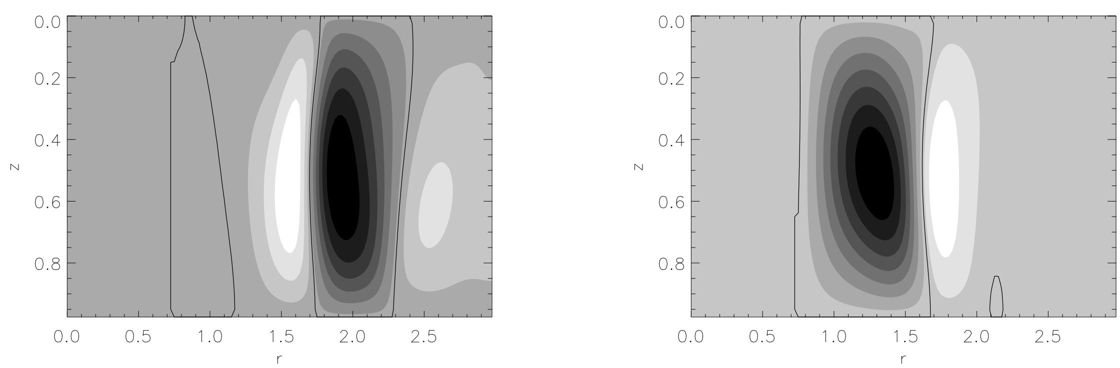

Fig. 3.— The eigenfunctions of the steadily growing linear modes in the (r, z) plane for $M_\phi = 4$, using the full PDEs (left) and the PDEs with azimuthal averages (right). The magnetic flux tube occupies approximately $0 < r < 1.42$ (Figure 1) while the enhanced dealiasing sets all nonaxisymmetric modes to zero between $0 < r < 0.75$. The azimuthal perturbation of the vertical velocity $(\tilde{v}_z)$ is plotted with the scale between contours logarithmic, and lighter shades positive and darker shades negative contours. The black line is the zero contour.

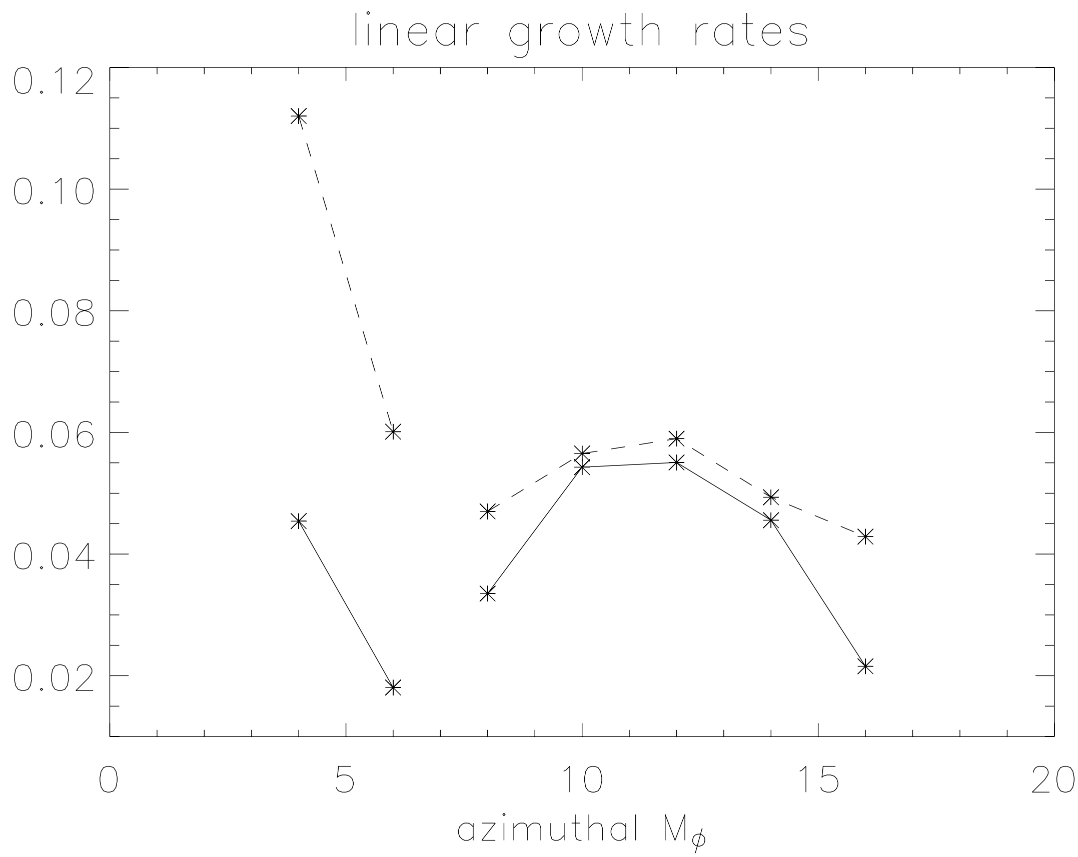

Fig. 4.— Growth rates for different wedge widths (M_ϕ), obtained from the full PDEs (solid line) and the PDEs with azimuthal averages (broken line) for $R = 10^5$ and $m = 1.495$. The modes for $M_\phi \leq 6$ are steady and for $M_\phi \geq 8$ oscillating, with their frequency increasing as M_ϕ increases. The oscillation periods obtained from the PDEs with azimuthal averages are approximately 25% longer compared to those obtained with the full PDEs.

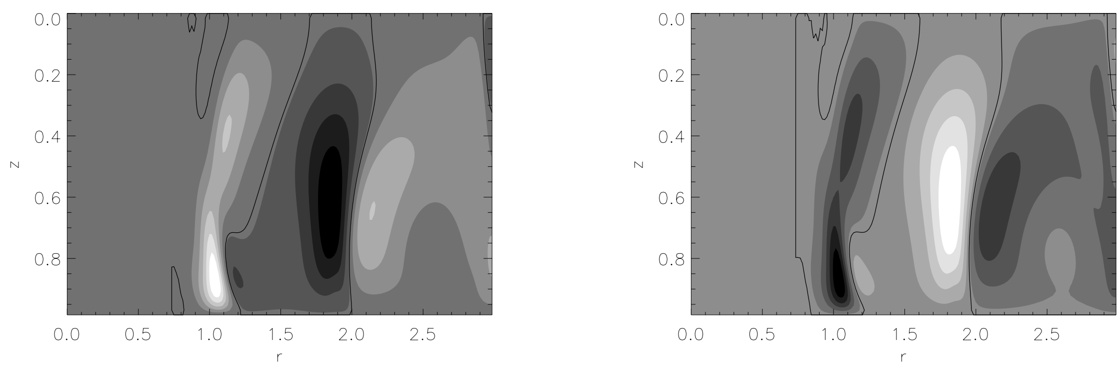

Fig. 5.— The eigenfunctions of the oscillating growing linear modes in the (r, z) plane for $M_\phi = 12$, using the full PDEs. The contour plot on the right is sampled 29.96 time units after the plot on the left and is approximately half a period later. The magnetic flux tube occupies approximately $0 < r < 1.1$ while the enhanced dealiasing sets all modes to zero between $0 < r < 0.7$. The contours have the same meaning as in Figure 3.