MATH0370: Introduction to Applied Mathematics 2, 2010–11
Examples 9: Circular motion

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Course web page: `http://www.maths.leeds.ac.uk/~alastair/MATH0370/`.

Section 1 will be covered in the example class on Friday 1st April. Hand in your answers to questions from **section 2** in the School of Mathematics Undergraduate Office (level 8) by 12:00 on **Tuesday 3rd May**. The office is open 10–12. Throughout, take the gravitational acceleration at the surface of the Earth as $g = 10\,\mathrm{m/s^2}$.

Section 1: to be attempted in the examples class

1. A toy motor boat of mass 2.5 kg is constrained to move in a circle by means of a string 4.9 m long, attached to a post in the centre of a pond. The boat circles the post at a constant speed, completing one circuit in 22 s. Calculate the tension in the string.

2. A satellite orbits the Earth in a circular orbit of radius equal to five times the Earth's radius. Show that the magnitude of its acceleration is $0.4\,\mathrm{m/s^2}$. Draw a diagram showing all the forces acting on the satellite.

3. The coefficient of friction between the road and the tyres of a car is 0.2. At what speed is it safe to drive round a corner of radius 200 m if the road is horizontal?

4. A road is banked at an angle α so that a car can round a corner of radius R at a speed of v without skidding, no matter how smooth the road is (see picture on next page). Show that the angle of the bank must satisfy $\tan\alpha = v^2/Rg$.

5. A pilot is flying an airplane while sitting on his bathroom scales in the cockpit, so that he can record any forces exerted. When the plane flies horizontally at 180 km/hr, the scales read 75 kg. He then flies in a vertical circular loop at the same speed. What is the radius of the loop if the maximum weight he reads on the scales is 450 kg.

Section 2: to be handed in

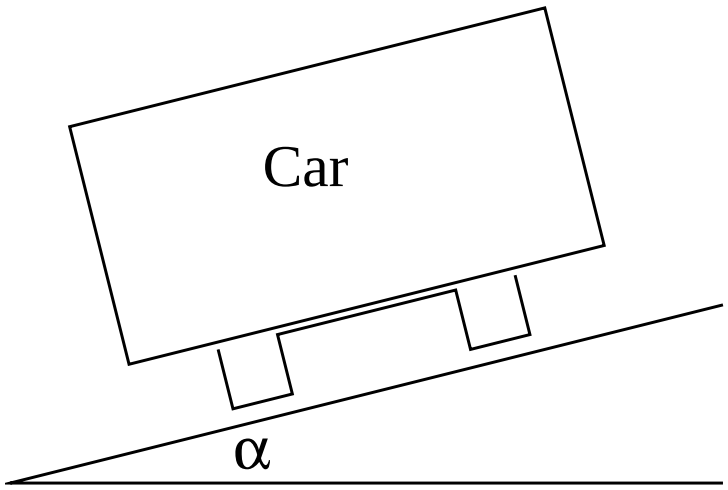
1. The radius of the Earth's orbit around the Sun is approximately 150 million kilometers. What is the circumference of the Earth's orbit, and how long does it take the Earth to travel this distance? Calculate the speed of the Earth in km/h, the angular speed in radian/s, and magnitude of the acceleration, in m/s^2. What is the direction of the acceleration?

2. The London Eye (see `http://www.londoneye.com/`) has diameter 135 m and takes 30 minutes to complete a full revolution. Calculate the forces acting on a passenger standing in a capsule at the top of the wheel. How long would one revolution take if passengers just left the floor of the capsule at the top of the wheel?

3. A fairground ride consists of a large drum (radius 5 m). People stand around the edge of the drum, which begins to rotate about a vertical axis. When the drum is rotating rapidly, the floor drops away, but participants do not fall: they feel that they are pressed against the wall of the drum, and are held in place by friction. If the drum rotates once every two seconds, and this is just fast enough to prevent the participants from slipping down, draw a diagram showing all the forces acting on each person, and find the coefficient of friction.

4. The coefficient of friction between the road and the tyres of a car is 0.4. At what speed can the car drive round a corner of radius 100 m without skidding, if the road is horizontal?

5. A road is banked at an angle α so that a car can round a corner of radius 150 m at a speed of 20 m/s without skidding, no matter how smooth the road is (see below). Calculate the angle of the bank.



6. A particle of mass m is attached by a string of length 0.7 m to a fixed point O. The particle is pulled to one side and then pushed horizontally, so that it rotates in a horizontal circle, whose centre A is on a vertical line through O. If the string makes an angle of 30° with the vertical, calculate the tension in the string, and find the time taken for the particle to make a complete revolution.