MATH0370: Introduction to Applied Mathematics 2, 2010–11
Examples 10: Work, energy and power

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Course web page: `http://www.maths.leeds.ac.uk/~alastair/MATH0370/`.

Section 1 will be covered in the example class on Friday 1st April. Hand in your answers to questions from **section 2** in the School of Mathematics Undergraduate Office (level 8) by 12:00 on **Tuesday 3rd May**. The office is open 10–12. Throughout, take the gravitational acceleration at the surface of the Earth as $g = 10\,\mathrm{m/s^2}$.

Section 1: to be attempted in the examples class

1. (a) A 10 kg mass has an initial speed of 1 m/s, and is subjected to a 20 N force in the same direction as its initial velocity. What is the Kinetic Energy at the start, and what is the Kinetic Energy after it has travelled a distance of 4 m? What is the speed at that point?
   (b) What is the Kinetic Energy of a 2 kg block that has fallen from rest through a height of 20 m? What is its speed at that point?

2. A mountaineer of mass 100 kg (including his backpack) climbs 360 m in one hour. What is the power he develops, assuming he is walking at a constant rate all the time?

3. A block of mass 10 kg starts from rest and slides 5 m down a rough slope, which is angled 36.87° below the horizontal. The coefficient of friction is $\mu = 0.1$. Calculate the work done by (a) the normal reaction force; (b) friction; (c) gravity. Hence find the speed of the block at the bottom of the slope.

4. A footballer (mass 80 kg) is moving at a speed of 2 m/s when he collides with a referee (mass 60 kg), who is stationary. If they cling on to each other on impact, at what speed do they move after the collision? Calculate the total mechanical energy before and after the collision. Is energy conserved?

5. A ball of mass m is dropped onto a horizontal pavement from a height H. If the coefficient of restitution is e, how high does it rise after the bounce? How much mechanical energy does the ball have before and after the bounce, and what fraction of the mechanical energy is lost in the bounce?

6. A bowling ball A (mass 1 kg), moving with speed 4 m/s, directly impacts on an identical bowling ball B, which is initially at rest. If the ball A is brought to rest, find the coefficient of restitution, and the speed of ball B after the collision. Calculate the total mechanical energy before and after the collision. Is energy conserved?

Section 2: to be handed in

1. A girl throws a ball (mass 2 kg) vertically upwards, with an initial speed of 10 m/s.
   (a) Calculate the maximum height of the ball.
   (b) What is the Kinetic Energy of the ball at the instant she throws it, and what is the Kinetic Energy of the ball when it reaches its maximum height.
   (c) Calculate the Work Done on the ball by gravity, using equation (27): Work = $\int F\, ds$.
   (d) Verify that the total energy of the ball is conserved.

2. A block is pushed so that it starts sliding up a smooth 30° slope with an initial speed of 5 m/s.
   (a) Use Newton's Laws of motion to calculate how far up the slope it goes.
   (b) Use the principle of conservation of mechanical energy to calculate how far up the slope it goes.

3. A horse is harnessed to a rope, which goes over a pulley and down a mineshaft. A basket containing 100 kg of coal is attached to the rope, and the horse is capable of pulling the coal up a distance of 45 m in one minute. Calculate the power developed by the horse. (This amount of power is approximately equal to one horsepower, a unit commonly used for expressing the power of car engines.)

4. A horizontal force is applied to a block of mass m, which moves across a rough horizontal surface, of coefficient of friction μ. The force is just strong enough to overcome friction and keep the block moving at a constant speed v. Find the power supplied by the force.

5. (a) A railway carriage of mass 1.5 tonnes travelling at 5 m/s hits a stationary carriage of mass 1.0 tonnes. The two carriages couple together automatically and move together after the collision. With what speed do they move? Calculate the total mechanical energy before and after the collision. Is energy conserved?
   (b) A billiard ball A of mass 4 kg is moving with speed 3 m/s to the right. It collides with a second ball B, which has mass 2 kg and which is moving at speed 6 m/s in the opposite direction. If ball A rebounds with speed 3 m/s to the left, calculate: the speed of ball B after the collision; the coefficient of restitution; the total Kinetic Energy before and after the collision. Is energy conserved?

6. A ball of mass 0.1 kg is dropped onto a horizontal pavement from a height of 3 m. How high will the ball bounce if the coefficient of restitution is 0.2? How much mechanical energy does the ball have before and after the bounce, and what fraction of the mechanical energy is lost in the bounce?