

MATH0370: Introduction to Applied Mathematics 2, 2010–11

Examples 1: Kinematics

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Course web page: <http://www.maths.leeds.ac.uk/~alastair/MATH0370/>.

Section 1 will be covered in the example class on Friday 4th February. Hand in your answers to questions from **section 2** at the **start** of the lecture on **Monday 7th February**. Throughout, take the gravitational acceleration at the surface of the Earth as $g = 10 \text{ m/s}^2$.

Section 1: for the examples class

1. A train leaves the station in such a way that the distance it has moved along the track is

$$s(t) = \frac{20t^2}{1+t},$$

where t is measured in seconds and s in metres. Find the average speed between $t = 0$ and $t = 4$ s. Find the instantaneous speed at $t = 9$ s.

2. A bus draws away from a bus stop and travels along a straight road until it gets to the next stop. Between the two stops, the distance it has travelled is

$$s(t) = \frac{1}{1000}t^2(150 - t),$$

where t is measured in seconds from the departure time, and s in metres. What is its speed at the second stop? At what time does it reach the second stop? How far away is the second stop?

3. A spring has a mass M on the end, which oscillates in such a way that its position $z(t)$ from its rest position is

$$z(t) = 3 \cos 2t$$

where t is measured in seconds and z in metres. Find the speed v and acceleration a of the particle, and find the maximum and minimum values of the acceleration. Show that $a + 4z$ is a constant.

4. A particle moves according to

$$\underline{r}(t) = (t^2 + t) \underline{i} + (3t - 2) \underline{j} + (2t^3 - 4t^2) \underline{k}.$$

Find its velocity and speed, its acceleration and the magnitude of its acceleration. Is there at time at which the particle is at rest?

5. A particle moves such that its acceleration is

$$\underline{a}(t) = (24t^2 + 1) \underline{i} + t \underline{j} + (6t^2 - 2t) \underline{k}.$$

Find its velocity and position, given that the particle starts from rest at the origin.

6. Find the formula for the average speed v between times t_1 and t_2 if the distance a particle moves is

$$(a) \quad s(t) = 1 + t - \frac{1}{t}, \quad (b) \quad s(t) = \sqrt{t}, \quad (c) \quad s(t) = \frac{3}{t^2}.$$

Section 2: to be handed in

1. A particle moves according to

$$\underline{r}(t) = \cos t \underline{i} + \sin t \underline{j}.$$

Find its velocity and speed, its acceleration and the magnitude of its acceleration. Is there at time at which the particle is at rest?

2. A particle moves such that its velocity is

$$\underline{v}(t) = (12t^3 + 2t) \underline{i} - 4t \underline{j} + (6t^2 - 4) \underline{k}.$$

Find its acceleration and position, given that the particle starts at $\underline{i} - \underline{j}$. Is there at time at which the particle is at rest?

3. Suppose that the distance a particle moves in time t is $s(t) = At^2 + Bt + C$, where A , B and C are constants. Find the distance travelled between times $t = 0$ and $t = T$, and hence find the average speed between $t = 0$ and $t = T$. Find the instantaneous speeds at times $t = 0$ and $t = T$. Show that the average speed between $t = 0$ and $t = T$ is equal to the average of the instantaneous speeds at $t = 0$ and $t = T$.
4. A ball is rolled down a sloping bowling alley 20 m long. The ball reaches a distance $8t + t^2$ metres from the top of the alley after a time t seconds. What is the speed of the ball after t seconds? At what speed does the ball meet the skittles at the far end of the bowling alley?
5. A stone is thrown up in the air in such a way that its height is $20t - 5t^2$. What is its instantaneous speed and acceleration? How long does it take to reach its maximum height, and how high does it go? How fast is it moving just as it returns to the ground?
6. If the speed of a particle travelling in one dimension is $2t + 12t^2$ m/s, find how far the particle travels between $t = 2$ s and $t = 4$ s.