Reconstruction of rainfall fields by spatio-temporal models using radar and raingauges measurements in Southern Italy

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1 Introduction

In response to the drought experienced in Southern Italy a Rain enhancement project has been set and developed during several years. The initiative was taken with the purpose of applying Israeli methods and rain enhancement technology in Puglia and other regions of Southern Italy. The aim of this study is to accomplish a support to the evaluation of the experimental part of the Rain Project which was conducted for the period 1986-1994. In particular our aim is to reconstruct rainfall fields combining two source of data: rainfall intensity and radar reflectivity. In 1992, a high density network of 80 automated raingauges with high time (10 minutes) and space ($\sim$10 km inter-gauge spacing) resolution was installed in the experimental area; moreover a C-band digital weather radar, scanning the whole area every five minutes, was introduced. Radar raingauge systems are increasingly used to reconstruct rainfall fields since they are able to provide spatially continuous images of precipitation for short and regular time intervals. On the other hand, ground raingauges monitoring network located in a set of sites within the study area provide more accurate, direct (punctual) estimates of rainfall intensity. However the coverage of the study area by such raingauges is spatially sparse in comparison to the grid on which radar reflectance is recorded. In this work we investigate the relationship between radar reflectance and rainfall intensity in order to build a model able to provide estimates of rainfall intensity over a region larger then the one covered by the ground raingauges network. We compare several approaches to the reconstruction of the rainfall field: the first one is based on the state-space approach proposed in (Brown et al. 2001), the second one uses a Kriging with external drift technique and the third one is built on a complex Bayesian model as proposed in Mardia and Sahu (2004).

2 Data quality control, spatial and temporal analysis of rainfall data

It is of fundamental importance to know how accurately rainfall has been measured by the specially designed raingauge network. In this work data have been pre-processed to remove anomalies and outliers. In order to explore the rainfall process and to address the development of statistical models, basic exploratory analysis have been carried out. This type of analysis has been useful in order to highlight dry or rainy periods. In addition the spatial and temporal structures of the rainfall process within the study area have been examined. This part required the choice of appropriate time and spatial “windows” to be used in the evaluation of spatial and temporal tools as: cross correlations, variograms, autocorrelations etc. The data quality control leads to the choice of reliable rain stations, one different set for each rain enhancement experiment and the number of "valid" station ranges from a maximum of 65 (January 1994) to a minimum of 42 (March 1994). The exploratory analysis suggested to carry on separate analysis for each experiment. In the following we'll present results for the 11th of April 1992 cloud seeding operation.

3 Weather radar calibration for rainfall estimation

The area covered by the radar include the experimental area and a region around it of about 200 km^2. Recorded values (the pixel size is of 0.5km x 0.5km) are in radar reflectivity units of dBZ. This is a measure of the power scattered back to the radar by precipitation particles in the atmosphere. Meteorologist commonly convert from a radar reflectivity factor Z to a rainfall rate R (in mm h^{-1}) using a power law that, unfortunately, is not exact. Actually, rainfall and radar measurements are sensibly different: on one hand the first ones give a direct measure, spatially punctual and time integrated, of ground rainfall; on the other hand, the second ones give an indirect measure of precipitation, on the air, and integrated in space and at a given point in time. Most of the conversion formulas, called $Z - R$ relationships, result in the following relation:

$$Z = aR^b \tag{1}$$

where a and b are unknown coefficients. In general, estimates of a can vary from about 30 to 500 and b can vary from about 1.2 to 2.0. A typical value of a and b are respectively 200 and 1.6, which give the well-known Marshal-Palmer relation. The Marshal-Palmer law provides a first approximation to the rainfall rates throughout the radar field.

The calibration factor. An attempt was made to apply the simple approach that meteorologist often use. They simply correct radar measurements by applying a calibration factor (CF). The CF is normally defined as being the ratio between rainfall measured at ground level R_g and rainfall measured by the radar R_r: $CF = \frac{\text{gauge rainfall}}{\text{radar rainfall}} = \frac{R_g}{R_r}$. CF is calculated for each raingauge and each time point and then values are summarized by averaging over time. This factor and its inverse, known as assessment factor AF, have been used to adjust all the rainfall rates produced by the first approximation. The method is equivalent to choose a new value of a in Eq.(1). It was possible notice that there is a general reduction in the errors in radar-derived rainfall estimates but large spatial errors remain. Furthermore several drawbacks in the use of CF are evident: (i)*The calibration factor* becomes infinite when the radar reading is zero and the raingauge value is not. (ii) The simple averaging does not allow for the variation in time of the two factors (a and b), that is the estimator has no "dynamic" memory. (iii) CF is not available for sites in the study which are not covered by the raingauge and to use just one factor for the whole area produces large calibration errors.

A space-time approach. The modelling strategy proposed reflects the fact that the ground data are spatially sparse, but temporally dense. Our approach starts from what proposed in Brown et al. (2001). A dynamic linear regression model is used to re-formulate Eq.(1) in a time-series regression form where the coefficients are assumed time-varying and can be represented as stochastic AR(1) processes. The Kalman filter provides an efficient algorithm for finding the estimates. Kriging surfaces of the coefficients have been obtained through Ordinary Kriging for each time-interval. Reconstruction of the rainfall field has been performed for the event that involves two seeding operations (11th of April 1992) plus three hours afterwards. In the fitting the procedure six sites are left out for validation purposes. Analysis has shown relatively small MSE values for this six sites (cfr Tab.1). As a farther comparison term we implemented a straight forward version of the model proposed in Brown et al. (2001). However there were no improvements in term of MSE.

The Kriging with External drift method. Calibration can be done using external drift method, that applied in the IRF-k framework (intrinsic random function of order k) produces maps of a variable when a secondary variable, correlated with the first one, is available on the whole field. The method consists in adding a supplementary equation to the kriging system (Wackernagel 1998). Reconstruction of the rainfall field has been performed (always for the 11th of April) leaving out the same six locations we used for the validation. Analysis showed a dramatic reduction of MSE values for this six sites (cfr Tab.1).

Validation Site	MSE from Brown et al. model	MSE from the state-space plus kriging	MSE from the Kriging with external drift
S107	0.79	0.20	0.03
S304	0.60	0.30	0.00
S606	0.31	1.04	0.01
S512	0.41	0.57	0.01
S211	1.05	0.45	0.05
S217	0.66	0.22	0.14

Table 1: MSE for the six validation sites

Discussing the different approaches. The considerable difference in the MSE between the state space approaches and the kriging with external drift may be due to several reason: (a) Our estimates of the time varying coefficients didn't show a AR(1) behaviour. Their time behaviour is very erratic and it changes considerably from one site to the other making difficult to choose another type of model structure. (b) The use of a constant spatial structure for all times (as in the Brown's model), then the assumption of a separable space-time model, is not appropriate for this problem. This is confirmed by the slight improvement in the MSE obtained using the state-space plus kriging approach, in this model we fit a different variogram model to each time. (c) The kriging with external drift estimates are less sensitive to time variation as they are computed separately for each time, adapting an appropriate generalized covariance function model for each temporal slice. Furthermore the use of generalized covariance functions allows us to treat data that are not necessarily second order stationary. However kriging type estimates are highly reliable only inside the domain in which ground measurements are taken. As we are interested in evaluating extra-area effects we need a further strategy to improve our estimates.

4 Conclusions: A new strategy

The model proposed as further comparison term is based on the proposal by Mardia and Sahu (2004). In their model the spatial drift is modelled by principal kriging functions and the time component is modelled by a vector random-walk process. The process considered is continuous in space and discrete in time. The full model is built in a hierarchical framework. This allows the inclusion of a "nugget" term in the spatial part of the model and of covariates in the drift modelling. It is non-linear and it incorporates a huge number of parameters. The model has been fitted and used by Mardia and Sahu for forecasting purposes in a unified computational framework using Markov Chain Monte Carlo methods (MCMC). Our aim is to develop a coherent Bayesian computational methodology implementing flexible hierarchical models for space-time prediction of rainfall surfaces during a seeding event. At present we are creating the C-code for the complete model.

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Essential References

Anderson B.D., Moore J. B. (1979). *Optimal filtering*. Englewood Cliff's:Prentice-Hall.

Brown P.E., Diggle P.J., Lord M.E., Young P.C.(2001) Space-time calibration of radar-rainfall data. JRSSC, *Applied Statistics* (C), 50(2):221-241.

Chiles J.P., DelfinerP. (1999). *Geostatistics, Modelling Spatial uncertainty*.Wiley, New York.

Collinge, V.K.(1993). Weather radar for rainfall measurement and forecasting, pages 710-714. *Concise Encyclopedia of Environmental Systems*. ed. P.C. Young, Oxford; Pergamon edition,

Mardia, K.V., Goodall, C.R., Redfern, E.J., Alonso, F.J.(1998). The kriged kalman filter (with discussion). *TEST*, 7:217-252.

Sahu, S. K., Mardia, K.V.(2004). A Bayesian kriged-kalman model for short-term forecasting of spatio-temporal processes. JRSSC *Applied Statistics*, to appear.

Wackernagel, H.(1998). *Multivariate Geostatistics*. Springer-Verlag, Berlin.

Wikle, C.K., Cressie, N.A.C. (1999). A dimension reduction approach to spacetime kalman filtering. *Biometrika*, 86:815-829.

Wood, S.J., Jones, D.A., Moore, R.J.(2000a). Accuracy of rainfall measurement for scales of hydrological interest. *Hydrology and Earth System Sciences*, 4(4):531-543.

Wood, S.J., Jones,D.A., Moore, R.J. (2000b). Static and dynamic calibration of radar data for hydrological use. *Hydrology and Earth System Sciences*, 4(4):545-554.

Zawadzki, I.(1975). On radar-raingauge comparison. *J. Appl. Meteor.*, 14:1430- 1436.