Irregularity and singular vector growth in the differentially heated rotating annulus

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Irregularity is a characteristic feature of the large-scale atmospheric flow [1]. The differentially heated rotating annulus, a laboratory experiment that is considered as an analogue of the mid-latitude atmospheric circulation [2], shares this characteristic behavior with the atmosphere. This has long been recognized, however, the possible routes to geostrophic turbulence is still an puzzling process for geophysical flows that can be investigated (and clarified) by laboratory experiments and low-order numerical simulation.

In contrast to the atmosphere, laboratory experiments are controllable and reproducible. We use a low-order model of the rotating annulus as well as new laboratory data to determine singular vectors (SVs) in different flow regimes. Usually, such regimes are characterized by the typical spatial structure of the most unstable linear mode. However, there where also attempts to characterize the regimes and their transitions by using other measures, for instance Lyapunov exponents [3]. Though different by definition, [4] compares Lyapunov and SVs for baroclinic waves and found many similarities. This suggests that SVs can also be used to quantify predictability. Moreover, besides their potential for quantifying predictability, SVs are also interesting from a dynamical point of view. It was speculated whether non-classical instability (i.e. growing SVs) adds additional irregularity to the annulus flow in the transition to geostrophic turbulence. Later, more general studies seem to confirm this. [5] investigated whether SVs in quasigeostrophic turbulence are excited sufficiently strong and frequently to account for the energy-containing eddies in the turbulent flow. He found positive results and suggests that these can be carried over to other systems.

Whether this holds also for the flow regimes of the real laboratory experiment is not clear yet. However, we show that the low-order model captures many features of the real flow. Moreover, we compute singular vectors empirically from the laboratory data [6] and compare them with the singular vectors of the low-order model. Again, a qualitative agreement can be found. This comparison gives us confidence that the results obtained for the low-order model can be trans-
ferred to the laboratory experiment and the real atmosphere. Our study might help to understand better the role of optimal perturbations in quasigeostrophic turbulence.

Figure 1: Growth factors of singular vectors as a function of the Taylor number, $Ta \sim \Omega^2$, and the Rossby number, $Ro \sim \Delta T/\Omega^2$, where $\Omega$ is the rotation rate of the annulus and $\Delta T$ the radial temperature difference. Rotation rate and temperature difference can be controlled experimentally. The diagram demonstrates the rich dynamical behavior of the annulus model. Different regimes of the flow can be quantified by the growth factor. The largest growth (red) corresponds with the irregular flow regime.

References
