Magnetorotational-type instability in Couette-Taylor flows of viscoelastic polymeric liquids

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The Magnetorotational Instability (MRI) is believed to play a crucial role in the angular momentum transport in planetary dynamos. Experimental demonstration of the MRI is challenging due to the requirement of a sufficiently large magnetic Reynolds number ($Rm \gg 1$), which is not even available for the most conducting liquid metals.

In [1] it was shown theoretically that in the limit of large magnetic Reynolds number ($Rm \gg 1$), the MRI equations bear strong analogy with the Oldroyd-B describing ViscoElastic Instabilities (VEI) in the liquid with constant viscosity in the limit of large values of Weissenberg number ($We \gg 1$).

$$ \lim_{We \to \infty} \text{(viscoelastic fluid)} = \lim_{Rm \to \infty} \text{(MHD fluid)}.$$  

A convenient choice of experiment geometry to demonstrate the MRI is the Taylor-Couette flow of a liquid metal between differentially rotating concentric cylinders, [2]. By setting the angular velocity of the cylinders in a Keplerian ratio and introducing an uniform magnetic field, the MRI becomes possible even when the centrifugal instability is suppressed according to Rayleigh’s criterion. Liquid metal MRI experiments employ a magnetic field and are not expected to be able to access the range of magnetohydrodynamic turbulence. In these respects the polymer experiment will offer a more faithful representation of the astrophysical systems of interest. Other major advantages are that the materials are cheap and safe, and that optical visualization or velocimetry can be carried out.

By means of a linear stability analysis an instability was described in [3] that is directly analog to the MRI and occurs instead in a simple viscoelastic liquid, with polymer molecules playing the role of magnetic field lines. A typical eigenfunction of the MRI analog at the onset of instability was found to be qualitatively similar to Taylor vortices and has a vertical wavelength comparable to the gap width but is nonaxisymmetric and would appear as a rotating spiral pattern. However, the existing experimental and theoretical studies of instabilities in viscoelastic Taylor-Couette flow with differentially rotating cylinders have not revealed the types of characteristic behavior relevant to the MRI analogy up to now. [3]. Only one experiment has been
claimed to describe this analogy [4], but the results have not been so conclusive. It is necessary to carry out detailed numerically simulations and careful experiments to prove the MRI analogy, under very precise conditions. The domain parameters of the viscoelastic Taylor-Couette flow used in linear stability analysis of the Oldroyd-B fluid model, in numerical works and in experimental one, are summarized in Fig. 1. Until now focus of the experimental, theoretical

Figure 1: Stability diagram accordingly to Rayleigh’s criterion for Newtonian fluids; the black dots correspond to existing studies of viscoelastic fluids in Taylor-Couette flows; the blue dots correspond to the domain parameter values proposed in [3]; the red dot corresponds to preliminary numerical tests with viscoelastic fluid models.

and numerical works concerning the pattern formation and stability of viscoelastic fluids in the Taylor-Couette system is mainly on flow studies for radius ratio \( \eta \) and angular velocities ratio \( \mu \) in the Newtonian inertial unstable (red) region of Fig. 1. This parameter range, for fixed outer cylinder and rotating inner cylinder (\( \mu = 0 \)), or values of (\( \mu, \eta \)) above the Rayleigh’s line is not of interest for the demonstration of the MRI analog instability. Only very few studies deal with Taylor-Couette system parameters in the inertial stable area, and that especially for the case of rotating outer cylinder and fixed inner cylinder(\( \mu = \infty \)). But these few experiments in which the inner and outer cylinders can rotate independently appear not to have examined the MRI analogy characteristic behavior up to now.

Aim of the project is to conduct a complementary numerical and experimental study of the MRI analogy with viscoelastic polymeric liquids in a differential rotating Taylor-Couette apparatus in a relevant parameter range.
References


