Modulated Couette-Taylor flow

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Abstract. We investigate the parametric excitation of newtonian fluid in Couette-Taylor system. The outer cylinder is stationary whereas the rotation of inner cylinder is modulated sinusoidally as $\Omega(t) = \Omega_0 [1 + \varepsilon \cos(2\pi f t)]$. It is observed that the destabilisation varies inversely as the amplitude of the modulation, eventually attaining insensitivity towards the modulation frequency. An increase in the vortex size can be achieved by increasing either the Taylor number or the modulation frequency.

1. Introduction

The study of modulated flows have been of much interest to several researchers owing to their various applications in the nature, for example blood flow into the aorta [1], and industry, for example the formation of droplets from a sinusoidally excited jet [2]. Donnelly [3, 4], in his pioneering experiments, showed that modulation destabilizes the flow; low modulation frequency produces large destabilisation. A similar observation was made in the theoretical work of Carmi & Tustaniwskyj [5]. Later the contradictory observation of low modulation frequency producing small destabilisation was made theoretically for example[10, 6, 7] as well as experimentally [8]. Barenghi & Jones [10] bridged the gap between the contradictory results by attributing it to experimental imperfections. According to them, experiments conducted by Donnelly were not sufficiently accurate to take into account the threshold shift of the critical parameter due to the modulation of inner cylinder about a non zero mean. This imperfection induced transient vortices thereby leading to the incorrect result of a positive destabilisation parameter. They also showed that the numerical work of Carmi & Tustaniwskyj [5] used large time step in the integrations of the Galerkin truncations leading to lower value of critical parameter.

2. Experimental setup and flow parameters

The Couette-Taylor system consists of two rotating coaxial horizontal cylinders of length $L = 601$ mm. The inner cylinder of radius $a = 44.60$ mm is made of black AU4G and the outer cylinder of radius $b = 50.80$ mm is made of glass. The dimensionless geometric parameters are gap size $d = b - a = 6.20$ mm, radius ratio $\eta = a/b = 0.878$ and aspect ratio $\Gamma = L/d = 97$. In our experiments the outer cylinder is fixed whereas the inner cylinder is driven by a DC servomotor with a modulated angular velocity $\Omega(t) = \Omega_0 [1 + \varepsilon \cos(2\pi f t)]$. Here $\Omega_0$ is the constant angular velocity, $f$ is the frequency of the modulation, $\Delta\Omega$ is the amplitude of the modulation and $\varepsilon = \Delta\Omega/\Omega_0$ is the relative amplitude of the modulation.

The working fluid is a solution made up of deionized water and glycerol in the ratio 7:3 and has kinematic viscosity $\nu = 0.020$ cm$^2$/s at 25C. For flow visualisation 2% of kalliroscope AQ 1000 is added to the solution. A 1392 x 1040 pixels CCD camera records the distribution of the reflected light.
intensity along the axial direction of the entire cylinder with a spatial resolution of 2.2 pixels per mm. Recorded images are captured with an average rate of 25 frames per second and stacked together to provide space-time diagrams of flow patterns. Further the spectral analysis is performed using post-processing techniques based on complex demodulation and fast Fourier transform (FFT).

The flow is characterized by three dimensionless parameters: the Taylor number $Ta = \left( \frac{\Omega_0 ad}{\nu} \right) \left( \delta \right)^{1/2}$, the modulation frequency $\sigma = 2\pi f d^2 / \nu$ and the relative amplitude of the modulation $\varepsilon$.

![Figure 1](image)

(a) TVD at the onset: $Ta_c = 34.11; \varepsilon = 0.83$

(b) UTVF: $Ta = 48.36; \varepsilon = 0.59$

(c) MUVF: $Ta = 85.44; \varepsilon = 0.33$

(d) Time power spectrum of MUVF

Figure 1: Flow patterns for $\sigma = 4.25$ and different values of $Ta$ and $\varepsilon$.

3. Results

The experiment is performed for various values of the parameter $\varepsilon \in [0.1, 15]$ and $\sigma \in [4, 192]$, much beyond the previously studied range (see for eg. [11, 6, 9]). In the non modulated case, the onset of stationary Taylor vortices is achieved at $Ta_c(\varepsilon = 0) = 44.37$ with the axial wave number $q_c = 3.12$ which is well within the range noted in the literature [12].

On excitation of the flow parameters, the flow departs from the base state eventually leading to a pattern called modulated undulating vortex flow (MUVF). The transition to MUVF can be characterized by the frequency of the modulation with no dependence on the Taylor number and the amplitude of the modulation. It is observed that for $\sigma \geq 16.98$ the transition leading to MUVF follows a definite scenario, viz., periodic vortex flow (PVF) to undulating periodic vortex flow (UPVF) to modulated undulating vortex
flow (MUVF). However, for $\sigma < 16.98$ no such a scenario was observed. With sufficient modulation, the base flow bifurcates to temporary appearance of dislocated vortices over a cycle. Following Donnelly [3], who has also observed such vortices at low frequency of modulation, we call them transient vortices with dislocation (TVD). At $\sigma = 4.25$ the TVD appears at $Ta_c = 34.11$ and $\varepsilon = 0.83$ (figure 1.a). On increasing the Taylor number the dislocation of the vortices disappears and another pattern which we call as undulating transient vortex flow (UTVF) is obtained. The UTVF appears at $Ta = 48.36$ and $\varepsilon = 0.59$ (figure 1.b). Further increase of the Taylor number to $Ta = 85.44$ exhibits a pattern with rigorous undulation and formation of vortices with temporally varying size at $\varepsilon = 0.33$. We term it as modulated undulating vortex flow (MUVF)(figure 1.c). It is clear from the time power spectrum that MUVF generates a temporal chaos in the modulated flow (figure 1.d). The critical parameters for the transition at $\sigma = 25.48$ is cited as an example. Here, the appearance of PVF is encountered at $(Ta_{c1} = 40.95$, $\varepsilon = 0.69$) followed by the
occurrence of UPVF at \((Ta_c^2 = 47.22, \varepsilon = 0.60)\). The amplitude of UPVF increases with further increase in the Taylor number arising MUVF at \((Ta_c^3 = 96.85, \varepsilon = 0.29)\).

The flow pattern from the base flow through MUVF experiences an increase in the vortex size. This is elucidated in figure 2.a which represents the variation of axial wave number \((q)\) with respect to the Taylor number \((Ta)\) at \(\Delta \Omega = 1.110\) rad/s. It could be emphasized that the axial wave number of the pattern decreases by increase in either the Taylor number or the modulation frequency. The evolution of the critical wave number \((q_c)\) with respect to the modulation frequency for several values of \(\Delta \Omega\) is plotted in figure 2.b. The inverse relation of \(q_c\) and \(\sigma\) at lower values of \(\sigma\) and the eventual stagnation of \(q_c\) at higher values of \(\sigma\) is apparent.

The effect of modulation on the stability can be characterized by the destabilisation factor \(\Delta(\varepsilon, \sigma) = \left[\frac{Ta_c(\varepsilon, \sigma) - Ta_c(\varepsilon = 0)}{Ta_c(\varepsilon = 0)}\right]\). The dependence of \(\Delta(\varepsilon, \sigma)\) on \(\varepsilon\) for various values of \(\sigma\) is shown in figure 3. The flow is increasingly destabilised with the modulation amplitude up to a certain value. Further the destabilisation is immune to \(\varepsilon\). We, thus, validate the results of Ganske et al [6].

4. Conclusion

We have investigated the transition from base flow to modulated undulating vortex flow (MUVF) in the modulated Couette flow. In the case of \(\sigma < 16.98\), transient vortices appear. At \(\sigma \geq 16.98\) different instability modes (PVF, UPVF, MUVF) were observed. In both cases the vortex size increases when the Taylor number or the modulation frequency increases. The destabilisation factor decreases with increasing value of the amplitude of modulation until insensitivity to the modulation frequency.

References