Mirror symmetric travelling wave solution in plane Poiseuille flow

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We consider the motion of an incompressible fluid in a channel bounded by two horizontal walls at \( z = \pm 1 \). The motion of the fluid is induced by moving the walls in the positive and negative \( x \)–directions with a constant speed and, in addition, by applying a constant pressure gradient in the \( x \)–direction. Our goal is to establish a homotopy continuation from the stationary and travelling hairpin vortex solutions in plane Couette flow (PCF) [1] to plane Poiseuille flow (PPF). For this purpose we consider the basic flow \( U_B(z) \) given by

\[
U_B(z) = -\sqrt{1 - \varepsilon R_C} z + \sqrt{\varepsilon R_P} (1 - z^2),
\]

where \( R_C \) and \( R_P \) represent the Reynolds numbers for PCF and PPF, respectively.

We can see two successful homotopy routes from PCF to PPF in Figure 1. It is found that

![Figure 1](image_url)

Figure 1: Homotopy from plane Couette flow (\( \varepsilon = 0, R_C = 250 \)) to plane Poiseuille flow (\( \varepsilon = 0, R_P = 1300 \)). The total energies \( E \) of the travelling wave solutions relative to the basic flow energy \( E_B \) are plotted.

the two solutions on \( \varepsilon = 1 \) merge to form a saddle-node bifurcation when \( R_P \) is decreased as shown by a thick curve in Figure 2. It is notable that this saddle-node bifurcation point occurs below the critical value known to date in PPF (see the dashed curve in Figure 2). The latter was obtained from Nagata’s solution [3] in PCF by a homotopy approach [2].

As seen from Figure 3, the flow field of the current solution possesses mirror-symmetries with respect to planes, \( y = \pm \pi / (2 \beta) \), perpendicular to the spanwise direction, as well as a mirror-symmetry about the mid-plane \( z = 0 \), in contrast to Waleffe’s solution[2] which is only mirror-symmetric about \( z = 0 \). The present solution is characterized by two quasi-streamwise...
Figure 2: The Reynolds number, $R_P$, vs. the bulk Reynolds number, $R_b$, for the wavenumbers, $\alpha$ and $\beta$ in the $x$- and $y-$directions, respectively, which gives the minimum value for $R_P$. The solid and dashed curves correspond to the present solution and Waleffe’s solution [2], respectively.

low-speed streaks in one spanwise period ($-\pi/\beta \leq y \leq \pi/\beta$) in the vicinity of each boundary as shown in Figure 4. The low-speed streaks are aligned with the planes of mirror symmetry, $y = \pm \pi/(2\beta)$, with their width varying in a varicose fashion in the streamwise direction. A pair of quasi-streamwise vortices forms a $\Lambda$-shaped vortex: vortices are up-lifted downstream while keeping their feet in the neighboring varicose bulges of the streamwise low-speed streaks. This vortex structure has a strong resemblance to the $\Lambda$-vortex pattern of Herbert-type (H-type) [4] observed in near wall turbulence in channel flows.

Figure 3: The velocity field projected on the $y - z$ cross section at $x = 0$ at the saddle-node point. (a): the present solution. (b) Waleffe’s solution [2].

Lastly, we briefly describe the connection of the mirror-symmetric disturbed flow found in the present paper to the exact coherent structures in Hagen-Poiseuille flow [5].
Figure 4: The flow pattern of the present solution at \((R_P, \alpha, \beta) = (937.1, 1.47, 3.06)\). Isosurfaces of the streamwise velocity at \(u = 300\) (gray), and the streamwise vorticity at \(\omega_x = 400/ - 400\) (red/blue).

References


