

CURRICULUM VITAE

John Colin WOOD

Personal details

Affiliation School of Mathematics, University of Leeds, Leeds LS2 9JT, G.B.
Nationality British
Date of Birth 14 March, 1949

University Education

1967–70 Open Scholarship in Mathematics, Wadham College, University of Oxford
1970–73 Research Student in Mathematics, University of Warwick

Qualifications

1970 B.A. (Hons) First Class (Oxford) in Mathematics
1971 M.Sc. with distinction (Warwick) in Mathematics
1974 Ph.D. (Warwick) in Mathematics

Positions

1973–77 Lecturer in Mathematics, Brighton Polytechnic (now University of Brighton)
1977–89 Lecturer in Mathematics, University of Leeds
1989–97 Reader in Differential Geometry, University of Leeds
1997– Professor of Differential Geometry, University of Leeds
2003–06 Head of Department, Pure Mathematics, University of Leeds

Visiting positions

1 Sept. **1980**–31 Aug. **1981** *Gastprofessor*, Sonderforschungsbereich *Theoretische Mathematik* (now Max Planck Institut), Universität Bonn.
1 Sept. **1990**–30 Nov. **1990** *Chercheur associé*, Ecole Polytechnique, Palaiseau, France.
1 Sept. **1990**–8 Aug. **1991** *Visitor*, Institut des Hautes Etudes Scientifiques, Bures-Sur-Yvette, France.
15 Feb. **1991**–14 Aug. **1991** *Professeur invité*, Université de Paris-Sud, Orsay, France (gave two courses, one at undergraduate level and one for the post MSc course *Diplome d'Études Approfondies*, both in French).
1 Dec. **2007**–31 July **2008** *Gulbenkian Professor*, University of Lisbon, Portugal.

Principal invitations

- 1976** Principal lecturer, *Summer Course in Complex Analysis*, International Centre for Theoretical Physics, Trieste, Italy.
- 1979** Invited speaker, *Topology*, Oberwolfach, Germany.
- 1980** Principal lecturer, *Summer Seminar on Complex Analysis*, International Centre for Theoretical Physics, Trieste, Italy.
- 1981** Invited speaker, *Contact Group in Differential Geometry*, Leuven, Belgium.
Invited speaker, *Invariant Metrics, Harmonic Mappings and Related Topics*, Rome, Italy.
- 1982** Invited speaker, *London Mathematical Society Symposium on Global Riemannian Geometry*, Durham, G.B.
- 1983** Invited speaker, *British Mathematical Colloquium*, Aberdeen, G.B.
Invited by the Scuola Normale Superiore, Pisa, to give a 20 lecture course on *Differential Geometry*, Cortona, Italy.
- 1984** Invited follow-up course entitled *Harmonic Maps and Differential Geometry*, Trento, Italy.
Invited speaker, *American Mathematical Society joint summer research conference on Complex Differential Geometry and Nonlinear Differential Equations*, Bowdoin, U.S.A.
Invited speaker, *Fifth International Colloquium on Differential Geometry*, Santiago de Compostella, Spain.
- 1985** Principal lecturer, *Sixth International Symposium on Differential Geometry and Differential Equations*, Shanghai, China.
Invited speaker, *Nordic Summer School*, Lyngby, Denmark.
- 1986** Invited speaker, *Applications harmoniques*, Luminy, France.
Invited speaker, *Workshop on Global Differential Geometry*, International Centre for Theoretical Physics, Trieste, Italy.
- 1987** Invited speaker, *Skyrmion Workshop*, Cambridge, G.B.
Invited to give another 20 lecture course on *Minimal Submanifolds and Harmonic Maps*, Cortona, Italy.
- 1988** Invited speaker, *Workshop on Harmonic Maps and Minimal Surfaces*, Mathematical Sciences Research Institute, Berkeley, U.S.A.
Invited to give a short course on *Harmonic Maps* (in Italian) at the University of Bari, Italy.
Invited speaker, *Workshop on Twistor Geometry and Related Topics*, Cortona, Italy.
- 1989** Invited speaker, *London Mathematical Society Symposium on Geometry of Low Dimensional Manifolds*, Durham, G.B.
27 November–1 December, Invited speaker, *College on Differential Geometry*, International Centre for Theoretical Physics, Trieste, Italy.
- 1990** 9–11 April, Invited speaker, *Workshop on Global Analysis*, University of Warwick, G.B.
- 1991** 4–11 September, Invited speaker, *Workshop on Minimal Surfaces*, Granada, Spain.
- 1992** 30 March–5 April, Invited speaker, *Workshop on Geometry and Differential Equations*, University of Warwick, G.B.
20–25 September, Invited speaker, *Workshop on Symplectic Topology and Related Topics*, Cortona, Italy.

- 22 October–4 November, Invited speaker, *Lobachevskii Semester*, Euler International Mathematical Institute, St. Petersburg, Russia.
- 1993** 12–23 July, Invited speaker, *First International Research Institute of the Mathematical Society of Japan: Geometry and Global Analysis*, Tohoku University, Sendai, Japan.
17–19 November, Invited speaker, *Belgian Contact Group in Differential Geometry*, Han-Sur-Lesse, Belgium.
- 1994** July, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
- 1995** July, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
- 1996** March–April, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
17–21 June, Invited speaker, *Interplays between Geometry and Topology*, International Centre for Mathematical Sciences, Edinburgh.
1–6 July, Invited plenary speaker, *First Brazilian–USA Workshop on Geometry, Topology and Physics*, Campinas, Brazil.
8–12 July, Invited speaker, *International Conference on Differential Geometry*, Instituto de Matematica Pura e Aplicada, Rio de Janeiro, Brazil.
- 1997** 17–21 March, Invited to give *A Short Course on Harmonic Morphisms*, University of Bucharest, Romania.
April, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
May–June, *Professeur invité* for 1 month, Université Libre de Bruxelles, Belgium.
27–29 August, Invited speaker, *Complex Methods in Differential Geometry*, International Centre for Mathematical Sciences, Edinburgh.
- 1998** 28 March–3 April, Invited to give a follow-up short course on harmonic morphisms, University of Bucharest, Romania.
July, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
- 1999** July, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
16–22 Sept, Main speaker, *Fourth International Workshop on Differential Geometry and its Applications*, Braşov, Romania.
- 2000** 17–21 February, Main speaker, *Winter School on Differential Geometry*, Keio University Conference Centre, Bury St. Edmunds, G.B.
1–5 February, Main speaker, *Workshop on Harmonic Maps and Minimal Immersions*, Caparide, Portugal.
May–June, *Professeur invité* for 1 month, Université Libre de Bruxelles, Belgium.
June–July, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
17–21 July, Main speaker, *Integrable systems in differential geometry*, Tokyo Metropolitan University, Japan.
- 2001** 28 May–1 June, Opening speaker, *Harmonic morphisms and harmonic maps*, Centre International de Rencontres Mathématiques, Luminy, France.
30 July–9 August, Invited speaker, *Special structures in differential geometry*, London Mathematical Society Durham Symposium.
- 2002** 10–11 September, Invited colloquium and seminar, University of Lund, Sweden.
- 2003** April, *Professeur invité* for 1 month, Université Libre de Bruxelles, Belgium.
July, *Professeur invité* for 1 month, Université de Bretagne Occidentale, Brest.
- 2004** 15–17 April, Invited speaker, *International Workshop on Global Analysis*, Çankaya University, Ankara, Turkey.
13–15 May, Invited special session speaker, *Joint meeting of the American and Mexican Mathematical Societies*, University of Houston, Texas, USA.

- 30 August–3 September, Invited speaker, *International conference on differential geometry and its applications*, Charles University, Prague, Czech Republic.
- 2005** 8–12 January, Invited main speaker, *UK–Japan Winter School*, Evesham, G.B.
4–7 April, Invited ‘morning speaker’, *British Mathematical Colloquium*, University of Liverpool, G.B.
29–31 August, Invited plenary speaker, *5th conference of the Balkan Society of Geometers*, Mangalia, Romania.
31 August–2 September, Invited sectional speaker, *Differential Geometry and Physics*, Eötvös Loránd University, Budapest, Hungary.
5–11 September, Invited plenary speaker, *Seventh International Conference on Differential Geometry and its Applications*, Deva, Romania.
14–17 September, Invited plenary speaker, *Symmetry in geometry and physics. In Honour of Dmitri Alekseevsky*, University of Rome 1, Italy.
- 2006** 11–21 August, Invited participant, *London Mathematical Society Durham Symposium: Methods of Integrable Systems in Geometry*, University of Durham.
21 August–1 September, Invited visitor, University of Lund, Sweden.
11–16 September, Invited speaker, *XV International Workshop on Geometry and Physics*, Puerto de la Cruz, Tenerife, Canary Islands.
- 2007** 24–25 May, Invited plenary speaker, *Irish Geometry Conference*, National University of Ireland, Galway.
15–28 July, Invited visitor, University of Campinas, Brazil.
29 July–3 August, Invited speaker, *26^o Colóquio Brasileiro de Matemática*, IMPA, Rio de Janeiro, Brazil.
16–23 September, Invited visitor, University of Southern Denmark, Odense.
- 2008** 27 June, Invited speaker, *Geometry Day in Memory of A. Sanini*, Università di Torino, Italy.
30 June–5 July, Main speaker, *Conformal Geometry*, CNRS, Roscoff, France.
1 Sept.–31 Oct. Invited visitor, University of Southern Denmark, Odense.
1 Nov.–13 Dec. Invited visitor, Università di Cagliari, Italy.
15–20 December, Main speaker, *The 16th Osaka City University International Academic Symposium “Riemann Surfaces, Harmonic Maps and Visualization”* Osaka City University, Japan.
- 2009** 27 June–11 July, Invited visitor, Université de Bretagne Occidentale, Brest, France.
7–10 September, Birthday boy and plenary speaker, *A Harmonic Map Fest in honour of Prof. John C. Wood on the occasion of his 60th birthday*, Università di Cagliari, Italy.
- 2010** 4–14 January, Invited visitor, Mathematics, and invited speaker, *CP3 origins*, University of Southern Denmark, Odense, Denmark.
video of talk at <http://cp3-origins.dk/a/1280>.
20–22 June, Invited speaker, *Durham Conference on Geometry and Topology in Honour of John Bolton and Cherry Kearton*.
1–7 November, Invited speaker, VI International Conference on *Finsler Extensions of Relativity Theory*, Bauman University, Moscow.
19 October, Invited speaker, King’s College, University of London.
- 2011** 28 February, Invited seminar, University of Leicester.
- 2012** 3–8 April, Invited speaker, *Differential Geometry*, Będlewo, Poland.

- 1–7 July, Invited speaker, *Geometric Structures on Manifolds and their Applications*, Rauischholzhausen, Germany.
- 6 December, Invited speaker, *Recent Understanding in Harmonic Maps*, Graduate Center, City University, New York.
- 2013** 27 January–8 February and 19–23 August, Invited visitor, University of Southern Denmark, Odense
- 2014** 11 February, Invited talk, Universidad Autonoma, Madrid.
31 March–11 April, LMS-funded visit to the University of Lisbon to research with Bruno Simões and Maria João Ferreira.
14 May, Closing speaker at *Differential Geometry Day*, Lund University, Sweden.
22–27 September, Invited visitor and colloquium speaker, Lund University, Sweden.
- 2015** 26–30 January, Invited visitor, Lund University, Sweden.
10–12 June, Opening speaker, *Differential Geometry Workshop*, University of Cagliari, Italy.
1–11 September, Invited visitor, University of Lisbon.
23–28 November, Invited visitor and colloquium speaker, Lund University, Sweden.
- 2016** 27 June–1 July, Invited visitor and colloquium speaker, University of Beira Interior, Covilhã, Portugal.
2–8 July, Invited visitor, University of Lisbon.
7–9 September, Opening speaker, *Quaternionic Differential Geometry and its Related Topics*, Ochanomizu University, Tokyo, Japan.

Conferences organized

- 1992** 11–15 May (with A. Fordy) ,
International Workshop on Harmonic Maps and Integrable Systems, University of Leeds (43 participants from 10 countries).
- 1994** 28 July–1 August (with A. West and S. Carter) ,
International Workshop on Harmonic Maps and Curvature Properties of Submanifolds, University of Leeds (over 80 participants from all over the world).
- 1997** 7–11 July, On Scientific Committee for the first international conference on
Harmonic Morphisms, Harmonic Maps and Related Topics, Université de Bretagne Occidentale, Brest, France.
- 2000** 11–14 April (with S. Carter and J.M. Speight)
International Workshop on Harmonic Maps and Curvature Properties of Submanifolds, 2, University of Leeds.
- 2001** 28 May–1 June, On scientific Committee for *Harmonic morphisms and harmonic maps*, Centre International de Rencontres Mathématiques, Luminy, France.
- 2008** 6–13 Jan, On scientific Committee for *Winter School in Geometry and Theoretical Physics*, University of Southern Denmark, Odense.
25–28 March, Co-organizer, special session on Differential Geometry and Geometric Analysis, *British Mathematical Colloquium*, York.
- 1977–2007** Organized one or two *Differential Geometry Days* at Leeds every year. These have frequently had speakers from abroad and have attracted participants from all over Britain. These developed into *Yorkshire and Durham Geometry Days*, held in turn in Leeds, York and Durham, attracting funding from the London Mathematical Society.

Principal Research Grants

- 1989–92** Part of EC Science Plan Twinning arrangement with 12 European Universities.
1993–97 Scientist in charge of Leeds “laboratory” for an EC Human Capital and Mobility network.
1999 LMS grant for conference at Leeds, April 2000: 4000 GBP.
2000 EPSRC research grant, May 2001 for 24 months, 81,387 GBP to employ Research Assistant (Radu Pantilie).

Teaching

I have taught undergraduate courses for Mathematicians and for scientists at all levels, including analysis, algebraic topology and differential geometry and an MSc course on *Infinite dimensional Manifolds*. In 1991, I gave the course on statistics to first year Biology students and a course for the (post Msc) *Diplome d'études approfondies* in Mathematics called *Applications harmoniques entres variétés riemanniennes*, at the Université de Paris-Sud, Orsay, France (both in French). I gave a short course on *Harmonic Maps* at the University of Lisbon in 2008.

Research

My main interests have been in *harmonic maps and morphisms*.

1. Harmonic maps

Harmonic maps are mappings between Riemannian manifolds which extremize the energy functional, a natural generalization of the Dirichlet integral. They include *geodesics* (paths of shortest distance such as great circles on a sphere), *minimal surfaces* (soap films) and *non-linear sigma models* in the physics of elementary particles. They also have applications to the theory of liquid crystals (see below) and robotics (see, for example [Y-J Dai, M. Shoji, H. Urakawa, *Harmonic maps into Lie groups and homogeneous spaces*, Differential Geom. Appl. 7 (1997), 143–160]).

In [5], I proved with J. Eells that *there is no harmonic map from the torus to the sphere of degree one*; this was the first non-existence result for harmonic maps, I proved the first such result in dimension more than two for the Dirichlet problem in [18], then generalized with H. Karcher in [19]. This is of interest in the theory of liquid crystals, see [K.S. Chou and X.P. Zhu, *Some constancy results for nematic liquid crystals and harmonic maps*, Ann. Inst. H. Poincaré — Anal. Non Linéaire, 12 (1995), 99–115], for other applications, see [J.F. Escobar, A. Freire and M. Minoo, *L^2 vanishing theorems in positive curvature*, Indiana Univ. Math. J., 42 (1993), 1545–1554] and [J.F. Grotowski, Y. Shen and S. Yan, *On various classes of harmonic maps*, Arch. Math. (Basel), 64 (1995), 353–358].

In [15] following work of Glaser–Stora and Din–Zakrzewski, I gave with Eells the classification of harmonic maps from the 2-sphere to complex projective space; for maps into certain other symmetric spaces, see [16,22,24,25,27,28,31]. These constructions have developed into a *twistor theory*, see F.E. Burstall and

J.H. Rawnsley, *Twistor theory for Riemannian symmetric spaces. With applications to harmonic maps of Riemann surfaces*. Lecture Notes in Mathematics, 1424. Springer-Verlag, Berlin, 1990], and into *integrable systems methods* for harmonic maps, see [42]. This is the non-linear sigma-model of elementary particle physics. For some applications, see [G. Dunne, *Chern–Simons solitons, Toda theories and the chiral model*, Comm. Math. Phys., 150 (1992), 519–535], [B. Piette and W.J. Zakrzewski, *Properties of classical solutions of the $U(N)$ chiral sigma-models in two dimensions*, Nuclear Phys. B 300 (1988), 223–237].

In [45] I described with L. Lemaire (Université Libre de Bruxelles) the space of harmonic 2-spheres in the complex projective plane \mathbf{CP}^2 as a smooth submanifold of the space of all C^k maps ($k \geq 2$). It is important to understand the *infinitesimal deformations* (*‘Jacobi fields’*) along harmonic maps between given Riemannian manifolds M and N , especially the question of whether all Jacobi fields are *integrable*, i.e. do they arise from variations through harmonic maps. An affirmative answer implies that the space of harmonic maps from M to N is a manifold with the Jacobi fields as tangent spaces; the case of harmonic 2-spheres is particularly important as it also has a bearing on the structure of the singular set of such maps from *any* manifold to N . We have just shown [70] that the case of harmonic 2-spheres in S^3 or S^4 is different, with not all Jacobi fields integrable.

My more recent work has been to revisit some constructions of harmonic maps from surfaces which were inspired by the seminal work of K. Uhlenbeck [*Harmonic maps into Lie groups: classical solutions of the chiral model*, J. Differential Geom. 30 (1989) 1–50]. In [72], with M.J. Ferreira and B.A. Simões, I found completely explicit formulae for the uniton factorization due to G. Segal; in [73], with M. Svensson, I showed how to do that for any uniton factorization and extended the method to harmonic maps into the classical Lie groups and symmetric spaces. In [76], we characterized when a harmonic map into a compact classical symmetric space has a twistor lift and gave an explicit geometrical construction of that lift; in [77] we extended this work to the exceptional symmetric space $G_2/\mathrm{SO}(4)$, finding twistor lifts into its three twistor spaces, one of them the quaternionic twistor space.

At present, I am working with Ferreira and Simões on explicit algebraic constructions of harmonic maps of finite uniton number into the orthogonal group, a subject which relates to the free Weierstrass representation of a minimal surface. Also, with A. Aleman (Lund), I have developed a simple criterion for finiteness of the uniton number which relates to the existence of closed circuits.

2. Harmonic morphisms

Harmonic morphisms are mappings of Riemannian manifolds which preserve solutions of Laplace’s equation; elementary examples are conformal transformations of the complex plane. In stochastic processes, harmonic morphisms are Brownian path-preserving transformations, see, for example [A. Bernard, E.A. Campbell and A.M. Davie, *Brownian motion and generalized analytic and inner functions*, Ann. Inst. Fourier (Grenoble) 29 (1979), 207–228]. Harmonic morphisms can be characterized as harmonic maps which satisfy an additional

condition called ‘horizontally weakly conformality’ or ‘semiconformality’, which is dual to the condition of weak conformality. I recently completed with P. Baird (Brest) the first account in book form [62] of this subject; this has rapidly become the standard text.

In [26] and [30], Baird and I constructed all harmonic morphisms from S^3 , \mathbf{R}^3 and H^3 to surfaces; for a re-proof of the \mathbf{R}^3 case, which is apparently simpler to probabilists, see [F. Duheille, *Une preuve probabiliste élémentaire d’un résultat de P. Baird et J.C. Wood*, Ann. Inst. H. Poincaré Probab. Statist. 33 (1997), 283–291]. For other 3-manifolds see [32].

In [37] for maps from an Einstein 4-manifold to a surface, I showed that the existence of a harmonic morphism implies the existence of a Hermitian structure and conversely; see [V. Apostolov and P. Gauduchon, *The Riemannian Goldberg-Sachs theorem*, Internat. J. Math. 8 (1997), 421–439] for related work. In the semi-Riemannian case, harmonic morphisms determine *shear-free ray congruences*, of interest in mathematical physics, see [49].

In [*Harmonic morphisms with fibers of dimension one*, Comm. Anal. Geom. 8 (2000), 219–265], R. Bryant showed that there are just two types of harmonic morphism with one-dimensional fibres from a space form of dimension four or more, the second type being induced from Killing fields. In [60], R. Pantilie and I extended Bryant’s result to Einstein manifolds of dimension 5 or more. We gave applications to finding new Einstein metrics in [61]. Work continues to understand harmonic morphisms from self-dual 4-manifolds, where a further type appears [63]. All the new types of harmonic morphisms satisfy a *monopole-type equation*.

In [75], Baird and I unified known some constructions of harmonic morphisms from 3-dimensional Euclidean or pseudo-Euclidean spaces by using the bicomplex numbers. This should help us to see more links between objects of interest to mathematical physicists such as shear-free ray congruences and geometric objects such as Hermitian structures.

My recent research has two strands. The first strand is to develop a general infinitesimal theory of harmonic maps and morphisms started in [59, 70]; as explained above, this will help us to understand *moduli spaces* of harmonic maps and morphisms as well as their behaviour at singularities. The first step is to start with cases where the harmonic maps are all constructed from holomorphic data by twistor theory, this includes harmonic 2-spheres in symmetric spaces. Because of the presence of branch points, these constructions are not, in general, smooth; and the idea is to understand the behaviour of the constructions at the branch points, and then relate the infinitesimal deformations of the harmonic maps to deformations in the space of holomorphic data. The next step is to understand those deformations by developing tools in the deformation theory for holomorphic maps.

The second strand is to understand harmonic maps from surfaces to symmetric spaces more explicitly. Svensson and I recently characterized when a harmonic map from a surface to a classical Riemannian symmetric space has a twistor lift [76] and are, at present, trying to extend that work to exceptional symmetric spaces, so far we have done that for $G_2/SO(4)$ [77].

Leadership

Whilst Head of Department, I was on the *School of Mathematics Management Committee* (2003–06). I am currently on the Head of Department's *Pure Mathematics advisory group* as the leader of the Differential Geometry Group.

I was the Pure Mathematics leader for the *Research Assessment Exercise (2008)*, responsible for the final submission in November 2007.

I have been on the following University committees: *University Library Consultative Subcommittee* (1985–87), *School of Mathematics Executive Committee* (1991–94), *Examinations Group of the Graduate Board of the University* (2001–03), *Peer Review of Teaching Working Group* (2011–): together with the Director for Undergraduate Studies, I have been running a pilot in Mathematics of the Teaching Enhancement Scheme devised by this committee.

For three years I was the *Quality Enhancement Officer* for the School of Mathematics and thus on the *Taught Student Education Committee* and the *Examinations Monitoring Group*.

I am regularly asked to sit on university appointment and promotion committees, often in other faculties.

Research Students Supervised

1980–1984 S. Erdem, PhD awarded 1984
1984–1988 A. Bahy-El-Dien, PhD awarded 1988
1988–1992 V. Parmar, PhD awarded 1992
1988–1992 S. Gudmundsson, PhD awarded 1992
1989–1992 T. Wheldon (left due to ill health)
1992–1995 M.T. Mustafa, PhD awarded 1995
1992–1996 E. Loubeau, PhD awarded 1996
1993–1997 S. Montaldo, PhD awarded 1997
1997–2000 R. Pantilie, PhD awarded 2000
1998–2002 A. Pambira, PhD awarded 2003
2003–2007 B. Simões, PhD awarded 2008
2015– J. Oliver

Other activities

External examiner for over 16 PhD theses including being on the jury twice at the Université de Paris-Sud, Orsay, France, twice at the Université de Bretagne Occidentale, Brest, once at Université de François Rabelais, Tours (as President of the Jury), and once at the Università di Cagliari, Italy.

I have been the Pure Mathematics External Examiner for the mathematics degree courses at the following universities:

1998–2002 University of Oxford.
2001–2004 University of York.
2003–2006 University of Durham.
2010–2014 University of Bath.

On editorial board of *Beiträge zur Algebra und Geometrie. Contributions to Algebra and Geometry*.

Referee for many journals and grant awarding bodies, including the EPSRC, the National Science Foundation (USA) and the Hong Kong Research Grants Council.

About one review a month written for *Mathematical Reviews* and several reviews for the London Mathematical Society.

Languages

English (native), French (reasonable), German (could be revived), Italian (comes and goes), Portuguese (improving), Spanish (learning), Mathematical Russian (with difficulty, when the need arises).