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Title: The use of decomposition in the study of graph classes defined by excluding induced subgraphs

Abstract:
We consider finite and simple graphs. We say that a graph G *contains* a graph F, if F is isomorphic to an induced subgraph of G. A graph G is F*-free* if it does not contain F. Let $\mathcal{F}$ be a (possibly infinite) family of graphs. A graph G is $\mathcal{F}$*-free* if it is F-free, for every $F \in \mathcal{F}$.

Many interesting classes of graphs can be characterized as being $\mathcal{F}$-free for some family $\mathcal{F}$. Most famous such example is the class of perfect graphs. A graph G is *perfect* if for every induced subgraph H of G, $\chi(H) = \omega(H)$, where $\chi(H)$ denotes the chromatic number of H and $\omega(H)$ denotes the size of a largest clique in H. The famous Strong Perfect Graph Theorem states that a graph is perfect if and only if it does not contain an odd hole nor an odd antihole (where a *hole* is a chordless cycle of length at least four, it is *odd* if it contains an odd number of nodes, and an *antihole* is a complement of a hole).

In the last 15 years a number of other classes of graphs defined by excluding a family of induced subgraphs have been studied, perhaps originally motivated by the study of perfect graphs. The kinds of questions this line of research was focused on were whether excluding induced subgraphs affects the global structure of the particular class in a way that can be exploited for putting bounds on parameters such as χ and ω, constructing optimization algorithms (problems such as finding the size of a largest clique or a minimum coloring), recognition algorithms and explicit construction of all graphs belonging to the particular class. A number of these questions were answered by obtaining a structural characterization of a class through their decomposition (as was the case with the proof of the Strong Perfect Graph Theorem). In this talk we survey some of the key results in this area.