Interpreting True Arithmetic in the Δ_2^0-Enumeration Degrees

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When working with a mathematical structure, it is natural to ask how complicated the first order theory of the structure is. In the case of structures that can be interpreted in first order arithmetic, this is equivalent to asking if the theory is as complex as possible, namely is it as complex as first order arithmetic. Restricting our attention to the language of partial orders, $\leq$, there have been many results in this direction. For example, it has been shown that the first order theory of both the Turing degrees (Simpson, 1977) and the enumeration degrees (Slaman, Woodin, 1997) are as complex as second order arithmetic. In addition, it is known that the theory of each of the following structures is as complex as the theory true arithmetic ($Th(\mathbb{N}, +, \times)$): Δ_2^0-Turing degrees (Shore, 1981), c.e.-Turing degrees (Nies, Shore, Slaman, 1998), m-degrees of c.e. sets (Nies, 1994), truth table c.e. degrees (Nies, Shore, 1995), and weak truth table c.e. degrees (Nies, 2001). The same result holds for the lattice of c.e. sets under inclusion (Harrington, Nies, 1998).

In this talk, we outline a proof that adds the Δ_2^0-enumeration degrees to this already lengthy list, and discuss what needs to be done to show the same result for the Σ_2^0-enumeration degrees.