

Interpolation in Abstract Model Theory

Jouko Väänänen
University of Amsterdam

One of the roles of logic is to serve as a tool for the study of structures. The best known tool, first order logic, cannot distinguish between cardinalities of infinite models. There are many extensions of first order logic where such and other sharper distinctions are possible. The most notable ones are the infinitary logics, axiomatizable finitary logics with generalized quantifiers, and higher order logics. There are also intermediate logics which do not fit well into these three categories, such as the equicardinality quantifier “there are as many x with $\phi(x)$ as there are y with $\psi(y)$ ”. It is an example of a *strong* logic, that is, a logic which has enough power to express properties of not only this or that model, but of the underlying set theoretical universe. The opposite is an *absolute* logic, that is, a logic the truth definition of which makes no reference to what kind of set there exists in the underlying universe.

I will describe the inherent difficulties in attempts to extend the Craig Interpolation Theorem to extensions of first order logic, and what can be done to overcome these difficulties.