

MID-TERM REVIEW REPORT

IHP Research Training Network title : Classical Analysis, Operator Theory,
Geometry of Banach spaces, their Interplay and their Applications

Network short title : Analysis and Operators
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Network co-ordinator : Professor Jean ESTERLE
Organisation : Université Bordeaux 1
Address : Laboratoire de Mathématiques Pures
Université Bordeaux 1
351, Cours de la Libération
33405-Talence (France)
Telephone : 33 556 84 6119
Telefax : 33 556 84 69 59
E-mail : esterle @ math.u-bordeaux.fr

Network home page : <http://maths.leeds.ac.uk/pure/analysis/rtn.html>

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Date and timing of the meeting : November 20, 2002, 8h30-18h.

Part A - Research Results

A1 . Scientific Highlights

During the first 27 months of network activity the network made significant progress with respect to all tasks. Some truly outstanding results have been obtained by team 3, which resolved long standing questions related to task 2. Also for other tasks results of high international level have been achieved. Here below these achievements are described in some detail

Task 1(Hardy and Bergman spaces, interpolation)

Teams 3 and 9 obtained [1] a characterization of interpolation sequences for the Bloch space, i.e. the space of analytic functions in the disc with bounded hyperbolic derivatives. They also obtained [5] a description of the Fourier frames of Duffin and Schaeffer, based on de Branges' theory of Hilbert spaces of entire functions. Using harmonic measure, teams 1 and 9 constructed noncyclic functions without zeroes in the disc for a very large class of weighted Bergman spaces [2].

Team 1 and team 7 [8] studied extremal functions as divisors for kernel of Toeplitz operators, with applications to the characterization of certain complete interpolating sequences (Milestone 2, task 1). Team 3 obtained a characterization of interpolating and sampling sequences for some spaces of entire functions.

Task 2(Cauchy integrals, Capacities, Harmonic approximation)

Team 3 solved a long standing problem by giving a complete characterization of planar Cantor sets of zero analytic capacity. Soon after that X.Tolsa, also from team 3, obtained outstanding further results concerning analytic capacity, including the subadditivity of analytic capacity and the characterization of the compact sets in the plane with zero analytic capacity (Painlevé's problem).

Team 3 and 4 [13] obtained new results concerning the representation of functions on the boundary of a domain Ω by sums of Poisson, Green or Martin kernels associated to a subset E of Ω (this relates to milestones 1 and 2 of task 2). Using harmonic approximation techniques and fine potential theory, team 4 identified the boundary sets where general harmonic functions may tend to infinity, and the Farrel and Mergelyan sets of the Dirichlet space were classified by team 7.

Task 3(Function models and applications of operator theory)

Teams 2 and 8 obtained [26] new results about the existence of a pair of solutions of an algebraic Riccati equation in the infinite dimensional case, using an invariant subspace approach. Teams 8 and 9 obtained [25] seminal results about the quadratic numerical range, including numerical codes to compute it. An analogue of the Sz-Nagy-Foias model for pairs of commuting operators on the Hilbert space having the symmetrized bidisc as a spectral set was constructed by team 5. Constrained von Neumann inequalities [18] were obtained by teams 1 and 6, with very interesting applications to Fourier series.

Team 2 contributed to a new version of the commutant lifting theorem of Nagy and Foias, with applications to interpolations problems in which the usual norm constraint is replaced by a relaxed version. Team 2 and 9 developed new finite section methods for inversion of differential and difference equations [20], [23], applied the

state space method to spectral and scattering problems for canonical differential systems [21], [22] and studied completeness problems for operators arising from delay and other functional differential equations. Teams 2 and 8 obtained new results about the generalized Schur algorithm using the theory of reproducing kernel Pontryagin spaces [14], [15], [16] and were able to establish a spectral theory of general matrix operators which unifies the study of elliptic eigenvalue problems with λ -depending boundary conditions, of delay differential equations and of block operator matrices from magnetohydrodynamics.

Task 4(The invariant subspace problem)

Teams 6 and 9 [30] showed by using smooth variational principles that a large class of operators on separable real Banach spaces have a nontrivial invariant subspace, which gives the existence of nontrivial translation invariant subspaces of $l_p^{\omega}(\mathbb{Z})$ for all symmetric weights ω and all $p \geq 1$ (the case $p > 1$ had been obtained previously by team 9). Jointly invariant subspaces for semigroups of translations and dilations were obtained by team 5.

Team 9 showed that a reflexive translation invariant space on an arbitrary locally compact abelian group which is symmetric, self adjoint and satisfies some natural conditions has a proper translation invariant subspace, and obtained new information on the invariant subspace structure for shifts and backward shifts of a wide class of Banach spaces of analytic functions in the unit disc. It was also shown in team 9 that the shift on l^1 has invariant subspaces of arbitrary multiplicity. Teams 5 and 6 obtained new results on the structure of invariant subspaces for isometric composition operators on $H^2(\mathbb{D})$ and $H^2(\mathbb{C}^+)$ [34], and gave constructive examples showing that the classes $\mathcal{A}_{m,n}$ of the Brown-Chevreaux-Pearcy theory are distinct [33]. Team 1 and 6, using the theory of asymptotically holomorphic functions, gave a complete classification of the translation invariant subspaces for a large class of dissymmetric weighted Hilbert spaces of sequences on \mathbb{Z} [40] and gave conditions on a weight ω which ensure that every nontrivial translation invariant subspace of the corresponding weighted Hilbert space contains a nonzero sequence $(u_n)_{n \in \mathbb{Z}}$ such that $u_n = 0$ for $n > 0$.

Task 5(Geometry of Banach Spaces and applications)

A young mathematician from team 6 showed that the classical Hardy space H^1 , which has as is well-known an unconditional basis, fails to have a *completely* unconditional one. Team 6 also showed that a large class of noncommutative L^p -spaces have a basis.

Extension results for operators defined on reflexive subspaces of L^1 or L^1/H^1 were obtained in team 10 using Interpolation theory. Team 10 also obtained compactness and Schatten class criteria for the differentiation operator acting on a star invariant subspace of the Hardy space H^2 , and team 1 showed that James' theorem fails for starlike bodies.

Mathematicians from Team 6 obtained a version of Grothendieck's theorem for operator spaces, and constructed an Hilbertian operator space for which every completely bounded endomorphism is the sum of a compact operator and a scalar multiple of the identity. It was also shown that under usual spectral conditions the sum of two operators having a bounded McIntosh H^∞ -calculus on a AUMD space also has a bounded McIntosh H^∞ -calculus (the AUMD class contains the L^1 -spaces and the UMD-spaces, and this question was open even for UMD spaces). Concerning interpolation for Hilbert spaces, it was shown in team 7 that Hilbert couples have the so-called Calderon property with constant 1. Orlicz norms estimates for eigenvalues of matrices were obtained in team 5, and teams 5 and 6 obtained in [41] applications to approximation of interpolation results between Hardy spaces on circular domains.

Task 6(Convex geometry,concentration of measures)

Convexity methods were used in teams 5 and 6 to obtain new results on the concentration of measures for Random matrices. A successful new approach [43] of duality for metric entropy of operators or convex sets was developed by teams 6 and 9. Functional geometric inequalities coming from Convexity theory (inequality of Brunn-Minkowski, theory of mixed volumes, inequalities of Young and Brascamp-Lieb) were used in team 6 to obtain new transportation and Log-Sobolev type inequalities. A young Mathematician from team 5 obtained results concerning almost sure weak convergence and concentration for the circular ensembles of Dyson. Euclidean structures on finite dimensional normed spaces were studied in team 9, and team 6 obtained a geometric localization theorem related to extreme s-concave probabilities.

A2.Joint publications and patents

Young Researchers are indicated using bold.

Task 1(Hardy and Bergman spaces,interpolation)

[1] **B.Boe**(team 7) and A.Nicolau(team 3), *Interpolation by functions in the Bloch space*, preprint(<http://www.mat.uab.es>).

[2] A.Borichev(team 1) , H.Hedenmalm(team 7) and A.Volberg, *Large Bergman Spaces : inversibility , cyclicity , and subspaces of high index* , preprint , 41 pp.
(H.Hedenmalm:<http://www.maths.lth.se/matematiklu/personal/haakan/bibliography.html>).

[3] A.Borichev(team 1) and M.Sodin(team 9), *Krein's entire functions and the Bernstein approximation problem* ,Illinois Journal of Mathematics 45 (2001), 167-185.

[4]**I.Chalendar**(team 6) and R.Mortini(team 1), *When do finite Blaschke products commute*, Bull. Austral. Math. Soc. 64 (2001), 189-200.

[5]**J.Ortega-Cerdà**(team 3) and K.Seip(team 9), *On Fourier frames*, Annals of Math. 155 (2002), 789-806.

[6] **A.Hartmann**(team 1) and X.Massaneda(team 3), *Interpolating sequences for functions of restricted growth*, Ill. J. Math., to appear.

[7] **A.Hartmann**(team 1), X.Massaneda(team 3) and A.Nicolau(team 3), *Interpolation in the Nevanlinna and Smirnov class*, preprint.

[8] **A.Hartmann**(team 1) and K.Seip(team 7), *Extremal functions as divisors for kernels of Toeplitz operators* 15 pp., preprint, (<http://www.math.ntnu.no/~seip/recentpapers.html>).

[9] R.Mortini(team 1) and A.Nicolau(team 3), *Frostman shifts of inner functions*, preprint.

[10] F.Nazarov, M.Sodin(team 9) and A.Volberg(team 6), *Local dimension-free estimates for volumes of sublevel sets of analytic functions*, Israel J. Math., to appear.

Task 2(Cauchy integrals , Capacities , Harmonic approximation)

[11] S.J.Gardiner(team 4) and **J.Pau**(team 3), *Approximation on the boundary and sets of determination for harmonic functions*, preprint.

[12] **I.Chalendar**(team 6), J.Lebond and J.R.Partington(team 5), *Approximation problems in some holomorphic spaces , with applications* , in A.A. Borichev and N.K. Nikolski: Systems, Approximation, Singular Integral Operators and Related Topics, Proceedings of IWOTA 2000, Birkhauser (2001), 143-168.

Task 3(Function models and applications of operator theory)

[13] G.R.Allan(team 5), G.Kakiko, A.G.O'Farrell(team 4) and R.O.Watson(team 4), *Localness in $A(\Psi)$ -algebras*, Proc. Royal Irish Acad. 101 (2001), 61-70.

[14] D.Alpay, T.Y.Azizov, A.Dijkmsa(team 2) and H.Langer(team 8), *The Schur algorithm for generalised Schur functions I : coisometric realizations*, Operator Theory : Advances and Applications 129 (2001), 1-36.

[15] D.Alpay, T.Y.Azizov,A Dijkmsa(team 2) and H.Langer(team 8), *The Schur algorithm for generalised Schur functions II : Jordan chains and transformations of characteristic functions*, preprint.

[?] D.Alpay, T.Y.Azizov,A Dijkmsa(team 2) and H.Langer(team 8), *The Schur algorithm for generalised Schur functions III : J -unitary matrix polynomials on the circle*, preprint.

[17] D.Alpay, A.Dijkmsa(team 2) and H.Langer(team 8) , *On the Loewner problem in the class N_κ* , Proc.Amer. Math.Soc.130 (2002), 2057-2066.

[18] **C. Badea**(team 1) and G. Cassier(team 6), *Constrained von Neumann inequalities*, Adv. Math. 166 (2002), 260-297.

[19] A.Dijkmsa (team2), P.Binding, R.Hryniv, **A. Batkai** (team 8), and H.Langer (team 8), *Spectral properties of block operator matrices* (submitted).

[20] I.Gohberg(team 9), M.A.Kaashoek(team 2) and F.van Schagen(team 2), *Finite section method for difference equations*, Operator Theory : Advances and Applications 130 (2001), 197-207.

[21] I.Gohberg(team 9), M.A.Kaashoek(team 2) and A.L.Sakhnovich, *Bound states of a canonical system with a pseudo-exponential potential*, Int. Eq. and Op. Th. 40 (2001), 268-277.

[22] I.Gohberg(team 9), M.A.Kaashoek(team 2) and A.L.Sakhnovich, *Scattering problems for a canonical system with a pseudo-exponential potential*, Asymp. An. 29 (2002), 1-38.

[23] I.Gohberg(team 9), M.A.Kaashoek(team 2) and F.van Schagen(team 2), *Finite section method for linear ordinary differential equations on the full line*, to appear in H.Dym Anniversary Volume.

[24] **S.Kupin**(team 1) and V.Vasyunin(team 10), *Criteria for similarity of a dissipative integral operator to a normal operator*, Algebra i Analiz 13 (2001).

[25] H.Langer(team 8), A.Markus, V.Matsaev(team 9) and C.Tretter(team 8), *A new concept for block operator matrices: The quadratic numerical range*, Linear Algebra Appl. 330 (2001) 89 -112.

[26] H.Langer(team 8), ACM Ran(team 2) and **B. van de Rotten**(team 2), *Invariant subspaces of infinite dimensional Hamiltonians and solutions of the corresponding Riccati equations*, Operator Theory : Advances and Applications 130 (2001), 235-254.

[27] H.Langer(team 8), H.de Snoo(team 2) and V.A.Yavrian, *A relation for the spectral shift function of two self-adjoint extensions*, Operator Theory : Advances and Applications 127 (2001), 437-455.

[28] H.Langer(team 8), A.Markus, V.Matsaev(team 9) and C.Tretter(team 8), *Self-adjoint block operator matrices with non-separated diagonal entries and their Schur complement*, J. Func. An., to appear

Task 4(The invariant subspace problem)

[29] E.Abakoumov(team 6) and A.Borichev(team 1), *Shift invariant subspaces in weighted l^p spaces*, J. Func. An. 188 (2002), 1-26.

[30] A.Atzmon(team 9) and G.Godefroy(team 6), *An application of the smooth variational principle to the existence of nontrivial invariant subspaces*, C.R. Acad. Sci. Paris Sér.I 332 (2001), 151-156.

[31] A.Atzmon(team 9), G.Godefroy(team 6) and N.Kalton, *Invariant subspaces and the exponential map*, preprint.

[32] C.Benhida(team 1) and D.Timotin(team 6), *Notes on dual algebras and perturbations of contractions*, Bulletin Romanian Math.Soc. 44 (2001), 193-198.

[33] G.Cassier(team 6), **I.Chalendar**(team 6) and B.Chevreau(team 1), *A mapping theorem for the boundary set X_T of a contraction*, preprint, 14 pp.

[34] **I.Chalendar**(team 6) and J.R.Partington(team 5), *L^1 Factorizations for some perturbations of the unilateral shift*, C.R. Acad. Sci. Paris Sér.1 Math. 332 (2001), 115-119 .

[35] **I.Chalendar**(team 6) and J.R.Partington(team 5), *Spectral density for multiplication operators with applications to factorization of L^1 -functions*, J. Op. Th., to appear.

[36] **I.Chalendar**(team 6) and J.R.Partington(team 5), *Constrained approximation and invariant subspaces*, preprint.

[37] **I.Chalendar**(team 6) and J.R.Partington(team 5), *On the structure of invariant subspaces for isometric composition operators on $H^2(\mathbb{D})$ and $H^2(\mathbb{C}^+)$* , Archiv.der Mat., to appear.

[38] **I.Chalendar**(team 6), J.R. Partington(team 5) and **M. Smith**(team 5), *Approximation in reflexive Banach spaces and applications to the invariant subspace problem*, preprint.

[39] J.Esterle(team 1) and A.Volberg(team 6), *Analytic left-invariant subspaces of weighted Hilbert spaces of sequences*, J. Op. Th. 45 (2001), 265-301.

[40] J.Esterle(team 1) and A.Volberg(team 6), *Asymptotically holomorphic functions and translation invariant subspaces of weighted Hilbert spaces of sequences*, Ann. Scient. Ec. Norm. Sup. 35 (2002), 185-230.

Task 5(Geometry of Banach Spaces and applications)

[41] **I.Chalendar**(team 6) and J.R.Partington(team 5) , *Interpolation between Hardy spaces on circular domains with applications to approximation*, Archiv Math. 78 (2002), 223-232.

Task 6(Convex geometry,concentration of measures)

[42] V.D.Milman(team 9) and A.Pajor(team 6) , *Entropy and asymptotic geometry of non-symmetric convex bodies* , Advances in Math. 152 (2000) , 314 -335.

[43]V.D.Milman(team 9) and S.J.Szarek(team 6) , *A geometric lemma and duality of metric entropy* C.R.Acad.Sci.Paris Sér.I 332 (2001), 157-162.

Part B -COMPARISON TO THE PROJECT PROGRAMME (Annex 1)

B1 . Project Objectives

The project objectives, as stated in Annex I of the contract, seem still realistic after two years of network activity. Progresses with respect to these objectives are described below.

In **Function Theory** The characterization of planar Cantor sets of zero analytic capacity in team 3 during the first year of network activity was a major progress in capacity theory. This was followed by a very spectacular breakthrough: the subadditivity of analytic capacity was established in the Summer 2001 by X.Tolsa, from team 3, who also solved other long standing questions concerning analytic capacities. These results go far beyond what was expected in the research objectives of the network concerning capacities. Besides these breakthroughs, analytic capacity was also successfully used by team 4 to characterize open subsets U (of the complex plane or of a Riemann surface) for which $A(U)$ is pervasive or real-pervasive.

A complete characterization of interpolating sequences was obtained by teams 3 and 7 for the Bloch space [1], and some necessary or sufficient conditions have been obtained in team 3 for Bergman spaces or related spaces in higher dimensions. Interpolating sequences for the Nevanlinna and Smirnov class, and for spaces of functions of restricted growth, have been successfully studied by teams 1 and 3 [6],[7] (characterization of interpolating and sampling sequences for classical function spaces was one of the research objectives of the network in function theory).

In Lund (team 7), there were some significant progress concerning z -invariant subspaces of the Bergman space (another important research objective of the network) obtained in relation to seemingly unrelated subjects, such as Hele-Shaw flow and biharmonic Green functions on hyperbolically curved surfaces.

Boundary sets where general harmonic functions may tend to infinity were identified by team 4, and the joint paper [11] between team 4 and team 3 is a significant contribution to approximation at the boundary.

The joint contribution [5] by teams 3 and 7 the network in function theory to the classical theory of Fourier frames of Duffin and Schaeffer is another indication of the pertinence of the research objectives of the network in function theory. Altogether there was some significant progress (outstanding concerning analytic capacities) on 70 % of the research objectives in function theory during the first 28 months of network activity.

New objectives: Results obtained at Lille (team 1) and Tel-Aviv (team 9) show that the network has the ability to contribute to the theory of thin sets in classical Harmonic Analysis and to the theory of Fourier series. Aspects of approximation theory more general than those described in the contract are successfully being developed by teams 5 and 6.

In **Operator Theory**, an important research objective of the network was to develop applications of operator theory and in particular to develop new operator theoretical methods to analyze problems arising from concrete classes of integro-differential and delay equations, and important results in this direction were obtained through collaboration between teams 2, 8 and 9 [20], [21], [22], [23]. Other important applications of operator theory were obtained in papers [14], [15], [16], [17], [26], [27] by teams 2 and 8, and in [28] by teams 8 and 9. The seminal paper [25] by teams 8 and 9 introduces the quadratic numerical range, a new powerful concept for operator block matrices. The development of the theory of function models was also an important research objective of the network. An analog of the Nagy-Foias function model for some pairs of commuting operators (see part A.1. for task 3) was obtained by team 5, and interesting results concerning the existence of

nontrivial compact or Schatten class operators in the commutant of a model operator have been obtained by a young mathematician from team 1. New criteria for similarity to a normal operator were obtained in paper [21] between teams 1 and 10, and promising new results about Schur multipliers were obtained in a joint work between two senior women from teams 1 and 10.

New applications of the Brown approximation scheme, another important research objective of the network, were obtained by teams 1 and 6 [33], and by teams 5 and 6 [34] for absolutely continuous contractions, and this scheme was also used in team 1 to obtain the hyperreflexivity of the bilateral weighted shift for a large class of irregular weights. There were other important results concerning translation invariant subspaces of the weighted spaces $l_\omega^2(Z)$ obtained by using the theory of asymptotically holomorphic functions by teams 1 and 6 [39], and by using real variable methods in team 9, were the existence of nontrivial translation invariant subspaces was proved for all symmetric weights (the initial proof worked for $p > 1$, but the use of variational methods [31] allowed teams 6 and 9 to extend it to the case $p=1$). The abundance of joint publications or preprints by members of different teams in this direction of research (tasks 3 and 4) is very encouraging.

New objectives: Significant progress was made on most research objectives in Operator theory during the first two years of network activity, and the main orientations of research in this area described in the contract remain valid. Applications of operator theory to Harmonic Analysis in the direction opened by [18] should be developed. Some work performed at Lille (team 1) shows that the network has the ability to contribute to moment problems, and there should be more emphasis on questions of similarity (see [24]) and on numerical ranges (see [25]).

Concerning **Geometry of Banach spaces and Convex Geometry**, the third main direction of research in the network, one important objective consisted in finding new applications of convexity theory. There was some significant progress in this direction by teams 6 and 9, with applications of various inequalities to methods of mass transport and problems of concentration of measure. Another objective consisted in finding new applications of variational principles, and teams 6 and 9 found an unexpected one to the invariant subspace problem [30]. The important results obtained in team 6 concerning Operator spaces in the second year of network activity meet the objective of "transferring the technology of Geometry of Banach spaces" to operator theory and function theory. There was also important progress in noncommutative Harmonic Analysis with values in \mathcal{M}_n or with values in $B(H)$ developed in teams 5 and 6, and significant progress on the Mc Intosh H^∞ -functional calculus was also reported from team 6. A senior Mathematician from team 6 obtained important results greatly improving the understanding of the Sherrington-Kirkpatrick model in Statistical Mechanics.

New objectives: A noncommutative version of the space BMO emerged from recent work in team 6 about noncommutative martingales. Full understanding of this new space (which is, roughly, the intersection of the space BMO -row with the space BMO -column) should be an important objective of the last two years of network activity. Also more network activity should concern the properties of the Mc Intosh H^∞ -functional calculus related to the Geometry of the underlying Banach space.

B.2 Research method and work plan

B.2.1. Contributions of the teams to the joint programme of work

The following table shows how the joint programme of work is broken into tasks, and for each task which team was supposed to coordinate the research, which teams were supposed to be involved in the task during the activity of the network and which teams did contribute during the first 28 months of network activity.

task n°	task title	coordinator	teams expected to contribute	contributing teams before 30/ 09 / 2002
1	Hardy and Bergman spaces, Interpolation	team 7	1, 3, 4, 7, 9, 10	1, 3, 7, 9, 10
2	Cauchy Integral, Capacities, Harmonic approximation	team 4	1, 3, 4, 7, 10	3, 4, 5, 6, 7
3	Function models and applications of Operator Theory	team 2	1, 2, 5, 7, 8, 9, 10	1, 2, 5, 6, 7, 8, 9, 10
4	The invariant subspace problem	team 9	1, 6, 7, 9	1, 6, 7, 9
5	Geometry of Banach spaces and applications	team 10	1, 5, 6, 7, 9, 10	1, 5, 6, 9, 10
6	Convex Geometry, Concentration of measure	team 6	6, 9	5, 6, 9

The table shows that, with very few exceptions, the various teams contributed to the tasks as expected in Annex 1 of the contract. Notice that in some cases more teams than expected did contribute to a task: teams 5 and 6 made a significant contribution to approximation [12](task 2), and G.Blower, a young mathematician from team 5, obtained interesting results about concentration of measure (task 6).

B.2.2 Methodological approach

The interplay between Function Theory, Operator Theory, Geometry of Banach spaces and Convex Geometry, and the research of application of these theories to other branches of Mathematics was the basis of the methodology described in Annex 1 of the Contract. This methodology has been working well during the first 28 months of network activity: tools as different as almost holomorphic functions, the Brown approximation scheme, variational principles and positive definite functions were used in teams 1, 2 and 9 to study translation invariant subspaces of weighted l^p spaces of sequences on \mathbb{Z} . The interplay between complex function theory and operator theory plays a central role in all the applications of operator theory developed by teams 2, 8 and 9, and the interplay between Operator Theory and Geometry of Banach spaces was the basis of the progress reported by team 6 concerning Operator Spaces and noncommutative Functional Analysis. The Geometric notion of *curvature of a measure* introduced some years ago by Melnikov and Verdera played a central role in the breakthroughs concerning analytic capacities performed in the summer 2001 in team 3, and probabilistic methods also played an important role in several results obtained in the network in function theory (team 3), harmonic analysis (team 1), problems of concentration of measure (teams 5, 6 and 9) and noncommutative Functional Analysis (team 6). At this stage of network activity it becomes possible to say more precisely that the interplay between Function Theory, Geometry, Operator Theory, Geometry of Banach spaces, Convex Geometry and Probabilistic methods is the basis of the network methodology.

B.2.3 Global progress of the work plan

It is of course impossible to predict accurately which significant Mathematical results will be obtained within a period of four years. For example there were rather unexpected outstanding breakthroughs about analytic capacities performed by team 3, while some problems appeared more difficult than expected. We will

discuss in detail below the progress with respect to the milestones mentioned in Annex 1 of the contract, but all in all, it seems fair to say that **from a global point of view, the network research progressed along the lines envisaged in the contract.**

B.3 Schedule and Milestones

As indicated above, the expected schedule was globally respected. Some milestones were pointed out in Annex 1 of the contract to serve as a sampling of the progress of the network workplan. We summarize in the following table the progress reported about these various milestones (the milestones have a 2-digit numeration, where the left number gives the task number: for example milestone 2.3 is the third milestone for task 2).

Progress concerning the Network Milestones			
Milestone number	Milestone title	Duration of research mentioned in Annex 1 of contract	Progress reported before 30/09/2002
1.1	Existence of zero-free noncyclic vectors for all weighted Hardy spaces in the log-convex case	48 months	Important progress reported in joint paper [2] by teams 1, 6, 7
1.2	Characterization of complete interpolating sequences in weighted Hilbert spaces of entire functions	24 months	Done by team 7 in important cases
1.3	Characterization of discrete random fields for which entire functions with zeroes in the field belong almost surely to the Fock space	24 months	No progress reported so far
2.1	Applications of capacities to approximation problems in spaces of analytic and harmonic functions	48 months	Decisive progress about capacities reported from team 3, important applications of capacities reported from team 4
2.2	Characterization of exceptional sets at the boundary for harmonic functions	24 months	Done by team 4
2.3	Computation of best constant inequalities for conjugate harmonic functions	24 months	No progress reported so far
3.1	Explicit computation of the characteristic function for new classes of contractions, and obtain resolvent tests for similarity to a normal operator	24 months	Tests for similarity obtained in team 1
3.2	Use of the band method to describe the set of all solutions for some Takagi type interpolation problems	48 months	Progress reported from teams 2 and 9
3.3	Use of the Agler-Young operator-theoretical method to make progress on the μ -synthesis problem in engineering	24 months	Progress reported from team 5
4.1	Reflexivity for pairs of commuting contractions with dominant Harte spectrum	24 months	Progress reported from team 1
4.2	Description of the lattice of invariant subspaces of the Bergman space counterpart of the Fourier binest algebra	48 months	No direct progress reported so far, but team 5 classified subspaces of $L^2(\mathbf{R})$ invariant by dilations and multiplication by functions in $H^\infty(\mathbf{R})$

Progress concerning the Network Milestones (continued)			
Milestone number	Milestone title	Duration of research mentioned in Annex 1 of contract	Progress reported before 30/09/2002
4.3	Complete description of translation biinvariant subspaces of $l_{\omega}^2(\mathbf{Z})$ for a large class of nonincreasing weights not bounded away from 0	24 months	Decisive existence results of translation biinvariant subspaces have been obtained by team 9 for even weights; the joint paper [40] gives results in this direction for weights equal to 1 on \mathbf{Z}^+
4.4	Reflexivity properties of the shift operator on $l_{\omega}^2(\mathbf{Z})$ in the case of thick spectrum	48 months	Partial results have been obtained in team 1; the general case still resists
5.1	Examples where Pisier's similarity satisfies $3 < d < +\infty$	48 months	No progress reported so far
5.2	Comparison between regularity and λ -regularity for sums of operators in U.M.D. spaces	24 months	No direct progress so far. Related results have been obtained in team 6
5.3	Formula for the Frechet subdifferential of the sum of two positive lower semicontinuous functions in smooth Banach spaces	24 months	Partial results have been obtained in team 1; the problem was then solved from outside the network
6.1	Improvement of the Kannan-Lovacz estimate $O(n^2)$ for the flatness of n -dimensional bodies	24 months	No progress reported so far; teams 6 and 9 obtained related results
6.2	Explicitation of a direct link between the Sherrington-Kirkpatrick model and questions related to the group of isometries of \mathbf{R}^n , n very large	24 months	Important results in this direction were obtained in team 6

B.4 Research effort of the participants

It is difficult at that stage to quantify precisely in person-months the research effort of the scientific staff from the different teams, and so the numbers given in column 2 of the table below are estimates. We give in bold the numbers corresponding to the first two years of network activity (for the third column it is the number of individuals for whom a concrete contribution to the research objectives of the network was reported), and we give between brackets what was indicated in Annex I of the contract for the four years of network activity. The contribution of predoctoral students is not quantified (it was not quantified either in Annex I of the contract).

Concerning young researchers appointed by the network **we give between brackets the number of person/months of appointments expected in the contract preparation forms during the first two years of network activity.** The appointment of young researchers will be discussed in more detail in section C.1 of the present report.

Participant	Young researchers financed by the contract (person month)		Researchers to be financed from other sources (person- month)		Researchers contributing to the project (number of individuals)	
1. UB1	10.5	[6] (36)	146	(294)	16	(24)
2. VUA	14	[18] (24)	92	(180)	9	(12)
3. UAB	18	[18] (36)	108	(176)	12	(16)
4. UCD	9	[6] (24)	80	(160)	8	(11)
5. ULeeds	8	[6] (24)	96	(192)	10	(14)
6. UPMC	26	[18] (36)	170	(312)	19	(31)
7. NTNU	9	[18] (24)	86	(160)	9	(12)
8. TU Vienna	9	[6] (24)	60	(120)	6	(8)
9. TAU	9	[12] (24)	60	(112)	7	(10)
10. POMI	0	[0] (0)	70	(112)	6	(8)
TOTAL	112.5	[108] (252)	968	(1818)	102	(134)

B.5 Networking activities

Networking activities were organised along four directions

- 1) Individual visits of participants of one node to participants from another node
- 2) Network participation to Conferences relevant to the network Research Plan, but not organised by the network
- 3) Network specific Annual Conferences and Workshops

We will describe these activities in the next subsections

B.5.1 Individual visits

The following table indicates the number of visits from one team (horizontal entries) to another (vertical entries). Twenty one of such visits, funded by the network, were reported between June 1, 2000, and September 30, 2002. The starred numbers correspond to the twelve other visits, directly related to network activities but funded from other sources, reported for the same period (this proportion of two visits funded by the network compared to one visit funded from other sources is not unusual, since the network was intended to stimulate already existing collaborations). The details about these individual visits can be found in the first two annual reports, available on the network homepage.

team N°	1	2	3	4	5	6	7	8	9	10
1	0	0	0	0	2	2	0	0	0	0
2	0	0	0	0	0	0	0	1 + 1*	0	0
3	1*	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0
5	1*	0	0	0	0	1 + 2*	0	0	0	0
6	1 + 1*	0	0	0	0	5 + 1*	0	0	0	0
7	1*	0	2	0	0	0	0	0	0	0
8	0	2*	0	0	0	0	0	0	0	0
9	1	0	1	0	0	1*	0	0	0	0
10	4 + 1*	0	1	0	2*	1*	0	0	0	0

B.5.2 Network participation to conferences not organised by the network

A significant amount of network activity was the participation of network members to Conferences not organised by the network, but relevant to the network research plan. The list of such Conferences was discussed at the meetings of network coordinators, with a few coorections between the meetings by email discussions. Priority was given to events organised in network nodes. The following two tables give a list of these Conferences.

1) Conferences not organized in network nodes

The following table gives a list of these Conferences, with the teams involved ¹

Conference Location	Country	Date of Conference	Conference Title	Teams involved (with network funding)	Teams involved (with funding from other sources)
Orléans	France	10/2000	Journées d'Analyse du GDR CNRS 2101	3 and 5	1 and 6 ²
Crete	Greece	08/2001	Convex Geometry Conference	6 and 9	
Odense	Denmark	08/2001	Balticon	1 and 6	
Umea	Sweedeen	06/2002	Analysis Conference	2 and 4	
Timisoara	Romania	06/2002	Operator Theory Conference	1, 5, 6 and 8	
Luminy	France	09/2002	Holomorphic Function Spaces and their Operators	1, 6 ³ and 7	1 and 6 ⁴

2) Conferences located in network nodes, but not organized by the network

The following table gives a list of these Conferences, with the teams involved

¹GDR CNRS 2101 is a French network, funded By the French National Research Center CNRS, to which most members of teams 1 and 6 participate. The coordinator of this French network is the coordinator of the network

Conference Location	Country	Date of Conference	Conference Title	Teams involved (with network funding)	Teams involved (with funding from other sources)
Bordeaux, team 1	France	06/2000	IWOTA	2, 3, 5, 7, 8 and 10	1 and 6 ⁵
Besancon, team 6	France	06/2000	Functional Analysis Summer School	5	1 and 6 ⁶
Saint Petersburg, team 10	Russia	08/2000	Analysis Conference	1	
Ambleside, team 5	England	09/2000	Operator Theory Conference	1 and 6	5
Amsterdam, team 2	Netherlands	10/2000	Operator Theory Conference	1 and 8	
Newcastle, team 5	England	06/2001	Conference in honor of B.E.Johnson	1 and 4	
Saint Petersburg, team 10	Russia	08/2001	Analysis Conference	1	
Lyon, team 6	France	10/2001	Journées d'Analyse GDR2101		1 and 6 ⁷
Belfast, team 4	United Kingdom	10/2001 Analysis Day	Belfast Functional	5	4
Saint Petersburg, team 10	Russia	08/2002	Analysis Conference	1	
Besancon, team 6	France	09/2002	Journées d'Analyse GDR2101		1 and 6 ⁸

B.5.3 Conferences and Workshops organized by the network.

1) Network workshops

A network workshop "Colloquium on Operator Theory" was held in Vienna on January 18-19, 2001. Besides the Vienna team including members from Regensburg(team 8) , 5 senior and 2 predocs members of the Amsterdam team (team 2) , and one member of the Tel-Aviv team (team 9) attended this workshop.

The second network workshop was held at Bordeaux on January 18-19, 2002 and all the postdocs appointed before that date were invited to give a one hour talk. This initiative, which will be discussed in more detail in the part of this report devoted to Training, was very popular among the postdocs appointed by the network and similar "postdoc workshops" will be organized during the third and fourth years of network activity. We summarize in the following table the information about these workshops.

Workshops organized by the network							
Location and organizing team	Country	Date	Title	Participating teams	Number of participants	Number of participants from network	Number of young participants from network
Vienna, team 8	Austria	January 17-18, 2001	Colloquium on Operator theory	2, 8 and 9	16	16	5
Bordeaux, team 1	France	January 18-19, 2002	Postdoc workshop	1, 2, 3, 4, 5, 6, 7, 8, and 9	20	19	12

The first annual network conference was held at Saint Petersburg(team 10) May 13 - 17 , 2001 , with 48 participants (43 participants from the network, and among them 31 from teams 1,2,3,4,5,6,7,8,9).All teams were represented , and 14 of the network participants were young mathematicians.Two series of morning lectures were devoted to Analytic Capacities and Harmonic Approximation (task 2 , with 3 lectures of 90 mn by A.O'Farrel

and 2 lectures of 90mn by S.Gardiner, both from team 4) and to Operator Theory (task 3, with 3 lectures of 90 mn by H.Langer,from team 8, and 2 lectures of 90 mn by S.Naboko,from team 10) and there were 24 afternoon talks of 30 mn.

The second annual network conference was organized by team 1 at the Casino Municipal of Biarritz from May 2 to May 7, 2002, with 68 participants. Only 2 of the 68 participants were exterior to the network. All teams were represented, and 32 of the network participants were young mathematicians. One sequence of morning lectures was devoted to Analytic Capacities (task 2, with 3 lectures of 90 mn by X.Tolsa, team 3, on his recent breakthroughs) and Interpolation (task 1, with 2 lectures of 90 mn by S.Kisliakov, team 10). Another sequence of morning lectures was devoted to Convex Geometry (task 6, with 2 lectures of 90mn by V.Milman, team 10) and to Operator Spaces (task 5, with 3 lectures of 90 mn by G.Pisier, team 6) and there were 30 afternoon talks of 20 mn. Informations about these Annual Conferences are summarized in the following table.

Annual Conferences of the Network								
Location and organizing team	Country	Date	Topic of main Lectures	Teams giving main Lectures	Participating teams	Number of participants	Number of participants from network	Number of young participants from network
Saint Petersburg, team 10	Russia	May 13-17, 2001	Capacities, Harmonic Approximation, Krein spaces, Operator Theory	4, 8 and 10	all	48	43	12
Biarritz, team 1	France	May 2-7, 2002	Interpolation, Capacities, Operator spaces, Convex Geometry,	3, 6, 9 and 10	all	68	66	32

B.6 Network organisation and management

There were 3 meetings of network coordinators between June 1, 2000, and September 30, 2002. The first one was held on June 12, 2000 at Bordeaux and the second one was held on May 12, 2001 at Saint Petersburg, the day before the beginning of the first annual conference of the network. The third meeting occurred on May 5, 2002 at Biarritz during the network annual conference. These meetings play a central role in network management : decisions are made about appointments of young mathematicians, the preparation of annual reports is discussed, and the location and the topics of the morning lectures of the next Annual Conference are chosen. Also these meetings are the place where decisions are made about the Conferences to which the network will officially participate and where the network workshops are planned.

All network participants are informed about network activities by messages from the principal coordinator (using the alias of all network participants). A global view of network activity is available on the network homepage, where the scientific part of the annual reports is available, as well as the schedule and abstracts of talks at the two network annual conferences at St Petersburg (May 2001) and Biarritz (May 2002) and at the postdoc workshop at Bordeaux(January 2002). The network strategy for dissemination of results and communication, besides visits, conferences and workshops also consists for each task to send by e-mail to all network participants a summary of results and a list of all relevant contributions , with addresses for all preprints available on line. This strategy was unfortunately not enforced so far, but we hope to be able to report some

progress in this direction at the Midterm Review Meeting. Also an exhaustive list of graduate students preparing a doctorate within the network is in preparation, with a summary of the subject thesis and a list of publications and preprints produced so far by these young mathematicians.

B.7 Cohesion with Less Favoured Regions and Associated States

The node of Dublin (team 4) and the subnode of La Laguna, Canarias (team 3) are located in less favoured regions of the community. There were no problems of integration, since the level of these groups of Mathematicians is very good. The same comment holds for the Norwegian node (team 7) and the Israeli node (team 9) which are located in Associated States.

B.8 Connections to Industry

R.Gay, who is an Emeritus Professor from team 1, pursued research with engineers from private companies in the Netherlands on concrete problems of signal theory arising from industrial problems, but the mainstream of network activity had no direct connexions with industry.

PART C-TRAINING

C.1 Appointment of Young Researchers

We summarise in the following table the number of young researchers (in person-months) whose appointment has been financed by the contract between June 1, 2000 and September 30, 2002, and we compare it to the overall deliverable specified in the contract.

Participant	Young researchers financed by contract until 30/09/2002 (person months)			Contract deliverable of young researchers to be financed by the contract (person months)		
	Pre-doc	Post-doc	Total	Pre-doc	Post-doc	Total
	(a)	(b)	(a+b)	(a)	(b)	(a+b)
1. UB1	1	12	13	12	24	36
2. VUA	0	18	18	0	24	24
3. UAB	0	18	18	12	24	36
4. UCD	0	12	12	0	24	24
5. ULeeds	0	13	13	0	24	24
6. UPMC	2	24	26	12	24	36
7. NTNU	0	13	13	0	24	24
8. TU Vienna	0	12	12	0	24	24
9. TAU	0	12	12	0	24	24
10. POMI	0	0	0	0	0	0
TOTAL	3	135	138	36	216	252

We see that the current situation of employment of young researchers is good. In fact **teams 2, 4, 5 and 7 have completed the hiring process**. In team 1 the hiring process is completed for the remaining 11 months of pre-docs: Michael Kopp, who is already at Bordeaux, will stay for eight more months and Helen Robinson, from York, will get a three months pre-doc position from May 1, 2003 to July 31, 2003. Sandra Pott, who is also from York, will get a three months post-doc position at Bordeaux from May 1, 2003 to July 31, 2003, and so **only 9 months of post-docs remain to be attributed in team 1**.

In team 3 a post-doc has been hired for a six-month position, and so **only 12 months of pre-docs remain to be attributed in team 3**.

Team 6 already had the 24 months of post-doc appointments expected in the contract and so **only 10 months of pre-docs remain to be attributed in team 6**. Also **6 months of post-docs remain to be attributed in team 8**.

The only serious appointment problem so far occurred in team 9: despite two calls for applications in the spring 2002, diffused to all network participants and to all recipients of the Banach algebra mailing list run by Marc Thomas in California, no applicants showed up for the two six months post-doc positions offered by Tel-Aviv. This problem is obviously related to the political situation in the Middle-East, and there is no easy solution to solve it. There will be a new call for applications in the Fall 2002.

Altogether the network still has to advertise 22 months of pre-docs and 27 months of post-docs, which gives 49 person-months of appointments of young researchers, which means that 80 % of the hiring for the overall deliverable specified in the contract has been completed on September 30, 2002, after only 28 months of network activity.

There might be a slight change in the distribution between pre- and post-docs from that foreseen in the contract: since Barcelona is now a Marie-Curie site in Analysis, there is an abundance of predoctoral positions available there and the possibility of having 8 months of post-docs instead of 12 months of pre-docs in team 3 will be discussed at a meeting of node coordinators during the afternoon of November 20, 2002.

Vacancies were advertised so far by messages sent to all network participants and all mathematicians registered in the Banach algebras mailing list, by calls for applications in the network homepage and through personal contacts. No major problems of integration were reported and all the young Mathematicians appointed by the network at the date of these events were given the opportunity to give a talk at the Saint Petersburg Conference (May 2001), at the Biarritz Conference (May 2002) and at the Bordeaux post-doc workshop (January 2002).

C.2 Training Programme

C.2.1 Networkwide training

The basis of the networkwide training program was the **two series of five 90mn morning lectures** delivered at the **first and second annual meetings of the network** (Saint Petersburg, May 2001, and Biarritz, May 2002). As indicated in the contract these lectures gave the state of the art on important directions of research directly related to the six main tasks of the network. At Saint Petersburg S.Gardiner and A.O'Farrel (team 4) gave a series of lectures devoted to Capacities and Harmonic approximation (task 2), and H.Langer (team 8) and S.Naboko (team 10) gave a series of lectures dedicated to Krein spaces and Operator Theory (task 3). At Biarritz S.Kisliakov (team 10) and X.Tolsa (team 3) gave a series of lectures devoted to Interpolation and Analytic Capacities (tasks 1 and 2) and G.Pisier (team 6) and V.Milman (team 9) gave a series of lectures devoted to Operator Spaces and Convex Bodies (tasks 5 and 6). These two annual meetings also gave the opportunity to many young mathematicians in the network to present their results and problems during afternoon talks and benefit from fruitful discussions with the various international experts attending these events.

This program was completed by the organization of a **2 days workshop at Bordeaux in January 2002** attended by all the young Mathematicians holding a postdoc in the network at that moment (Lehner, whose appointment at Paris had terminated on July 2001 also attended). These 12 young Mathematicians were all invited to give a 1 hour lecture and had the opportunity to have positive mathematical discussions together and with the Bordeaux team. This initiative, very popular among postdocs will be repeated during the third and fourth year of network activity. The second workshop of network postdocs (and predoc) will be organised at Paris on November 21-22, 2002, just after the midterm review meeting, with all the young mathematicians appointed by the network at that time and several former postdocs invited to this meeting. The presence of most network coordinators will help to organize when relevant visits of young mathematicians appointed by nodes of the network to some other nodes.

C.2.2 Local training

Local training of young mathematicians appointed by the network was organised through participations to seminars, individual collaborations with senior mathematicians from the appointing node and development of contacts with senior mathematicians from other nodes. For example G.Kozma, postdoc at Bordeaux, started an interdisciplinary collaboration with E. Strouse, where he uses his ability to deal with Fourier series to attack

problems about Hankel operators in several variables. N.Marco, postdoc at Barcelona, developed collaborations with mathematicians from Barcelona (and with a mathematician from team 1) and two joint papers arising from these contacts are submitted and the third one is in the final stage of preparation. J.Pau, postdoc at Dublin, and A.Hartmann, postdoc at Trondheim, wrote a joint paper with the local node coordinator during their appointment [11], [8]. F.Lehner, postdoc at Paris, developed contacts with mathematicians from Leeds through visits to Leeds and visits from Mathematicians from Leeds to Paris during his appointment, and A.Pelczar, postdoc at Paris, visited Bordeaux during her appointment (A.Pelczar and A.Buchholtz were also invited to give a talk at "Journées du GDR Analyse Fonctionnelle et Harmonique" at Lyon during their appointments at team 6). A.Baktai developed a collaboration with the local node coordinator and with mathematicians from team 1, see the joint paper [19]. In all cases the young mathematicians appointed by the network were enabled to enlarge the scope of their mathematical research, as shown for example by the talks at the Biarritz conference by A.Olofsson, postdoc at Amsterdam, on Operator theory, and by F.Oravec, postdoc at Tel-Aviv, on zeroes of trigonometric polynomials.

C.2.3 Possible improvements

All in all, the training of young Mathematicians appointed by the network went rather well, and other young mathematicians from the network nodes participated actively to the annual conferences (there were 32 network participants under the age of 35 at the Biarritz Conference). But the network should certainly play a more active role in the training of predoctoral student: only two visits of predoctoral students to other nodes were reported (M.Kopp, from Cambridge in team 5, to Bordeaux in team 1 in September 2001 and F.Bayart, from Lille in team 1 to Lund in team 7 in february 2002). The network management should to stimulate a more active program of visits of predoctoral students to other nodes in order to use more efficiently the complementarity between the teams in the predoctoral training program of the network.

C.3 Factual Information on the Young Researchers

The table below (followed by an index explaining the abbreviated names of the specialities of the young researchers) provides factual information about young researchers appointed so far by the network.

Name	Nationality	Age at App.	Start of App.	End of App.	Category	Speciality Work	Place of Work	Country of Network	Previous Exp. in Network (months)
Popovici	Romania	32	06/12/2001	05/05/2002	Postdoc	Op.Th.	Bordeaux	France	None
Kozma	Israel	30	08/01/2002	08/06/2002	Postdoc	Four. An.	Bordeaux	France	None
Kopp	Germany	27	01/09/2002	31/05/2003	Predoc	Ban. Alg. Series	Bordeaux	France	None
Olofsson	Sweden	29	15/01/2001	14/01/2002	Postdoc	Harm. An.	Amsterdam	Netherlands	None
Olofsson	Sweden	30	01/04/2002	30/03/2003	Postdoc	Harm. An.	Amsterdam	Netherlands	12 mths, Amst.
Marco	France	?	01/12/2000	31/05/2001	Postdoc	Func. Th.	Barcelona	Spain	None
Marco	France	?	01/09/2001	31/08/2002	Postdoc	Func. Th.	Barcelona	Spain	6 mths, Barc.
Pau	Spain	29	01/09/2001	31/08/2002	Postdoc	Func.Th.	Dublin	Ireland	None
Gustafsson	Sweden	30	01/10/2002	30/09/2003	Postdoc	Func.Th.	Dublin	Ireland	None
Michels	Germany	29	01/10/2001	30/09/2002	Postdoc	Ban. Sp.	Leeds	England	None
Jaeck	France	34	01/09/2002	31/08/2003	Postdoc	Inv. Spce.	Lancaster	England	None
Lehner	Austria	30	01/07/2000	30/06/2001	Postdoc	Noncom. Prob.	Paris	France	None
Pelczar	Poland	29	10/09/2001	09/03/2002	Postdoc	Ban. Sp.	Paris	France	None
Buchholz	Poland	33	01/09/2001	28/02/2002	Postdoc	Noncom. Prob.	Besancon	France	None
Artstein	Israel	23	06/04/2002	05/06/2002	Predoc	Conv.	Paris	France	None
Hartmann	Germany	34	01/09/2001	31/08/2002	Postdoc	Func. Th., Op. Th.	Trondheim	Norway	None
Harlouchet	France	31	01/09/2002	31/08/2003	Postdoc	Func.Th.	Trondheim	Norway	10 mths, Pisa
Batkai	Hungary	29	01/10/2001	30/09/2002	Postdoc	Op. Th.	Vienna	Austria	None
Lasarow	Germany	32	01/10/2002	31/03/2003	Postdoc	Comp. An., Op.Th.	Vienna	Austria	None
Oravec	Hungary	34	01/10/2001	30/09/2002	Postdoc	Op. Th.	Tel-Aviv	Israel	None

Speciality	Short name of Speciality
Op.Th.	Operator Theory
Four. An.	Fourier Analysis
Ban. Alg.	Banach and Fréchet algebras
Harm. An.	Harmonic Analysis
Func. Th.	Function Theory
Ban. Sp.	Geometry of Banach spaces
Inv. Spce.	Invariant subspaces and Dual Algebras
Noncom. Prob.	Noncommutative Probability
Conv.	Convex Geometry

Part D- SKETCHES OF THE YOUNG RESEARCHERS

D.1

Sketch of Anders Olofsson(postdoc, team 2):

I am originally from Stockholm, where I worked with professor Jan-Erik Björk at Stockholm University. I would classify my scientific background as harmonic analysis. Currently I work as network postdoc in Amsterdam under the supervision of professor M. A. Kaashoek. My experiences from the network are positive. The conferences and workshops organized by the network have to me been scientifically very interesting with a good mixture of topics.

Sketch of Frédéric Jaeck, (postdoc, team 5):

Ph. D. in Bordeaux (France) on Dual algebras and N-tuples of contractions Now at Lancaster university (UK)

My experience in this network has been really positive. The administrative part seems to be fast and efficient, the information is well spread through the different nodes of the network and this creates good work conditions. Here in Lancaster I was warmly welcomed and introduced to the department and local/semi local mathematical events; I have the opportunity to enlarge the field of my research and to develop new collaborative projects (S. Power and I are currently working on reflexivity and distance formula in operator algebras.)

Sketch of Iban Harlouchet(postdoc, team 7):

My working areas in mathematics are: Complex, functional and harmonic analysis. I defended my thesis in July 8, 1999, at Bordeaux. Its title is "Traces de Cauchy, fonctions asymptotiquement holomorphes et algebres de Beurling quasianalytiques", and I prepared it under the supervision of Professor Jean Esterle. Last year, I did a post doc within the network "Complex analysis and analytic geometry" (ANACOGA) of Professor Henri Skoda, at Scuola Normale Superiore di Pisa, with Professor Giuseppe Tomassini. So far, I have one published article: "Ideaux fermes de certaines algebres de Beurling quasianalytiques sur le cercle unite", at Journal de Mathematiques Pures et Appliquees (2000), and one submitted article: "Trace de Cauchy pour certaines fonctions localement integrables sur un ouvert borne de \mathbb{C} ". This year I'm working with Professor Kristian Seip in the Trondheim's node of the network "Analysis, Operator Theory, Geometry of Banach Spaces, Their Interplay and Their Applications". I intend to work mostly on Bergman spaces. I have no responsibilities in the Network. Until now my experiences are very positives. I attended a "Survival course in Norwegian" free of charge. The work environment is nice. I was helped to find an accomodation.

Sketch of Andreas Lasarow(postdoc, team 8)

1. Scientific background

From September 1990 till August 1996 I studied mathematics at University of Leipzig where I was dealing with nonnegative definite functions, measures, and their interplay for the first time. My education was focused on Stochastic, Harmonic analysis, and Prediction theory since 1994. Although learning was first priority during this time I decided to continue the researchs I began in my diploma thesis after finishing studies. During the following years I went on dealing with orthogonal rational matrix-valued functions on the unit circle. After having obtained a PhD in mathematics in May 2000 my research work was led primarily to refine this topic and furthermore on Schwarz-Pick-Potapov block matrices which play a key role by solving some interpolation problems. To briden my knowledge re Operator Theory, indefinite structures, and their applications I applied at the European Research Network "Analysis and Operators" for a postdoctoral position at the Institute for Analysis and Technical Mathematics of Vienna University of Technology starting in October 2002.

2. Responsibility in the Network

Being a part of the Network I have to study the connections between orthogonal rational functions and multiple Nevanlinna-Pick interpolation problems. In particular I have to investigate the Schur algorithm in definite or indefinite setting. In fulfilling these tasks I have to cooperate with the work group in Vienna. Discussions, lectures, and seminars give me the opportunity to learn new aspects and to get practical experience, too.

3. Experiences on the Network to date

Starting to prepare my stay in Vienna I get to know several people from my host institute, led by Prof. Langer, Mrs. Schuster and my predecessor Andras Batkaj. This way I want to express to these persons my gratitude for helping me to find my way through my adopted town for the next six months. After this warm welcome I ran into several obstacles re everyday life. This should be quite normal. In the first days I found it a bit confusing not to know where and how to start my research work. I felt that it was very strenuous to concentrate on my topic. The way communication and cooperation were performed on the host institute slightly differs from what I know. But I think that with a little bit of flexibility I should get over these teething troubles soon. And flexibility is one of the major issues why I wanted to join the Network...

Sketch of Michael Kopp(predoc, team 1)

Scientific background

*1995-1998 undergraduate degree (BA) at Trinity College Cambridge *1998-1999 Part III mathematics at Cambridge *1999-? Ph.D. in Cambridge under the supervision of Graham Allan.

Responsibilities in the network: -

Experiences (positive and negative) to date:

To date, I am in the fortunate position of not being able to report on any negative experiences. On the positive side, I was very warmly received and immediately integrated at Bordeaux university. Furthermore, I was given every assistance imaginable to helping me settle down.

Sketch of Jordi Pau(ex-postdoc, team 4)

My scientific background is essentially in one complex variable function theory, like the theory of Hardy spaces, interpolating sequences on some spaces of analytic functions and also for harmonic functions, and classical potential theory.

As a young researcher of the network in Dublin, I had great conditions for work, and the opportunity to meet a great number of mathematicians of my area. As a consequence of this, I had the opportunity to study some hard problems that I had no time before, and to learn new concepts of Potential Theory that I hope it would be useful in the next years. I also obtained some partial results in order to generalize Carleson interpolation theorem to general planar domains, and I wrote a paper with the local coordinator, Stephen J.Gardiner, that is now submitted for publication in a journal.

Finally I want to thank the local coordinator for his help in order to find accomodation in Dublin, and his helpful suggestions on the areas to find it.

Part E - NETWORK FINANCING

The following table gives an estimate of the amount spent by each team on Personnel Costs, Networking costs and Overheads, compared to the amounts foreseen in the contract. This estimate is based on the amounts declared to the Commission up to May 31, 2002 and to the estimated spending on personnel costs from June 1, 2002 to September 30, 2002, together with the related overheads.

Participant	Cost categories (spent until 30/09/2002)				Estimated Eligible Costs (foreseen in the contract)			
	Per- sonnel	net- working	Over- heads	Total	per- sonnel	Net- working	Over- heads	Total
1. UB1	44590	22834	13303	80727	117000	52000	33800	202800
2. VUA	64751	3294	10810	78855	97140	32000	25828	154968
3. UAB	49800	14484	13036	77320	111180	36000	29346	176616
4. UCD	37946	9975	9854	57775	86100	32000	23620	141720
5. ULeeds	44265	18053	12463	74781	86100	38000	24820	148920
6. UPMC	89410	13631	20608	123649	117900	56000	34780	208680
7. NTNU	62062	11732	11570	86364	97740	32000	25948	155688
8. TU Vienna	47438	13330	9346	70114	98040	32000	26008	156048
9. TAU	35862	12854	9743	58459	91800	32000	24760	148560
10. POMI	0	0	0	0	0	0	0	0

The following table gives an estimate of the overall spending of the network from June 1, 2000, to September 30, 2002, compared to the amounts foreseen in the contract.

Cost categories (spent until 30/09/2002)				Estimated Eligible Costs (foreseen in the contract)			
Per- sonnel	net- working	Over- heads	Total	per- sonnel	Net- working	Over- heads	Total
476124	120187	110733	707044	903000	342000	249000	1494000

These estimates show that so far Personnel Costs represented 67,34% of the total costs, Networking Costs 17% and overheads 15,66% . The corresponding proportions foreseen in the contract were 60,44% , 22,90% and 16,66% , but the difference seems rather technical, and the rules of the Commission (minimum of 60% for personnel costs, maximum of 16,66% for overheads) are respected. The estimates also indicate that 47,33 % of the total budget has been spent after 28 months of network activity, which seems to be a reasonable amount.

Part F- PROPOSED REVISION OF THE CONTRACT

There is no need to modify the research objectives, the management methods and the training program described in the contract. The only slight modification necessary now would consist in adding the University of Leiden to team 2, the Royal Institute of Technology of Stockholm to team 7 and the University of Bremen to team 8. These additions are due to the fact that Mathematicians already very active in the research and training activities of the network did move to these places. The Project Officer has been already informed about this problem.

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