NORTH BRITISH FUNCTIONAL ANALYSIS SEMINAR

A meeting of the North British Functional Analysis Seminar will be held at the Department of Mathematics at Queen’s University Belfast from 11.30am to 3.30pm, on Monday, 5 April 2004.

Prof. Erling Størmer
Oslo University, Norway

Positive Linear Maps of Operator Algebras

and

A Survey of Noncommutative Entropy

11.30am and 2.00pm on Monday, 5th April 2004

All interested are welcome to attend.

The talks precede the opening of the BMC in Belfast, which will also feature many talks of interest to the North British Functional Analysis seminarian.

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Prof. Erling Størmer

**Positive Linear Maps of Operator Algebras**

**Abstract:** In this lecture we shall consider positive linear maps of a von Neumann algebra $M$ into itself. In analogy with the representation of Hilbert-Schmidt operators on a Hilbert space $\mathcal{H}$ as the tensor product of $\mathcal{H}$ with itself, we represent finite rank maps $\phi : \mathcal{H} \to \mathcal{H}$ as operators in $M \otimes M$, and more general maps to be in a closure of the tensor product. If $\phi$ is positive unital of finite rank and invariant with respect to a faithful normal state, let $N$ denote the linear span of the eigenoperators of $\phi$ corresponding to eigenvalues of absolute value 1. Then $N$ is a Jordan algebra, and we get a version of a recent theorem of Arveson that $\phi = aP + b$, where $P$ is a positive projection on $N$, $a$ is a Jordan automorphism of $N$, and $b$ is a nilpotent map. This result is applied to the concept of capacity $C(\phi)$ from quantum information theory when $M$ is finite dimensional and $\phi$ is invariant with respect to a trace. Then we get

$$C(\phi) \geq \lim_{n} C(\phi^n) = C(P) = \log \operatorname{rank} N.$$ 

**A Survey of Noncommutative Entropy**

**Abstract:** This lecture will be a survey of the theory of noncommutative dynamical entropy, especially that developed by Connes, Strmer; Connes, Narnhofer, Thirring; and Voiulescu, in that order. I’ll describe some of the main results of the theory and emphasize how some abelianness of the $C^*$-dynamical system, like asymptotic abelianness, is necessary to get nontrivial results. As special examples I’ll consider the variational principle and examples from subfactor theory which relate entropy to index of subfactors.