1. The dynamical systems:

   (a) $\dot{x} = xy, \quad \dot{y} = -y - x^2,$

   (b) $\dot{x} = x^2y - x^5, \quad \dot{y} = -y + x^2$

   each have a non-hyperbolic fixed point at the origin. Calculate the first two nontrivial terms in the local expansion for the centre manifold $y = h(x)$ at the origin for each of these systems and use these to determine the stability of these fixed points.

2. (a) Sketch the curves in (x, μ) space of the equilibria of the following dynamical system:

   $$\dot{x} = (x^2 + \mu^2 - 1)(x - \mu),$$

   indicating the stability of each of the branches in the diagram. Identify the type of bifurcation at each of the bifurcation points.

   (b) Repeat this investigation for the dynamical system:

   $$\dot{x} = x(\mu - x - x^2)$$

3. A system is described by the pair of equations

   $$\frac{dx}{dt} = \alpha - (\beta + 1)x + x^2y\,, \quad \frac{dy}{dt} = \beta x - x^2y.$$

   Show that a Hopf Bifurcation occurs at the equilibrium $\left(\alpha, \frac{\beta}{\alpha}\right)$ at the critical value $\beta = 1 + \alpha^2$.

4. A system is described by the equations

   $$\frac{dx}{dt} = \alpha x + y - x^3\,, \quad \frac{dy}{dt} = -x + \alpha y + 2y^3.$$

   Show that a Hopf Bifurcation occurs at $(0, 0)$ at the value $\alpha = 0$ and determine whether or not the limit cycle which is created is attracting. Is the cycle present when $\alpha > 0$ or when $\alpha < 0$? For this example, change to complex co-ordinates and calculate the coefficient of $|z|^2z$.