1. Find the matrix e^{tA} for each of the following

$$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}, \quad \begin{pmatrix} 3 & 0 \\ 2 & 3 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}, \quad \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}.$$

2. A is an $n \times n$ matrix. Show that if A has eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$, then

$$\sum_{i=1}^{n} \lambda_i = \text{trace}\, A, \qquad \sum_{i=1}^{n} \lambda_i^m = \text{trace} A^m.$$

3. For the following 3×3 matrices, find the eigenvalues, the formula for $c_i(t)$ and hence e^{tA}:

   (a)
$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

   (b)
$$A = \begin{pmatrix} 3 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

4. Write the following linear differential equation as a first order system and find the characteristic equation of the corresponding matrix:

$$\frac{d^4x}{dt^4} + 5\frac{d^3x}{dt^3} + 13\frac{d^2x}{dt^2} + 19\frac{dx}{dt} + 10x = 0.$$

   Show that this has roots $\lambda_1 = -2$ and $\lambda_2 = -1$ and deduce the other two eigenvalues. Determine whether the origin is stable or unstable.

5. Consider the Lorentz system: $\dot{x} = \sigma(y - x)$, $\quad \dot{y} = rx - y - xz$, $\quad \dot{z} = xy - bz$, with $\sigma = 10$, $r = 2$, $b = 4$. Find the 3 fixed points, determine their stability and describe their stable and unstable manifolds.

   Use LOCBIF to plot the projections of trajectories in the $x - y$ and $x - z$ planes. Choose ranges for the phase variables so that all 3 fixed points are seen.